Quasiadiabatic three-body dynamics of deuteron stripping and breakup reactions

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A convenient new method, the quasiadiabatic approximation, is developed for the standard three-body model of deuteron-induced stripping and breakup. The approximation gives the breakup wave function at "coincidence" ($\vec{r}_n=\vec{r}_p$) directly as the solution of a simple differential equation with a source term that depends on the elastic channel wave function. The derivation assumes only that the internal Hamiltonian of the broken up deuteron can be replaced by a constant, $\bar{\epsilon}_L$, whose value depends on the c.m. angular momentum L. No approximation restricting the relative n-p angular momentum is needed. The quasiadiabatic approximation reduces to the Johnson-Soper adiabatic approximation if $\bar{\epsilon}_L$ is replaced by $-\epsilon_d$, the internal energy of the bound deuteron. The adiabatic approximation for the elastic channel wave function gives an estimate of the quasiadiabatic approximation source term. The behavior of the coincidence breakup wave function in the nuclear interior (previously calculated using a coupled channels method) is well explained by the quasiadiabatic approximation with the use of a simple prescription for $\bar{\epsilon}_L$. The separation of internal and external breakup, and the "$L=9$" effect are easily explained by the use of the quasiadiabatic approximation. The decrease of the coincidence breakup wave function at large distances is not reproduced by this approximation. In applications to stripping calculations the quasiadiabatic approximation coincidence wave function gives a marked improvement over the Johnson-Soper adiabatic wave function, and it seems promising for practical calculations. This is tested by means of a previously described distorted-wave Born iteration applied to the adiabatic wave function. The rather lengthy distorted-wave Born iteration adiabatic calculations agree with the stripping derived by coupled channel calculations and give a good description of the long-range features of the coincidence breakup wave function. The stripping comparisons are complicated by the necessity of allowing for "closed" breakup channels, which are absent from the coupled-channel comparison, optional in the distorted-wave Born iteration adiabatic approximation, and fully included in the quasiadiabatic approximation. The empirical use of phenomenological local deuteron optical potentials is discussed.

I. INTRODUCTION

Deuteron stripping theory shares the inherent vagueness of other direct reaction theories,[1] caused by the approximate restriction of the dynamical analysis to a very small model space. A further traditional question in criticisms of the standard distorted waves (DW) theory of stripping concerns the large, loose structure of the deuteron. Thus, in the DW approach the proton-neutron relative motion is taken to be that of an unmodified deuteron. To remedy this defect, improved analyses of deuteron-nucleus reactions usually apply some three-body picture, in which the relative motion is affected by the interactions with the target nucleus.

Several truncated coupled channel (CC) calculations have been constructed[2-8] which omit rearrangement from the three-body model and which introduce discretized representations of the deuteron breakup continuum. However, these calculations are sufficiently lengthy so that it has not been possible to obtain a general survey of breakup effects from them, not even from the recent extensive work by the Kyushu group.[6,7] The CC calculations typically analyze the three-body continuum with respect to proton-neutron relative momentum. Great complications are caused by long range continuum-continuum interactions among different k values.

We present in this paper an approximate closed-form version of the CC three-body theories, which is simple enough to clarify many aspects of stripping and perhaps even to be used for the practical analysis of experiments. Our new approach emphasizes the three-body wave function $\psi(\vec{r},\vec{R})$ at coincidence, with the relative coordinate (see the notation in Sec. II) chosen to have the value $r=0$. By this restriction, and by use of an average energy $\bar{\epsilon}$ for the breakup continuum, we obtain a closure theory for $\psi(0,\vec{R})$ that entirely avoids the complications and detail of the CC method. In particular, discretization is avoided.

The coincidence wave function $\psi(0,\vec{R})$ only has relative orbital angular momentum $l=0$. Although it certainly does not describe all aspects of the three-body dynamics, it suffices for the calculation of stripping. Because the n-p interaction $V(r)$ has short range, the exact post form matrix element for (d,p) reactions,

$$\langle \chi_p^{(-)}(\vec{r}_p)\phi_n(\vec{r}_n) \mid V(r) \mid \psi(\vec{r},\vec{R})\rangle \ , \tag{1.1}$$

is dominated by the function $\psi(0,\vec{R})$. The coincidence wave function also does not describe the asymptotic regions of configuration space that contribute to the deuteron breakup cross section. But here again, the exact post form matrix element for breakup,

$$\langle \chi_p^{(-)}(\vec{r}_p)\chi_n^{(-)}(\vec{r}_n) \mid V(r) \mid \psi(\vec{r},\vec{R})\rangle \ , \tag{1.2}$$

generates all the required asymptotic amplitudes. Once again, this matrix element contains the short range operator $V(r)$ and it is dominated by $\psi(0,\vec{R})$. We can expect an improved theory of $\psi(0,\vec{R})$ to yield interesting modifications of post form breakup calculations.

Because our closure theory represents the continuum by a single, average excitation energy (in each partial wave), it resembles the adiabatic theory of stripping.[9–11] It also uses the standard adiabatic theory as a starting point for calculation. For these reasons we refer to the new theory as *quasiadiabatic* (QAD).

To some extent the QAD theory goes beyond previous CC work. It incorporates some effects of breakup angular momenta $l>0$. It also incorporates some effects of *closed channels,* which have breakup energies $\epsilon(k)>E$. The latter effect is seen to cause sensitive modifications of stripping cross sections. Phase relations in the construction of the cross section are particularly strongly affected by closed channel contributions.

The previous CC and adiabatic theories are reviewed in Sec. II A. The QAD theory is derived in Sec. II B, leading to Eq. (2.26), on which all our applications are based. Numerical calculations and comparisons with the CC results of Farrell, Vincent, and Austern (FVA) (Ref. 4) appear in Sec. II C. Section II D describes a simple improvement of the QAD theory by averaging with respect to the excitation energy $\bar{\epsilon}$. A more involved improved theory is also described there, which restores much of the CC momentum analysis in an iterative framework (DWBIA). Section III applies these theories to a (d,p) stripping example. Section IV describes several qualitative aspects of breakup effects in deuteron dynamics including the sensitive consequences of rather weak coupling to breakup and the plausibility of phenomenological "deuteron optical potentials." Section V is a summary.

