PHYSICAL REVIEW C VOLUME 29, NUMBER 3 MARCH 1984

Effective range parameters in nucleon-nucleon scattering

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(Received 19 October 1983)

We have calculated effective range parameters for the S and P states for proton-proton and neutron-proton scattering and compared them with results from other authors. The calculations were done in the framework of a phase shift analysis. Different effective range expansions (with and without inclusion of the one-pion-exchange cut) were investigated. Taking into account features of the deuteron and of one pion exchange we give a low energy parametrization for the mixing parameter ϵ_1.

I. INTRODUCTION

The results of nucleon-nucleon phase shift analyses from various groups[1–8] show significant agreement for most energies up to ~ 350 MeV laboratory energy.[9–11] Nevertheless there exist some contradictory results between various analyses in the low energy region. In the proton-proton $^1S_0$ partial wave the values of the effective range parameters from an analysis of Arndt *et al.*[2] do not agree with values given by earlier and later authors.[12–14] The low energy behavior of the triplet P waves given by Sher, Signell, and Heller (in the following referred to as SSH)[12] differs from those extracted from earlier and later analyses of the Livermore group.[1,3] Also, different analyses have yielded different values (and even different structures) for the mixing parameter ϵ_1 at energies up to ~ 100 MeV.[2,7] Finally, since the low energy region in nucleon-nucleon scattering is influenced by one-pion exchange (OPE), several authors included features of OPE in the extraction of effective range parameters[15] and of properties of the two-nucleon bound system, the deuteron.[16]

This paper tries to resolve the problems stated above within the framework of a phase shift analysis and with respect to the inclusion of the OPE. By this we mean that our extracted low energy parameters are adjusted to represent a fit to an extensive multienergy data set. It also means that our parameters depend on the form of the underlying phenomenological representation for the phases. Therefore, our (model dependent) parameters might differ from the "true" values, but the differences should be small compared to the above-mentioned discrepancies. The error bars we show for our effective range parameters are (i) taken from the error matrix[17] directly for the S waves, or (ii) calculated from the error values (not matrix) of the coefficients of the energy dependent phase parameter basis functions[17] otherwise.

II. EFFECTIVE RANGE EXPANSIONS AND PARAMETERS

A. $^1S_0$ proton-proton

In analyzing the proton-proton data for phase shifts, Arndt *et al.*[2] searched also for the scattering length a and the effective range r of the $^1S_0$ partial wave, and they obtained the results

$$a = -7.76 \pm 0.0098 \text{ fm} ,$$
$$r = 2.687 \pm 0.0146 \text{ fm} .$$

In their program they used the "shape-independent" effective range expansion

$$X(k^2) = -1/a + \tfrac{1}{2} r k^2 , \tag{1}$$

where k is the momentum of either nucleon in the center of mass system and the function $X(k^2)$ is given by

$$\frac{1}{k} X(k^2) = \frac{1}{1+\tan^2\tau_0} C_0^2 (1+\chi_0)[(1+\chi_0)\cot\delta_0 - \tan\tau_0] + 2\eta h(\eta) + 2\eta l_0 , \tag{2}$$

with

$$C_0^2 = \frac{2\pi\eta}{e^{2\pi\eta}-1} , \quad \eta = \frac{me^2}{2\hbar^2 k}$$

(where m is the mass of the proton), and

$$h(\eta) = \eta^2 \sum_{l=1}^{\infty} \frac{1}{l(l^2+\eta^2)} - 0.57721 - \ln\eta .$$

The energy dependent functions τ_0, χ_0, and l_0 are defined and given by Heller,[18] and the phase δ_0 is of the electric type according to SSH.[12] In contradiction to the parameters given by Arndt[2] are results from several authors. In an earlier analysis SSH (Ref. 12) used a "shape-dependent" form for the expansion of the function $X(k)^2$,

$$X(k)^2 = -1/a + \tfrac{1}{2} r k^2 - P r^3 k^4 + Q r^5 k^6 , \tag{3}$$

and found

$$a = -7.821 \pm 0.004 \text{ fm} ,$$
$$r = 2.830 \pm 0.017 \text{ fm} ,$$
$$P = 0.051 + 0.014 \text{ fm} ,$$
$$Q = 0.028 \pm 0.013 \text{ fm} .$$

From Table II of Heller[19] one can see how the neglect of the shape parameters, P and Q, changes the values of a and r, respectively. It would be incorrect therefore to compare the values from Arndt *et al.*[2] to those calculated directly from models or wide energy phase parametrization, or to those determined by fitting with a shape dependent form.