II. TREATMENT OF THE THREE-BODY MODEL

A. Theoretical background

Our analysis concerns the simple three-body model in which a proton and a neutron move with respect to a stationary target "nucleus" according to the Hamiltonian

$$H=T_{\rm p}+T_{\rm n}+V(|\vec{r}_{\rm p}-\vec{r}_{\rm n}|)+U_{\rm p}(r_{\rm p})+U_{\rm n}(r_{\rm n})\ . \tag{2.1}$$

Relative and center of mass coordinates are introduced,

$$\vec{r}=\vec{r}_{\rm p}-\vec{r}_{\rm n}\ ,\quad \vec{R}=\tfrac{1}{2}(\vec{r}_{\rm p}+\vec{r}_{\rm n})\ , \tag{2.2}$$

and H is rewritten in the convenient form

$$H=T_R+h_r+U_{\rm p}(|\vec{R}+\tfrac{1}{2}\vec{r}|)+U_{\rm n}(|\vec{R}-\tfrac{1}{2}\vec{r}|)\ , \tag{2.3}$$

for which the Schrödinger equation is

$$(E-T_R-h_r-U_{\rm p}-U_{\rm n})\psi(\vec{r},\vec{R})=0\ . \tag{2.4}$$

The above model is often truncated to relative angular momentum $l=0$, for which

$$[E-T_R-h_r-U(r,R)]\psi(r,\vec{R})=0\ , \tag{2.5}$$

with

$$U(r,R)\equiv\frac{1}{4\pi}\int d\Omega_r(U_{\rm p}+U_{\rm n})\ . \tag{2.6}$$

Our approach to the analysis of (2.4) or (2.5) is related to the adiabatic approximation, therefore it is helpful to begin by describing that approximation. For the $l=0$ case, the model Schrödinger equation (2.5) is rewritten

$$[E+\epsilon_{\rm d}-T_R-U(r,R)]\psi(r,\vec{R})=(h_r+\epsilon_{\rm d})\psi(r,\vec{R})\ , \tag{2.7}$$

where

$$h_r\phi_{\rm d}(r)/r=-\epsilon_{\rm d}\phi_{\rm d}(r)/r\ , \tag{2.8}$$

and $\phi_{\rm d}(r)$ is the deuteron radial wave function. The entire set of n-p eigenstates is defined to be

$$\{\phi_{\rm d}(r),\phi(k,r)\}\ , \tag{2.9}$$

with

$$h_r\phi(k,r)/r=\epsilon(k)\phi(k,r)/r=(\hbar^2k^2/M)\phi(k,r)/r\ . \tag{2.10}$$

In the adiabatic approximation it is considered plausible that the higher-energy terms of the set (2.9) occur in ψ with low probability; on this basis the right-hand side (RHS) of (2.7) is expected to be small, and it is omitted. We get

$$[E+\epsilon_{\rm d}-T_R-U(r,R)]\psi^{\rm AD}(r,\vec{R})=0\ . \tag{2.11}$$

This is now an elastic scattering problem for the coordinate $\vec{R}$, in which r appears only as a parameter. This problem is solved separately for each value of r, using partial waves in the vector $\hat{R}$ and outgoing wave boundary conditions. Evidently the r dependence of $\psi^{\rm AD}$ defines a rather complicated wave packet composed of the states (2.9).

In a CC approach to the $l=0$ case, ψ is expanded immediately in the set of eigenstates (2.9), in the form

$$\psi(r,\vec{R})=\sum_{LM}a_{LM}[Y_{LM}(\hat{R})/rR]\left[\phi_{\rm d}(r)f_L(R)+\int_0^\infty dk\,\phi(k,r)g_L(k,R)\right]\ , \tag{2.12}$$

in which a_{LM} are coefficients determined by the incident plane wave part of ψ. Substitution in (2.5) gives the coupled equations

$$[E+\epsilon_{\rm d}-T_L(R)-\langle{\rm d}|U|{\rm d}\rangle]f_L(R)=\int_0^\infty dk\langle{\rm d}|U|k\rangle g_L(k,R)\ , \tag{2.13}$$

$$[E-\epsilon(k)-T_L(R)]g_L(k,R)-(2/\pi)\int_0^\infty dk'\langle k|U|k'\rangle g_L(k',R)=(2/\pi)\langle k|U|{\rm d}\rangle f_L(R)\ . \tag{2.14}$$

The potentials in (2.13) and (2.14) are matrix elements of $U(r,R)$ between the n-p eigenstates of the set (2.9). It is typical of three-body calculations that these potentials are somewhat long ranged. This is especially true of the continuum-continuum potential $\langle k \mid U \mid k' \rangle$, which decreases as R^{-2} if $k = k'$. In most calculations by the CC method[2-8] the continuum integrals in the coupled equations are converted to sums, by some process of discretization that approximates the integrands in a finite series of momentum bins. Such sums have a finite upper limit, often chosen so that $E - \epsilon(k) \geq 0$.

The coincidence wave function required in stripping calculations, as explained in the Introduction, is obtained from the CC calculation as

$$\psi(0,\vec{R}) = N \sum_{LM} a_{LM} R^{-1} Y_{LM}(\hat{R})[f_L(R) + h_L(R)] , \quad (2.15)$$

where

$$N \equiv \lim_{r \to 0} [r^{-1}\phi_d(r)] , \quad (2.16)$$

$$h_L(R) \equiv \int_0^\infty dk\, t(k) g_L(k,R) , \quad (2.17)$$

$$t(k) \equiv \lim_{r \to 0} [\phi(k,r)/\phi_d(r)] . \quad (2.18)$$

B. Quasiadiabatic approach

This new approach provides an immediate approximate calculation of the coincidence breakup functions $h_L(R)$ in (2.15), without evaluating the individual continuum functions $g_L(k,R)$ of the CC method, without discretization, and without truncating the spectrum of breakup energies.

For a derivation we first distinguish elastic and nonelastic (breakup and rearrangement) parts of $\psi(\vec{r},\vec{R})$ by means of projection operators

$$P \equiv |\phi_d\rangle\langle\phi_d| , \quad Q = 1 - P . \quad (2.19)$$

Then the exact Schrödinger equation (2.4) for the three-body model becomes

$$(E - T_R - h_r - U_p - U_n)[P\psi(\vec{r},\vec{R}) + Q\psi(\vec{r},\vec{R})] = 0. \quad (2.20)$$

If the elastic channel wave function is described in terms of an exact optical potential (perhaps quite different from phenomenological fits) by the equation

$$[E - T_R - h_r - U_{opt}(R)]P\psi = 0 , \quad (2.21)$$

then (2.20) can be rearranged in the form

$$(E - T_R - h_r - U_p - U_n)Q\psi = [U_p(r_p) + U_n(r_n) - U_{opt}(R)]P\psi . \quad (2.22)$$

The inhomogeneous equation (2.22) determines the breakup and rearrangement function $Q\psi$, if by some means we have available a suitable approximation for $P\psi$ or U_{opt}.

One simple way to convert (2.22) to a quasiadiabatic (QAD) equation for the coincidence wave function is to approximate the operator h_r by an average excitation energy $\bar{\epsilon}$ for the breakup continuum $Q\psi$ and then set $r = 0$. This yields

$$[E - \bar{\epsilon} - T_R - U_{JS}(R)]Q\psi_{r=0} = [U_{JS}(R) - U_{opt}]P\psi_{r=0} , \quad (2.23)$$

where

$$U_p(R) + U_n(R) \equiv U_{JS}(R) = U(0,R) \quad (2.24)$$

is the familiar Johnson-Soper potential (JS) of adiabatic stripping theories. It is interesting that the strong, short-ranged JS potential on the left-hand side (LHS) of (2.23) is related to the long-range, continuum-continuum coupling terms, introduced in the CC equations (2.14) by the use of $\vec{r},\vec{R}$ coordinates. It is the restriction to $r = 0$ that isolates the short-range part of these interactions when (2.23) is derived. The strength of the JS potential on the LHS of (2.23) emphasizes the importance of continuum-continuum coupling.[3,4,12]

Under the restriction $r = 0$, Eq. (2.23) only treats the $l = 0$ part of the breakup spectrum, rather like our sketch of the CC approach in Sec. II A. The $r \to 0$ limit suppresses the $l \neq 0$ parts of the exact equations (2.20) and (2.22). On the other hand, $l \neq 0$ effects are not omitted from our theory; in principle they influence the determination of the optical potential introduced in (2.21).

A more complete QAD analysis introduces the $l = 0$ partial wave expansion of (2.12) in the exact equation (2.22). Using the notation of the CC approach, this exact equation becomes

$$[E - T_L - U_{JS}(R)]h_L(R) - \int_0^\infty dk\, t(k)\epsilon(k) g_L(k,R) = [U_{JS}(R) - U_{opt}]f_L(R) . \quad (2.25)$$

In our QAD approximation $\epsilon(k)$ in (2.25) is now replaced by $\bar{\epsilon}$ and removed from the integral, which then reduces by definition [see (2.17)] to $h_L(R)$. Since the breakup spectrum is affected by centrifugal repulsion (see Sec. II C), we allow $\bar{\epsilon}$ to depend on L, and we therefore get

$$[E - \bar{\epsilon}_L - T_L - U_{JS}(R)]h_L(R) \approx [U_{JS}(R) - U_{opt}]f_L(R) . \quad (2.26)$$

Equation (2.26) is the basic equation of the QAD approach. The remainder of this paper is devoted to applications and tests of (2.26). Throughout this article we substitute $l = 0$ adiabatic elastic radial wave functions $f_L^{AD}(R)$ for $f_L(R)$, with U_{opt} determined from $f_L^{AD}(R)$ by the equation of motion.

C. Calculations

Numerical solutions of the exact CC equations are given by FVA for the case $E = 22.9$ MeV, with

$$U_p(r_p)=U_n(r_p)$$
$$=-(50\ \text{MeV})(1+0.1i)e^{-\alpha^2 r_p^2}, \quad (2.27)$$

with $\alpha=0.25\ \text{fm}^{-1}$. Five continuum bins are used, and the relative energy $\epsilon(k)$ is cut off at $\epsilon_{\max}=E$. The Coulomb potential is omitted. It is convenient to apply the new analysis to this same case, for ease of comparison. In fact, we will see that comparison with FVA has some limitations, because the new QAD theory has no upper cutoff of the breakup energies $\epsilon(k)$.

It is already known that the adiabatic approximation of (2.11) is in reasonable agreement with $f_L(R)$ from the CC calculation.[11,13] We therefore take $f_L(R)$ and the associated operator U_{opt} from the adiabatic wave function $\psi^{\text{AD}}(r,\vec{R})$, by projection on the ground state wave function $\phi_d(r)$. Thus the source term on the RHS of (2.26) is obtained from an adiabatic model in which U_{JS} and U_{opt} are included consistently. Indeed, with this source term the solution of (2.26) would return the adiabatic coincidence breakup function $h_L^{\text{AD}}(R)$, if $\bar{\epsilon}_L$ were replaced by $-\epsilon_d$; this serves as a test of the numerical calculation. More appropriate values of $\bar{\epsilon}_L$ then yield improvements of $h_L(R)$.

Our choice of the average excitation $\bar{\epsilon}_L$ is guided by FVA experience. The range of excitations in a CC theory is in the first place controlled by the bound-continuum matrix element $\langle k \mid U \mid d\rangle$, which drops strongly for $\epsilon(k)$ above about 20 MeV. We see further that as L increases the centrifugal repulsion in the kinetic energy operator $T_L(R)$ limits the contributions from continuum functions that have small net energies $E-\epsilon(k)$. On this basis, we estimate $\bar{\epsilon}_L \approx 10$ MeV for small L, decreasing gradually as L increases. A suitable expression for the FVA case is

$$\bar{\epsilon}_L \approx (10\ \text{MeV})\exp(-L/12)\ . \quad (2.28)$$

This rough estimate is easily modified, as needed.

Figure 1 shows the source potential

$$U_{\text{source}}=U(0,R)-U_{\text{opt}}\ ,$$

required in the RHS of (2.26), calculated from $f_L(R)$ under the assumption that U_{opt} is treated as a (trivial) local equivalent potential.[11] Of course, such a local equivalent U_{opt} fluctuates with respect to R and it has some L dependence. The calculation shows that (a) U_{source} is weak, with a maximum depth of about 10 MeV, (b) the real part of U_{source} changes sign at the nuclear surface, at $R=5$ fm, (c) the L dependence of U_{source} is mild, primarily a gradual weakening of the small imaginary part as L increases.

Figure 2 compares calculations of $|h_L(R)|$ for several variations of $\bar{\epsilon}_L$ and from the CC calculation. Although the approximate $h_L(R)$ functions do not decrease appropriately at large R, it can be noted that in the nuclear interior the detailed structure of the approximate functions improves progressively as $\bar{\epsilon}_L$ becomes more realistic. The exact $h_L(R)$ decreases at large R, as we see in (2.17), because the coincidence functions are mixtures of $g_L(k,R)$ functions with different momenta. Since the mixtures consist of functions that start in phase near $R=0$, they decrease in magnitude as R increases. In the approximation, each outgoing function $h_L(R)$ has a single momentum $\bar{k}_L$, and therefore its magnitude remains constant at large R.