A different expansion of (2) has been proposed since the OPE branch cut causes the series expansion of Eq. (2) [see Eq. (3)] to lack nice convergence properties above 9.71 MeV.[15] Explicitly taking into account the OPE cut in the expansion would extend the range of convergence to ~ 38 MeV. This inclusion of the OPE cut can be done exactly, but Noyes and Wong have shown that the simpler method suggested by Cini *et al.*[20] (replacing the S wave OPE amplitudes with the full OPE amplitude at 90°) is a very good approximation.[15] The result of this method is a modified effective range expansion,

$$X(k^2) = -1/a + \tfrac{1}{2} r k^2 - \frac{pk^4}{1+qk^2} , \tag{4}$$

where p and q are given by parameters of the OPE (the pion mass and pion-nucleon coupling constant). For the $^1S_0$ proton-proton case, Noyes and Lipinski[21] included also the Coulomb interaction and got for the OPE parameters

$$p = 0.64788 \text{ fm}^3 ,$$

$$q = 3.41611 \text{ fm}^2 ,$$

and for the scattering length and effective range

$$a = -7.826 \pm 0.010 \text{ fm} ,$$

$$r = 2.803 \pm 0.015 \text{ fm} .$$

In 1977 Naisse[14] repeated the calculation following Noyes and SSH but eliminated some minor errors and included new data, and he got for the OPE parameters

$$p = 0.7621 \text{ fm}^3 ,$$

$$q = 3.3149 \text{ fm}^2 ,$$

and for the effective range parameters

$$a = -7.828 \pm 0.008 \text{ fm} ,$$

$$r = 2.80 \pm 0.02 \text{ fm} .$$

Naisse claimed[14] that the discrepancy between his effective range parameters and those of Arndt[2] is due to the use of different expansion functions [Eq. (1) vs Eq. (4)]. To check this we have plotted in Fig. 1 the $^1S_0$ phase shift calculated with the parameter sets of Arndt and Naisse inserted in both expansions [Eqs. (1) and (4)]. Since the phase shift curves that each author was fitting (the two middle curves) are quite close, Naisse's claim would seem to be correct. That is, the data fairly well fix the phase shift, but the different expansion functions [Eqs. (1) and (4)] provide fits to this phase with different parameters.[22]

The crucial point, however, is that this statement is true just for the energy range from 1 to 10 MeV. If we add the experimental data of Brolley *et al.*[23] near the Coulomb interference minimum at 0.3 to 0.4 MeV (these data were not included in the Arndt analysis[24]), the fit to the experimental data up to 25 MeV with the effective range expansion (1) gives

$$a = -7.822 \pm 0.003 ,$$

$$r = 2.775 \pm 0.006 .$$

This analysis was done up to 25 MeV since we wanted the maximum energy in the analysis to be well above the region of interest, which was 0 to 10 MeV. We also varied the maximum energy in the analysis from 15 to 25 MeV and found the $^1S_0$ a and r to be essentially independent of this choice. We searched the parameters a and r for the $^1S_0$ phase with a particular effective range expansion as well as two expansion parameters for each of the ^3P_J phases. The ^3P_J results will be discussed later. The higher partial waves were fixed at the SM81 solution of Arndt, which is essentially OPE at these energies except for the $^1D_2$ phase, which rises to a value of 0.70° at 25 MeV. The uncertainties of the $^1S_0$ a and r come directly from the error matrix of this fit. These new parameters lead to a curve similar to the dashed one in Fig. 1. This new phase deviates from the "data produced" one (solid or dashed-dotted curve) and resulted in the decrease of the average χ^2 per datum from 1.08 for expansion (1) to 0.63 for expansion (4). The resulting parameters for the fit with Eq. (4) are

$$a = -7.844 \pm 0.003 \text{ fm} ,$$

$$r = 2.859 \pm 0.006 \text{ fm} ,$$

with Naisse's results used for p and q. In Table I the effective range parameters for the $^1S_0$ proton-proton case are summarized.