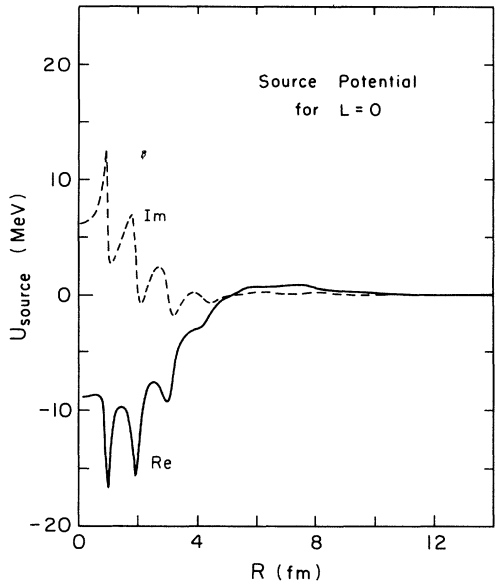

FIG. 1. Local equivalent source potential $U(0,R)-U_{\text{opt}}$ for the QAD calculation, evaluated for partial wave $L=0$.

D. Averaging, iteration

The mixture of momenta in the exact $h_L(R)$ is produced by the energy integral term in (2.25), the exact equation derived from the CC theory. The mixture of energies in this term introduces a mixture of momenta in h_L. We lose this effect when all the $g_L(k,R)$ functions are given the same energy. A possible *ad hoc* compensation for this defect is to calculate breakup functions $h_L(R,\bar{k})$ for several values of $\bar{k}$ and then average with respect to $\bar{k}$. We use the expression

$$\langle h_L(R,\bar{k})\rangle=\left[\frac{1}{2n+1}\right]\sum_{j=-n}^{n} h_L(R,\bar{k}_L+jb)\ , \quad (2.29)$$

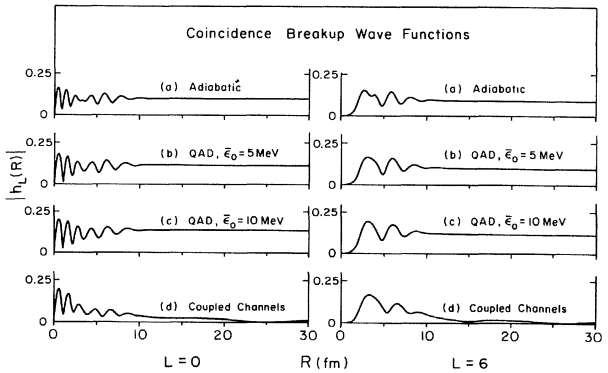

FIG. 2. Coincidence breakup functions $|h_L(R)|$ for partial waves $L=0$, 6 from (a) adiabatic theory, (b) QAD with $\bar{\epsilon}_L=(5\ \text{MeV})\exp(-L/12)$, (c) QAD with $\bar{\epsilon}_L=(10\ \text{MeV})\exp(-L/12)$, (d) CC theory.

where b is some plausible small increment of momentum. The strong potential U_{JS} in the nuclear interior helps to keep $h_L(R,\bar{k})$ in that region stable as $\bar{k}$ varies. We see in Fig. 3 that $\langle h_L(R,\bar{k})\rangle$ at large R is damped by the averaging.

A more fundamental improvement of the k mixture in h_L is obtained from the original CC equation (2.14), which we consider in the form

$$[E-\epsilon(k)-T_L(R)]g_L(k,R)=\frac{2}{\pi}\langle k \mid U(r,R) \mid \psi_L(r,R)\rangle \ , \tag{2.30}$$

with

$$\psi_L(r,R)\equiv\phi_d(r)f_L(R)+\int_0^\infty dk'\phi(k',r)g_L(k',R) \ . \tag{2.31}$$

Equation (2.30) can be used as a starting point for iteration[11] of any approximate ψ_L that is inserted on the RHS. But first, a kind of "distorted waves" modification of (2.30) is introduced, by subtracting $U(0,R)g_L(k,R)$ from both sides, so that

$$[E-\epsilon(k)-T_L(R)-U(0,R)]g_L(k,R) = \frac{2}{\pi}\langle k \mid U(r,R)-U(0,R) \mid \psi_L(r,R)\rangle \ . \tag{2.32}$$

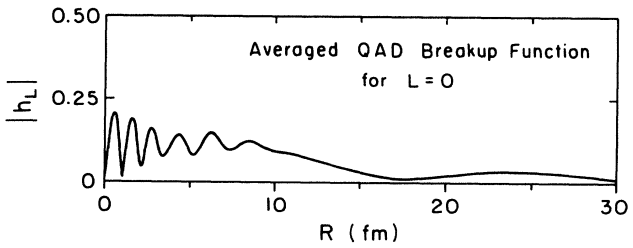

FIG. 3. Averaged QAD function $|\langle h_L(R,\bar{k})\rangle|$ from Eq. (2.29), using $n=4$, $b=0.05$ fm^{-1}, $\bar{\epsilon}_L=(10\text{ MeV})\exp(-L/12)$, for partial wave $L=0$.

The Johnson-Soper potential $U(0,R)$ in (2.32) largely cancels the continuum-continuum coupling from the RHS of (2.30) and transfers it to the LHS of (2.32). At this stage (2.32) has a tempting formal resemblance to the basic equation (2.26) of the QAD approximation. But the resemblance is limited. For example, if we put adiabatic wave functions on the RHS of these equations, then (2.32) contains the entire wave function ψ_L^{AD}, whereas (2.26) only contains the elastic f_L^{AD} term.

A formal solution for $h_L(R)$ in the *once-iterated* distorted-wave Born adiabatic method (DWBIA) is

$$h_L(R)\approx\frac{2}{\pi}\int_0^\infty dk\, t(k)[E^+-\epsilon(k)-T_L(R)-U(0,R)]^{-1}\langle k \mid U(r,R)-U(0,R) \mid \psi_L^{AD}(r,R)\rangle \ . \tag{2.33}$$

It is important that (2.33) collapses back to the solution of (2.26) if $\epsilon(k)$ is replaced by a constant, $\epsilon(k)\to\bar{\epsilon}_L$. This replacement allows the use of closure to express $h_L(R)$ as the operation of a simple Green's function on the quantity

$$(1-P)[U(r,R)-U(0,R)]\psi_L^{AD}(r,R)$$

evaluated at $r=0$. Then there is considerable cancellation between the large potentials $U(r,R)$ and $U(0,R)$ in the evaluation of this expression. Obviously, use of the correct $\epsilon(k)$ in (2.33) affects this cancellation; this is the principal dynamical effect that is lost by the $\bar{\epsilon}$ approximation in the QAD method. Thus at the cost of a rather tedious calculation, the DWBIA iterated method can incorporate much of the k dependence of the exact coupled equations. It seems of sufficient accuracy to use as a standard to test the simple quasiadiabatic approach. Figure 4 gives an example of an $h_L(R)$ function computed by this method.

It is helpful to recognize that the k integration in (2.33) can be separated into two parts, with $\epsilon(k)<E$ and $\epsilon(k)>E$. We obtain *open* and *closed* channel parts of $h_L(R)$, respectively. By this procedure, since the replacement of $\epsilon(k)$ by $\bar{\epsilon}_L$ in the Green's function of (2.33) returns the QAD approximation, we are able to *isolate the open channel part of the QAD calculation* of $h_L(R)$,

$$\frac{2}{\pi}\int_0^{k_{max}} dk\, t(k)[E^+-\bar{\epsilon}_L-T_L(R)-U(0,R)]^{-1} \times\langle k \mid U(r,R)-U(0,R) \mid \psi_L^{AD}(r,R)\rangle \ . \tag{2.34}$$

This truncated, open channel part of $h_L(R)$ is more suitable than the full QAD $h_L(R)$ for comparisons with previous CC work that omitted closed channels.[4] All our DWBIA calculations, with or without the $\bar{\epsilon}$ approximation, limit the k integration in the above manner.

III. STRIPPING CROSS SECTION

We calculate stripping cross sections for the same case treated by FVA.[4] The various radial wave functions under consideration, such as $F_L(R)$, are inserted in place of the deuteron distorted wave in a suitable zero range DWBA code. The proton distorted wave is computed with the (non-Coulomb) optical potential of (2.27). The captured neutron is chosen to have $l=2$, with a wave function computed with a real potential that has the same

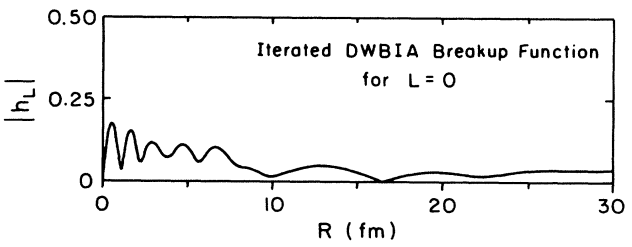

FIG. 4. Coincidence breakup function $|h_L(R)|$ from DWBIA iterated theory of Eq. (2.33), with closed channels excluded by an upper cutoff of the integration at $k=0.72$ fm^{-1}, for $L=0$.

geometry as (2.27), but a depth adjusted to the neutron binding energy $E_{Bn}=6.6$ MeV, for which $Q=4.4$ MeV.

The stripping angular distribution is a sensitive test of the three-body model, because it is affected by phase relations among the component wave functions of our approximations. Our standard for comparison is the self-consistent CC calculation of FVA,[4] checked more recently by Shepard, Rost, and Kunz.[14] Figure 5 gives the CC stripping cross section and the corresponding cross section obtained using $F_L^{AD}(R)$ from the JS adiabatic approximation. Figure 6 is an analysis of the JS cross section into separate contributions from the elastic f_L^{AD} and breakup h_L^{AD} components. Figure 7 is the stripping cross section obtained with the $h_L(R)$ functions from Eq. (2.26), using (2.28) for $\bar{\epsilon}_L$. Figures 8 and 9 use the "improved" $h_L(R)$ functions of Sec. II C. Figure 10 applies the DWBIA method of Sec. II C to construct the open channel part of the QAD approximation of Eq. (2.26). We recall that throughout Figs. 7–10 the elastic wave function $f_L(R)$ is taken from the adiabatic approximation, and only the $h_L(R)$ functions vary.

In all of Figs. 7–10 the breakup contribution to stripping is much larger than in the adiabatic calculation of Fig. 6, in agreement with CC calculations by Iseri *et al.*[7] It is interesting to see in Figs. 7–10 that the differential cross sections calculated from the $h_L(R)$ breakup functions alone agree well with each other. Since the stripping cross sections obtained with the total radial functions

$$F_L(R)=f_L(R)+h_L(R) \tag{3.1}$$

do not agree so simply, we see the possibility of a discrepancy in the relative phase between $f_L(R)$ and $h_L(R)$ at the nuclear surface. In this regard, the cross section of Fig. 7, for the simple QAD method, does not agree with the CC result, and the averaging introduced in Fig. 8 does not help very much. However, the DWBIA iteration in Fig. 9 does agree well with the CC calculation.

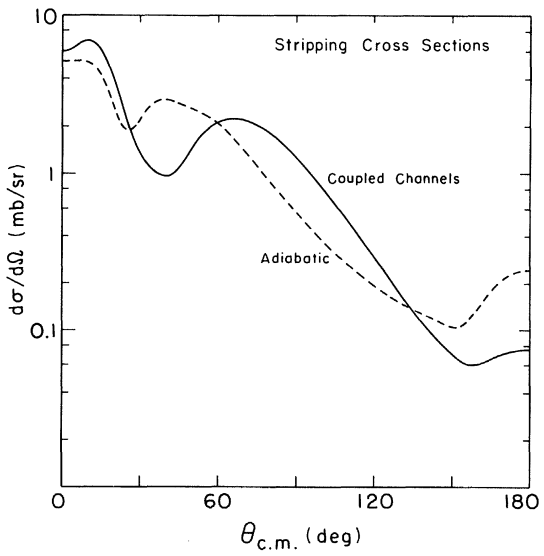

FIG. 5. Stripping cross sections for the case described in the text taken from the CC theory (Ref. 4) (solid line), and the adiabatic theory (Ref. 11) (dotted line).