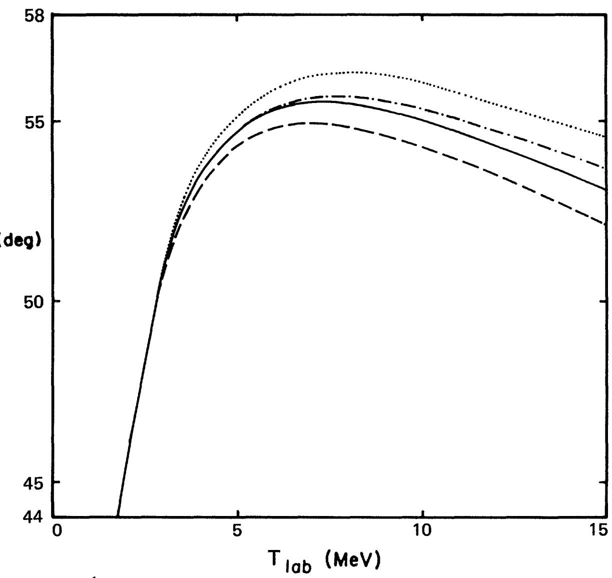

FIG. 1. $^1S_0$ (pp) phase calculated via the effective range expansions with the parameters sets given by Naisse (Ref. 14) [— — — with Eq. (1), —·—·—· with Eq. (4)] and by Arndt (Ref. 2) [—— with Eq. (1), ··· with Eq. (4)].

TABLE I. Effective range parameters for the proton-proton $^1S_0$ phase.

	Equation for expansion	a	r
Arndt *et al.* (Ref. 2)	(1)	-7.76 ± 0.0098	2.687 ± 0.0146
SSH (Ref. 12)	(3)	-7.821 ± 0.004	2.830 ± 0.017
Noyes (Ref. 21)	(4)	-7.826 ± 0.01	2.803 ± 0.015
Naisse (Ref. 14)	(4)	-7.828 ± 0.008	2.80 ± 0.02
Our results	(1)	-7.822 ± 0.003	2.775 ± 0.006
	(4)	-7.844 ± 0.003	2.859 ± 0.006

In conclusion, one can say that the difference between Arndt's effective range parameters and those of prior[12] and more recent investigations[21,14] is probably due to Arndt's omission of the Brolley and Seagrave data. (Arndt did not have vacuum polarization corrections available for those data.) One could also say that, for this augmented data set, Eqs. (1), (3), and (4) still give answers that differ from each other by up to 14 standard deviations (see the last two lines of Table I). In addition, Eq. (4) gives a better fit than Eq. (1). We did not investigate the quality of fit of Eq. (3).

B. $^1S_0$ neutron-proton

For the $^1S_0$ neutron-proton partial wave, the function $X(k^2)$ is given by

$$k\cot\delta = X(k^2)\ . \qquad (5)$$

Table II shows that there are no great differences in the effective range parameters given by several authors[13,25,26] (perhaps with the exception of the effective range of Naisse[25]). To study this channel we fit the combined np and pp data up to 25 MeV (this energy being chosen for the same reasons as mentioned for the pp data) by varying only a and r for the n-p $^1S_0$ using the effective range expansion of interest and leaving all other phases fixed at the SM81 solution. Again the uncertainties were taken directly from the error matrix of the analysis. In our fit to the experimental data we get values of

$$a=-23.721\pm0.020 \text{ fm}\ ,$$

$$r=2.658\pm0.062 \text{ fm}\ ,$$

using the effective range formula Eq. (1). If we include the OPE [Eq. (4)] we get

$$a=-23.720\pm0.017 \text{ fm}\ ,$$

$$r=2.665\pm0.070 \text{ fm}\ ,$$

(with a slight increase of χ^2 per datum). The diminished importance of the OPE part for the effective range expansion is a result of the parameters $p=0.554$ and $q=3.055$; this decreased value of p is caused by the larger absolute value of the scattering length and by the larger effective mass of the exchanged pion.[27]

C. P waves

In discussing the low energy behavior of the P waves, let us start with P waves relevant to the proton-proton case, the $^3P_0$, $^3P_1$, and $^3P_2$ partial waves. The shape independent effective range expansion [Eq. (1)] is applied to the function $X(k^2)$,

$$k^3(1+\eta^2)[C_0^2\cot\delta_{1j}+2h(\eta)]=X(k^2)\ , \qquad (6)$$

for $J=0$ and 1, respectively. The functions C_0^2, η, and $h(\eta)$ are the same as in Eq. (2). In the $^3P_2$ case the threshold behavior is k^5 instead of k^3, and therefore [following SSH (Ref. 12)] we insert the one-pion-exchange-substracted phase, $\tilde{\delta}_{12}$, into (6):

$$\tilde{\delta}_{12}=\delta_{12}-C_0^2(1+\eta^2)\delta_{12}^{\text{OPE}}\ . \qquad (7)$$

In the past there were disparate solutions for the low energy behavior of the triplet P waves given earlier by McGregor *et al.*[1] and later by Arndt *et al.*[2] compared to the results of SSH.[12] This discrepancy can be seen more clearly in the spin and space combinations of the phases called central, tensor, and spin orbit:[12]

$$\begin{aligned} \Delta_C &= \tfrac{1}{9}(\delta_{10}+3\delta_{11}+5\delta_{12})\ ,\\ \Delta_t &= \tfrac{5}{72}(-2\delta_{10}+3\delta_{11}-\delta_{12})\ ,\\ \Delta_{LS} &= \tfrac{1}{12}(-2\delta_{10}-3\delta_{11}+5\delta_{12})\ . \end{aligned} \qquad (8)$$

Holdeman *et al.*[28] claimed that the disagreement is a consequence of parameter boundedness in the analysis of McGregor, whereas Arndt[2] states that the discrepancy is a

TABLE II. Effective range parameters for the neutron-proton $^1S_0$ phase.

	Equation for expansion	a	r
Noyes (Ref. 13)	(4)	-23.715 ± 0.015	2.73 ± 0.03
Dilg (Ref. 26)	(1)	-23.749 ± 0.009	2.77 ± 0.05
	(4)	-23.749 ± 0.009	2.78 ± 0.05
Naisse (Ref. 25)	(1)	-23.717 ± 0.011	2.58 ± 0.10
Our results	(1)	-23.721 ± 0.020	2.658 ± 0.062
	(4)	-23.720 ± 0.017	2.665 ± 0.070

TABLE III. Effective range parameters for the P waves.

		SSH (Ref. 12)	Naisse (Ref. 14)	Barker *et al.* (Ref. 29)	Our result	Nagels *et al.* (Ref. 31)
$^1P_1$	a				2.4 ± 1.3	3.023
	r				-12.6 ± 2.2	-6.895
$^3P_0$	a	-2.6 ± 2.0	-3.12 ± 0.01	-4.82 ± 1.11	-2.84 ± 0.02	-2.841
	r	4.3 ± 2.0	3.93 ± 0.02	7.84 ± 0.93	4.45 ± 0.05	2.459
$^3P_1$	a	2.8 ± 1.3	1.99 ± 0.005	1.78 ± 0.1	1.90 ± 0.01	1.99
	r	-9.0 ± 1.0	-7.64 ± 0.001	-7.85 ± 0.52	-7.56 ± 0.05	-7.563
$^3P_2$	a	-0.45 ± 0.28	-0.282 ± 0.002	-0.317 ± 0.023	-0.31 ± 0.01	-0.294
	r	$15\pm10.$	a	7.5 ± 2.9	7.59 ± 0.28	4.402

[a]Not given by the author. In calculating Δ_C, Δ_{LS}, Δ_T we assumed a value of 5, which is an average of some solutions given by Naisse (Ref. 14).

reflection of the choice and handling of the data.

Very recently Barker *et al.*[29] measured the analyzing power at energies of 5.05 and 9.85 MeV very precisely. In analyzing these data Barker *et al.*[29] gave phase shift predictions at these energies and effective range parameters for the triplet P waves. We included these new data (and also the other analyzing power data by Bittner and Kretschmer[30]) into the data base of Arndt and searched for the S and P wave phase shifts using data from 0—25 MeV as discussed previously. The P wave effective range parameters were extracted from these fits by expanding the basis functions of the fits in powers of momentum and then relating the expansion coefficients to a and r. Similarly, the uncertainties of the expansion coefficients were combined in quadrature to obtain estimates for the uncertainties of a and r. In Table III we give the effective range parameters from our analysis compared to the results given by Naisse,[14] SSH,[12] and Barker *et al.*[29] For comparison we also show values obtained with a one boson exchange model by Nagels *et al.*[31] In Figs. 2, 3, and 4 we give the corresponding results in terms of Δ_C, Δ_{LS}, and Δ_T, respectively. From both the table and figures one can see that the new results (Naisse, Barker, and our solution) differ from those given by SSH and Nagels *et al.*

Our effective range parameters are especially in good agreement with that given by Barker *et al.*,[29] with the exception of the $^3P_0$ case. The central part of the P waves (see Fig. 2) shows higher values than the solution of SSH; this is caused mainly by the inclusion of new cross-section data[32,33] which are in contradiction to the older data by the Berkeley group[34] (Bugg[10] had already proposed discarding the Berkeley data; this was done by Barker *et al.* and also in our analysis). The result of Barker *et al.* for Δ_C at 5.05 MeV is relatively high, but the absolute value of Δ_C is so small that a small variation of one of the ^3P_J phases can change Δ_C from positive to negative values. The tensor part (Fig. 3) shows consistent results from the various analyses with our lower values favored by the polarization measurements.[29,30,35] Figure 4 (Δ_{LS}) shows more disparity, but again relative agreement of our solution with those from Refs. 29, 30, and 35.