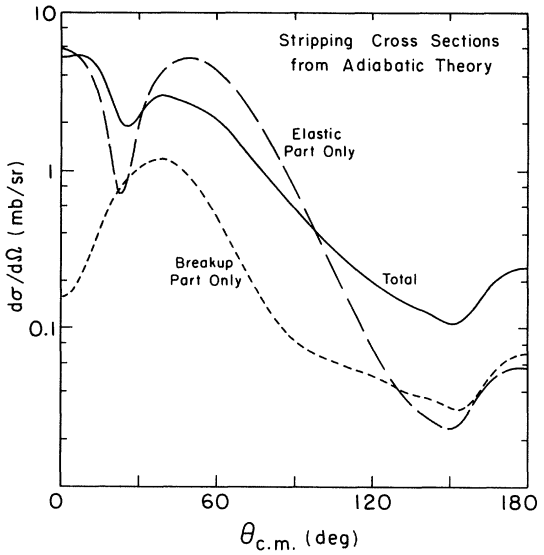

FIG. 6. Stripping cross sections as in Fig. 5, computed with the total adiabatic wave functions F_L^{AD} (solid line), only the elastic adiabatic functions f_L^{AD} (dashed line), and only the breakup adiabatic functions h_L^{AD} (dotted line).

We trace the discrepancy between the QAD and CC in Figs. 5 and 7 to the closed channel inconsistency discussed earlier. Figure 10 shows that good agreement with CC is restored, if (2.34) is used to remove closed channel parts from the QAD theory. Of course, in both cases some closed channel contributions correctly belong in the theory. It is not clear at this stage whether the discrepancy between Figs. 5 and 7 should be attributed to the absence of closed channel parts in Fig. 5, or to some exaggeration of closed channel parts in Fig. 7. Further calculations would be needed to settle this question.

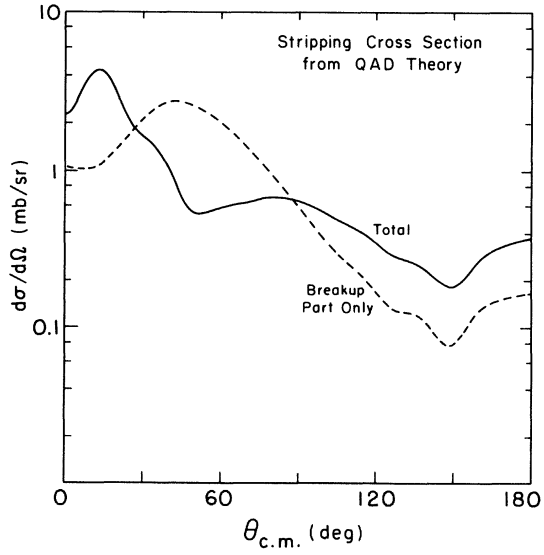

FIG. 7. Stripping cross section as in Fig. 5, computed with the QAD theory, using $\bar{\epsilon}_L=(10\text{ MeV})\exp(-L/12)$. Total wave function (solid line), only breakup wave function (dotted line), and elastic wave function same as in Fig. 6.

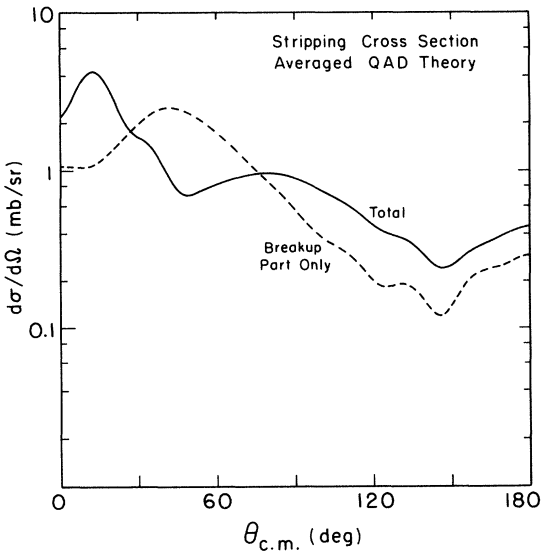

FIG. 8. Stripping cross section as in Fig. 5, computed with the averaged QAD wave functions of Eq. (2.29), as in Fig. 3. Total wave function used (solid line), only breakup wave function (dotted line), and elastic wave function same as in Fig. 6.

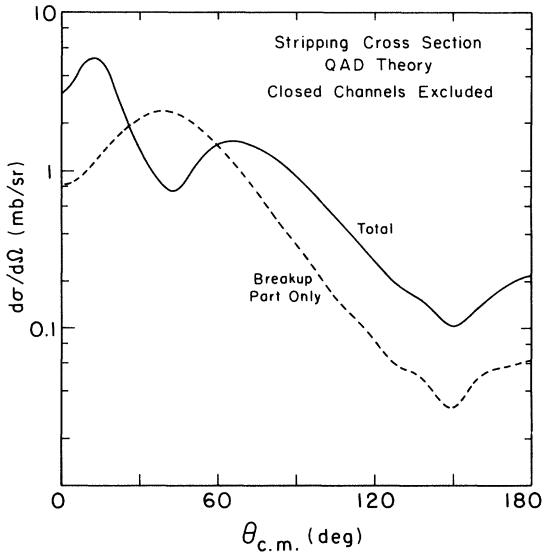

FIG. 10. Stripping cross section as in Fig. 5, computed for the QAD theory, using Eq. (2.34) with closed channels excluded by an upper cutoff at $k=0.72$ fm^{-1}. Total wave function used (solid line), and only breakup wave function used (dotted line).

IV. DISCUSSION

A. Aspects of the coupling to breakup

The elastic channel is coupled to the coincidence breakup wave function by the rather weak difference potential

$$U_{\text{source}}=U_{\text{JS}}-U_{\text{opt}}\ ,$$
$$=U_{\text{p}}(R)+U_{\text{n}}(R)-U_{\text{opt}}\ , \tag{4.1}$$

as we see initially in Eq. (2.23). This potential is weakly attractive inside the nucleus, and it has an even weaker long range repulsive tail in the exterior region. These properties are understandable: The JS potential $U(0,R)$ tends to be strong and short ranged; it is the sum of the short-ranged interactions of the nucleus with a neutron and proton that remain near each other. On the other hand, either in the calculation of U_{opt} or of the rather similar folded potential $\langle \text{d} \mid U \mid \text{d} \rangle$, the separation of the proton and neutron assumes large values, of the order of the size of the deuteron.[11] This geometrical looseness produces potentials U_{opt} or $\langle \text{d} \mid U \mid \text{d} \rangle$ that are more diffuse than $U(0,R)$ and that are weaker in the nuclear interior. This characteristic geometry of U_{source} produces the distinction between *interior breakup* and *exterior breakup,* noted before.[4,11] The zero of U_{source} at the nuclear surface leads to the "$L=9$ effect" of FVA;[3,4,11] breakup becomes very small at $L=9$, because at that angular momentum the first maximum of $f_L(R)$ falls near the zero of U_{source}.