In Table III we give effective range parameters also for the $^1P_1$ case calculated with expansion (1), where the function $X(k^2)$ is given by

$$k^3 \cot\delta_1 = X(k^2) \ . \qquad (9)$$

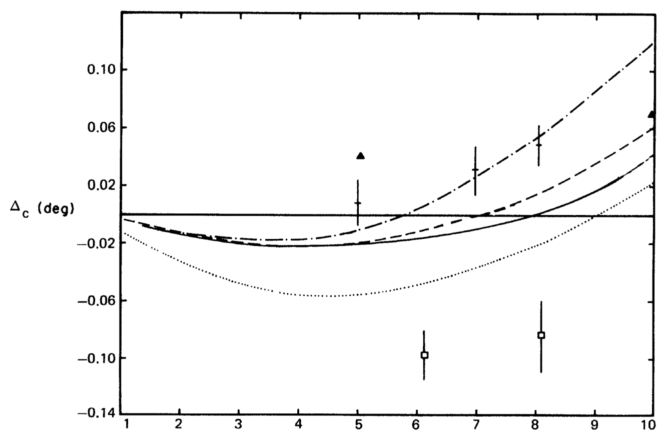

FIG. 2. Central part of the P waves Δ_C. The solid line gives our result, the dotted curve shows the calculation of SSH (Ref. 12), the dashed curve that of Naisse (Ref. 14), and the dashed-dotted curve that of Nagels *et al.* (Ref. 31). The triangles are the results of Barker *et al.* (Ref. 29), the squares are those given by the Berkeley group (Ref. 34), and the bars correspond to Imai *et al.* (Ref. 32).

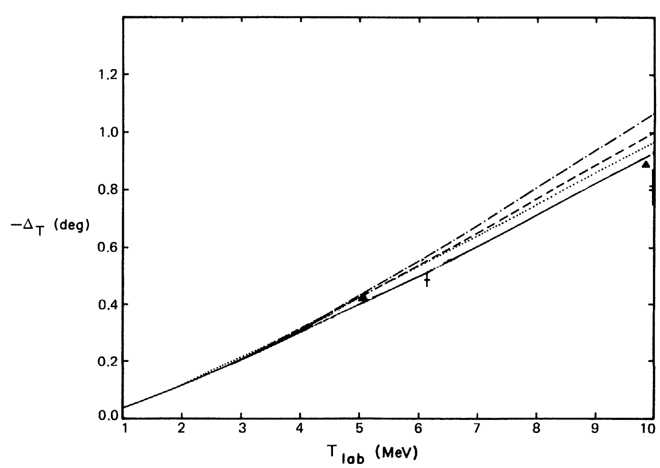

FIG. 3. The negative tensor part ($-\Delta_T$) of the P waves. The curves are the same as in Fig. 2. The (+) values are given by Bittner and Kretschmer (Ref. 30) at 6.14 MeV and by Hutton *et al.* at 5 and 10 MeV.

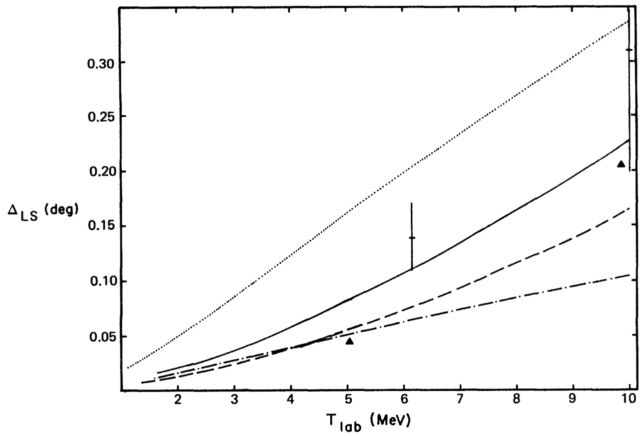

FIG. 4. The spin orbit part (Δ_T) of the P waves. The curves and data are the same as Fig. 3.

The parameters were obtained by a fit to the $^1P_1$ phase given by Arndt and VerWest[8] in the energy range from 1 to 10 MeV. The errors for the $^1P_1$ phase were estimated from the variation in $X(k^2)$ computed from the results of the single energy analyses at 10, 25, and 50 MeV, and are only approximate. For comparison we also show values given by Nagels.[31] Since the $^1P_1$ phase is connected to the $^3S_1$-$^3D_1$ channel, the discrepancy shown in Table III is possibly correlated to disparate results for the mixing parameter ϵ_1 which we will discuss next.

D. The $^3S_1$-$^3D_1$ channel

A low energy parametrization in the coupled $^3S_1$-$^3D_1$ channel is influenced strongly by the existence of the deuteron bound state therein (the bound state pole is located at a laboratory energy of ~ -4.4 MeV). The importance of the bound state for an effective range expansion shows up in the existence of three different effective range parameters depending on at which energy point the slope of the function $k\cot\delta_0$ is taken:

$$\tfrac{1}{2}\rho(0,0)=\frac{d}{dk^2}(k\cot\delta)\,|_{k^2=0}\,,$$

$$\tfrac{1}{2}\rho(-\epsilon,-\epsilon)=\frac{d}{dk^2}(k\cot\delta_0)\,|_{k^2=-\alpha^2}\,, \tag{10}$$

where δ_0 is the $^3S_1$ phase shift (we are working with the eigenphases by Blatt and Biedenharn[36] throughout this section) and α gives the location of the pole. The "mixed" effective range $\rho(0,-\epsilon)$ is defined by the slope of the line going through the points of $k\cot\delta_0$ at $k^2=0$ and $k^2=-\alpha^2$. By this definition the mixed effective range is given by

$$\tfrac{1}{2}\rho(0,-\epsilon)=\frac{1}{\alpha}-\frac{1}{\alpha^2 a}\,. \tag{11}$$

It is clear that for the shape independent form, Eq. (1),

$$\rho(0,0)=\rho(0,-\epsilon)=\rho(-\epsilon,-\epsilon)=r_t\,.$$

$\rho(-\epsilon,-\epsilon)$ can also be expressed in terms of the deuteron wave functions and is related to the deuteron S-wave normalization constant A_S by[37]

$$A_S^2(1+\eta^2)=\frac{2\alpha}{1-\alpha\rho(-\epsilon,-\epsilon)}\,, \tag{12}$$

where $\eta=A_D/A_S$ is the asymptotic D/S ratio of the wave functions.

Including the OPE into the effective range expansion and taking into account the bound state, Noyes[38] defined a modified effective range expansion for the $^3S_1$ case,

$$X(k^2)=-1/a+\tfrac{1}{2}rk^2-\frac{pk^2(k^2+\alpha^2)}{1+qk^2}\,. \tag{13}$$

The parameters p and q, obtained by Noyes,[38] are

$$p=0.1147\ \text{fm}^3\,,$$

$$q=3.861\ \text{fm}^2\,,$$

and show that the effect of the inclusion of the OPE on the effective range expansion is less than in the other channels. We fit the combined np and pp data up to 25 MeV, varying the a and r parameters for the $^3S_1$ phase while keeping all other parameters fixed (like what was done in the $^1S_0$-np case) and used Eqs. (1) and (13). In these fits the effective range parameters changed just slightly from

$$a=5.425\pm0.018\ \text{fm}\,,$$

$$r=1.711\pm0.053\ \text{fm}\,,$$

for Eq. (1) to

$$a=5.434\pm0.012\ \text{fm}\,,$$

$$r=1.694\pm0.023\ \text{fm}\,,$$

for Eq. (13). Table IV shows a compilation of the effective range parameters for the $^3S_1$ phase given by several authors.

One of the most uncertain quantities in low energy nucleon-nucleon scattering is the mixing parameter ϵ_1 (connected to the strength of the tensor part of the force between the two nucleons). An early analysis[2] gave negative values for ϵ_1 for $E_{\text{lab}}\sim 20$ to 90 MeV which were

TABLE IV. Effective range parameters for the $^3S_1$ phase.