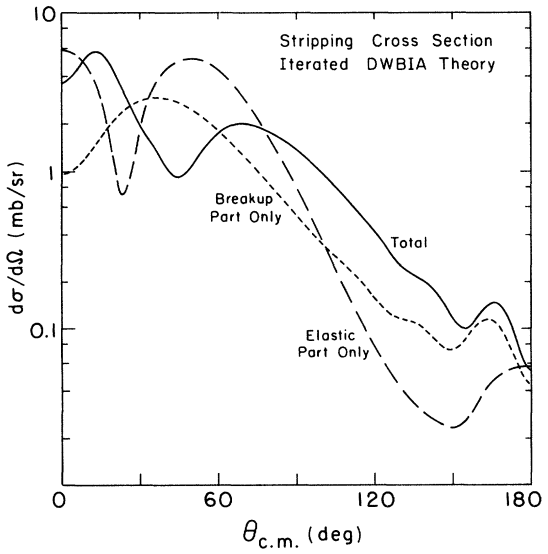

FIG. 9. Stripping cross section as in Fig. 5, computed with the DWBIA iterated theory of Eq. (2.33), with closed channels excluded by an upper cutoff at $k=0.72$ fm^{-1}. Total wave function used (solid line), only breakup wave function (dotted line), and only elastic wave function (dashed line).

The significance of the weak tail (~ 1 MeV) of U_{source} is tested by imposing a cutoff of U_{source} at $R=5$ fm, where the real part goes through zero. We find this cutoff entirely eliminates breakup in partial waves with $L>9$. Omission of the repulsive tail does not affect $h_L(R)$ in the interior region in lower partial waves, but it does slightly increase the asymptotic amplitudes of the (unaveraged) $h_L(R)$ functions for low L. Omission of the tail also eliminates the oscillations of $|h_L(R)|$ in low partial waves in the region $R>5$ fm, these oscillations therefore must reflect the standing wave patterns in the scattering wave functions $f_L(R)$ in the region $R>5$ fm.

Although, U_{source} is weak, the coincidence breakup wave functions $h_L(R)$ for penetrating partial waves are very large in the nuclear interior, as much as one-half or two-thirds of the elastic wave functions $f_L(R)$. We attribute this to a "resonance" effect. Thus we see in the propagators in the LHS of Eqs. (2.21) and (2.26) a tendency for

$f_L(R)$ and $h_L(R)$ to have nearly equal kinetic energies in the nuclear interior, with the potential difference $U(0,R)-U_{\rm opt}$ compensated by the energy shift $\bar{\epsilon}$. Therefore oscillations of $f_L(R)$ and $h_L(R)$ tend to remain in phase throughout the interior, and the effect of $U_{\rm source}$ is enhanced. This close momentum matching does not continue outside the nucleus. [A related effect in more complete CC calculations[3,4] is that momentum *mismatch* in the exterior causes channel functions $g_L(k,R)$ for large k to be confined to the interior region.]

The resonance coupling of elastic and breakup channels implies that the effects of breakup might be sensitive to small changes of the physical model, perhaps to absorption or to Pauli modifications. Questions such as these belong in future work. However, straightforward adiabatic calculations have been used already[11] to test changes of the absorption strength in the single-particle potentials $U_{\rm p}$ and $U_{\rm n}$. Increased absorption reduces $f_L(R)$ and $h_L(R)$ in the nuclear interior, but it leaves their ratio essentially unaltered.

B. Relation to phenomenology

Phenomenological DWBA calculations of stripping use local deuteron optical potentials that are fitted to deuteron elastic scattering data. Let us ask how this procedure is related to the three-body theory. Because $h_L(R)$ in the three-body theory vanishes at large R, the asymptotic amplitudes of the elastic radial wave functions $f_L(R)$ and the complete coincidence functions $F_L(R)$ of (3.1) must be identical. Our question about phenomenology is then whether a local potential fitted to the asymptotic data accurately generates one of these wave functions, and if so, which one? The adiabatic approximation gives a clear answer to this question: The $F_L^{\rm AD}(R)$ functions (which are asymptotically invalid) are exactly governed by the local potential $U_{\rm JS}(R)$; however, the $f_L^{\rm AD}(R)$ functions are fitted by the somewhat nonlocal[11] adiabatic potential $U_{\rm opt}$ used in this paper. How far does this conclusion persist when the more accurate QAD method is used? Let us attempt an answer by deriving the (trivial) equivalent local potential $\mathscr{U}_L(R)$ for the improved $F_L(R)$ functions obtained here.

If we define $F_L(R)$ to be the solution of the Schrödinger equation

$$[E+\epsilon_{\rm d}-T_L(R)-\mathscr{U}_L(R)]F_L(R)=0\ , \tag{4.2}$$

then

$$\mathscr{U}_L(R)=(f_L+h_L)^{-1}[E+\epsilon_{\rm d}-T_L(R)](f_L+h_L)\ , \tag{4.3}$$

where (3.1) has been used. From the definition of $U_{\rm opt}$, and from (2.25) and the definition of $U_{\rm source}$, we obtain an exact expression

$$\mathscr{U}_L(R)=U(0,R)+(f_L+h_L)^{-1}\times[\epsilon_{\rm d}h_L+\int_0^\infty dk\, t(k)\epsilon(k)g_L(k,R)]\ . \tag{4.4}$$

Using the $\bar{\epsilon}$ approximation of (2.26) this local equivalent potential simplifies to

$$\mathscr{U}_L(R)\approx U(0,R)+(\epsilon_{\rm d}+\bar{\epsilon}_L)h_L/(f_L+h_L)\ , \tag{4.5}$$

which we further improve by inserting the localized, averaged $\langle h_L\rangle$ functions discussed in Sec. II C. Thus

$$\mathscr{U}_L(R)\approx U(0,R)+(\epsilon_{\rm d}+\bar{\epsilon}_L)\langle h_L\rangle/(f_L+\langle h_L\rangle)\ . \tag{4.6}$$

Figure 11 shows the second term of (4.6), which is a correction to $U(0,R)$, using $\bar{\epsilon}_L$ from (2.28). We see especially in (4.3) that the fluctuations in Fig. 11 are indications of nonlocality in the calculation of $F_L(R)$, because nonlocality displaces the zeroes of the numerator and denominator from each other.