	Equation for expansion	a	r
Noyes (Ref. 13)	(13)	5.423±0.005	1.748±0.006
Dilg (Ref. 26)	(1)	5.423±0.004	1.760±0.005
Naisse (Ref. 25)	(1)	5.411±0.004	1.718±0.045
Our results	(1)	5.425±0.018	1.711±0.053
	(13)	5.434±0.012	1.694±0.023

caused by an erroneous constraint in the analysis.[39] More recent phase shift analyses by several groups have all yielded positive values for ϵ_1. These results are $\epsilon_1 \sim 2.8°$ (3.5°) at $T_{\rm lab}=25$ (50 MeV) by Bystricky *et al.*[7], $\epsilon_1=0.64°$ (0.90°) from the single energy analyses and $\epsilon_1=0.94°$ (0.71°) from the energy dependent analyses of Arndt and VerWest,[8] and $\epsilon_1=1.03°$ at 25 MeV by Bohannon *et al.*[40] (these values are given in the Stapp parametrization[41]).

Like Eq. (12) there exists also a relation between the mixing parameter ϵ_1 and the deuteron pole asymptotic D/S ratio

$$\tan\epsilon_1 \,|_{k^2=-\alpha^2} = -\eta \ . \tag{14}$$

Attempts were also made to relate the low energy part of ϵ_1 with properties of the deuteron: The first one relates the slope of ϵ_1 with the quadrupole moment (Q) of the deuteron. Blatt and Weisskopf[42] gave the approximation

$$\epsilon_1 = \sqrt{2} Q k^2 \ , \tag{15}$$

and Blatt and Biedenharn[36] derived

$$\epsilon_1 = \sqrt{2}(1-\alpha r_t)^2 Q k^2 \ . \tag{16}$$

Signell[43] has shown that Eq. (15) is in better agreement with some existing potential models than is Eq. (16). But Kermode *et al.*[44] have proven with the aid of unitary transformation that the connection of ϵ_1 with the quadrupole moment is very loose [one can see this also in the derivations of Eqs. (15) and (16)].

Wong[16] set up a dispersion relation for the mixing amplitude α^1 of the coupled channel taking into account the deuteron pole, the one pion exchange cut, and the low energy S-wave phase shift

$$(\alpha^1/ik) = \left[-\frac{1}{a}+\frac{r_t}{2}k^2+ik\right]\left[-\frac{2A_S^2\eta}{\left[\frac{1}{a}+\frac{r_t}{2}\alpha^2+\alpha\right](k^2+\alpha^2)} + \frac{f^2 m}{\sqrt{2}}\int_{-\infty}^{-1/4} \frac{\frac{1}{k'^2}+\frac{3}{4k'^4}}{(k'^2-k^2)\left[-\frac{1}{a}+\frac{r_t}{2}k'^2+ik'\right]}\, d(k'^2)\right] . \tag{17}$$

With $a=5.457$ fm, $r_t=1.714$ fm, the pion nucleon coupling constant $f^2=0.08$, and the requirement that (α^1/ik) be zero at zero momentum, Wong[16] was able to calculate the D/S ratio η to be 0.029.[45]

In recent years a lot of work has been done[46–50] to determine η from various experiments. Most of the predictions give a value of η between $\sim$0.026 and 0.027, whereas derivations from p-d scattering including Coulomb corrections yield slightly larger values ($\eta=0.0272$) (Ref. 49), and an analysis from photodisintegration data shows $\eta=0.023$.[48] One can see that Wong's prediction deviates less than 20% from these results. Since this is far less than the uncertainty in the mixing parameter ϵ_1, we will extract a low energy behavior for ϵ_1 from Eq. (17). [Comparisons of values of ϵ_1 calculated with Eq. (17) with values from nucleon-nucleon models and phase shift analyses have already been done by Noyes[51] and by Signell.[43]]

Contrary to Wong we fix η, and to obtain a zero value for ϵ_1 at zero energy we vary the pion-nucleon coupling constant, which has some uncertainty (see, for instance, the compilation in Ref. 52). With the most recent value from p-d experiments ($\eta=0.0272$) (Ref. 49) and with $a=5.424$ fm, $r=1.761$ fm, and $A_S=0.8842$ fm$^{-1/2}$, we got $f^2=0.073$, and with the data set given by Allen and Kermode[50] ($a=5.412$ fm, $r=1.733$ fm, $A_S=0.8883$ fm$^{-1/2}$, $\eta=0.0264$) the result was $f^2=0.072$, both values not excluded by Ref. 52, though slightly low. For $\eta=0.023$ (Ref. 48) we got the more unrealistic value $f^2=0.062$. The resulting values for ϵ_1 are plotted in Fig. 5 together with the result of the energy dependent and the single energy analysis of Arndt and VerWest[8] and with the result from the Paris potential.[53]

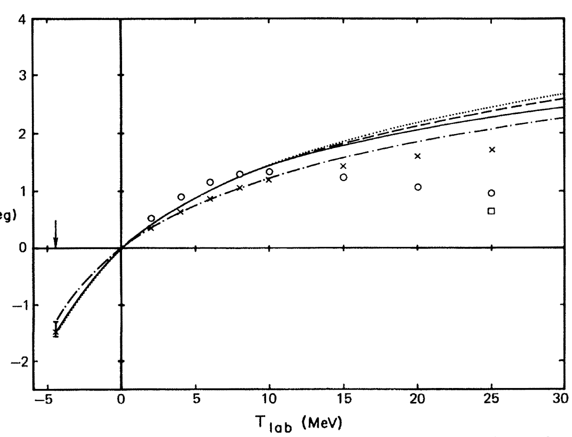

FIG. 5. The mixing parameter ϵ_1 (in Blatt-Biedenharn parametrization) calculated using Eq. (17) with $\eta=0.023$ (—·—·—·), $\eta=0.0272$ (· · ·), with parameters given by Allen (Ref. 49) (— — —), and using Eq. (18) (——). Also shown are the predictions of the Paris potential (Ref. 52) (× × ×) and the results of the energy dependent (○) and single energy (□) analyses of Arndt and VerWest (Ref. 8).

To use this result as input or a constraint in a phase shift analysis we parametrized ϵ_1 via the expression

$$\epsilon_1 = \frac{sk^2}{1+tk^2} \ . \tag{18}$$

The values

$$s = 0.347 \text{ fm}^2 \ ,$$

$$t = 5.5 \text{ fm}^2$$

give a very good approximation to the ϵ_1 curves with $\eta=0.0264$ and 0.0272 up to $E_{\rm lab}\sim 20$ MeV (see Fig. 5). Inserting this constraint into the analysis, the χ^2 increased (compared to the solution of Ref. 8). If one keeps s fixed and fits t to data up to 25 MeV, t tends to large values (>100 fm^2) and therefore ϵ_1 tends to small values. The data which push ϵ_1 to small values are the measurements of the spin correlation parameter A_{yy} (or C_{nn}) at 23.1 MeV.[54] It is known that A_{yy} is correlated strongly to ϵ_1.[54–56] On the other hand, ϵ_1 is connected very loosely to the other phase shifts (i.e., to the other data): In varying ϵ_1 at 25 MeV from 2.30° to 0.14° the χ^2 per datum of A_{yy} at 23.1 MeV dropped from 9.2 to 0.15, whereas for all the other data it remained nearly fixed (1.18 to 1.19).

Therefore one has to say that this measurement is "solely responsible" for a small value of the mixing parameter ϵ_1 and furthermore, this small value is in contradiction to predictions from OPE and also from most potential model calculations [see, for instance, the Paris potential[53] (Fig. 5)]. Further measurements of C_{NN} at energies around 20 MeV and below would be very useful to clarify this open question.

III. CONCLUSIONS

We have shown that some discrepancies in low energy results of the NN scattering are resolved. In the $^1S_0$ proton-proton case, measurements below 1 MeV nearly fix the effective range parameters, and the inclusion of the one pion exchange not only enlarges the range of convergence of the expansion but also improves the reproduction of the data between 1 and 20 MeV laboratory energy. The effect of this inclusion is minor for the $^1S_0$ neutron-proton and the $^3S_1$ case. Effective range parameters for the P waves are given and are in better agreement with an analysis by Naisse than were the results of the predecessors of both analyses. Following the work by Wong we have given a low energy parametrization of the mixing parameter ϵ_1. However, this parametrization is in contradiction to a polarization measurement (C_{NN}), and we suggest that only more experiments of this type will clear up this controversy.

ACKNOWLEDGMENTS

We are thankful for useful discussions and communications with Prof. R. Bryan, Prof. L. Heller, Prof. P. Signell, and Prof. D. Sprung. One of us (B.J.V.) would also like to thank ARCO Oil and Gas Company for aid in preparing this manuscript. This work was supported in part by the United States Department of Energy under Contract No. DE-AS05-76ER05223 while the authors were in the Physics Department at Texas A&M University.

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