Of course, all the above expressions tend to be dominated by the strong, local JS potential. Nevertheless introduction of the weak correction potential shown in Fig. 11 suffices to generate agreement with elastic scattering data and to produce a good approximation to the total coincidence wave function $F_L(R)$. The correction slightly reduces the real potential and the imaginary potential in low partial waves. As to nonlocality, the fluctuations in Fig. 11 are of the same magnitude as fluctuations[11] in the corresponding equivalent potential for $f_L(R)$. It is not obvious that phenomenology generates either function more accurately than the other.

V. CONCLUSIONS AND OUTLOOK

Our principal conclusion is that the QAD approximation provides a good, simple treatment of breakup effects in the dynamics of deuteron stripping and breakup. This approach focuses on small neutron-proton separations, with $r\approx 0$ and with relative angular momentum $l=0$. It produces the coincidence three-body wave function $\psi(0,\vec{R})$, which tends to dominate the calculation of the matrix elements for breakup and stripping, as discussed in the Introduction. We primarily considered stripping. Of course, approximate finite-range improvements of our ex-

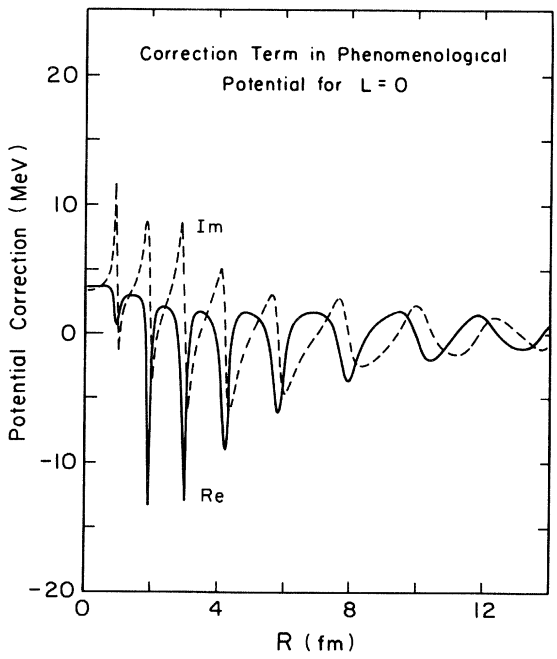

FIG. 11. Correction term in local equivalent potential of Eq. (4.6), with $\bar{\epsilon}_L=(10\ {\rm MeV})\exp(-L/12)$, for partial wave $L=0$.

treme zero-range theory are available.[1]

The QAD method clarifies our qualitative understanding of properties of the three-body model of deuteron-induced reactions. For example, complete CC calculations,[2–8] which analyze the breakup wave function with respect to relative momentum k, need to deal with long range, continuum-continuum interactions, with tails that decrease as R^{-2}. However, although we do find that interactions in the continuum are very important, our emphasis on $r=0$ avoids the long range interactions. Our treatment of the continuum is dominated by the strong, short-range JS potential $U(0,R)$. The QAD method also shows that the elastic-breakup coupling reverses sign at the nuclear surface; this clarifies, for example, the familiar "$L=9$ effect."

It is clear that the $h_L(R)$ coincidence breakup wave functions are largest in the nuclear interior, where they have nearly the same momenta as the elastic wave functions $f_L(R)$. At larger radii the $h_L(R)$ are essentially decoupled from the elastic channel. As we proceed further outward, the coincidence functions $h_L(R)$ decrease in magnitude, because as the separately moving nucleons travel onward, they gradually move apart, an obvious property of a three-body system.

The importance of the $f_L(R)$ and $h_L(R)$ functions in any actual stripping calculation depends on the degree of momentum and angular momentum matching between the entrance and exit channels. Good matching emphasizes contributions from the nuclear surface region and the exterior, poor matching allows greater importance to interior contributions. Since the $h_L(R)$ functions are largest in the interior, they tend to play a limited role in well-matched stripping reactions. A further suppression of $h_L(R)$ contributions in stripping occurs because it is in the grazing partial waves that dominate stripping that we find the sign reversal in the h_L, f_L coupling, mentioned earlier. Additional suppression of $h_L(R)$ occurs if the U_p and U_n optical potentials are given more realistic, i.e., larger absorption strengths than in (2.27). Since the $h_L(R)$ for low partial waves originate in the nuclear interior, more absorption in that region reduces these h_L functions overall. On the whole, the breakup continuum seems to give rather modest contributions in well-matched stripping reactions; presumably poor matching requires a more careful treatment.

The QAD theory depends significantly on the average energy parameter $\bar{\epsilon}_L$. For example, as compared with the adiabatic theory, use of plausible $\bar{\epsilon}_L$ values increases the magnitude of $h_L(R)$ and it more than triples the purely breakup cross section for stripping, in agreement with CC calculations.[7] On the other hand, although a correct correspondence with CC calculations requires $\bar{\epsilon}_L$ to decrease with respect to L, the precise details of this decrease do not seem to matter much. A more careful analysis of the $\bar{\epsilon}_L$ parameter would be helpful.

We noted a difficulty in comparing stripping calculations using the QAD theory with previous CC results, apparently because of a discrepancy in the relative phase between $f_L(R)$ and $h_L(R)$ at the nuclear surface. We trace this to closed channel effects in the QAD theory, which are missing from the comparison CC calculation. However, while it is interesting to find these additional effects in the approximate QAD theory, their accuracy is uncertain. It would be helpful to compare the QAD approximation with more complete CC calculations. Such a comparison might be easier at higher bombarding energies, because as E increases the entire important range of $\epsilon(k)$ tends to be included in the open channels, with $E-\epsilon(k)>0$.

Our study of the basis for phenomenological deuteron optical potentials in Sec. IV B is inconclusive. It is not yet clear from this work whether local phenomenological fits to deuteron elastic scattering more nearly generate the elastic wave function $f_L(R)$ or the entire coincidence wave function $F_L(R)$. However, a possible improved phenomenology seems indicated: to fit elastic data by means of a JS potential with L-dependent corrections, as in (4.5) and (4.6).

One advantage of a closed-form theory is that the effects of parameter changes are easily seen. For example, we see that an increase of ϵ_d, to make the "deuteron" more tightly bound, would tend to make U_{opt} more like $U(0,R)$ and therefore would reduce the breakup source function $U(0,R)-U_{opt}$. Increasing the bombarding energy seems to have more complicated effects. An increase of E on the RHS of (2.26) tends to weaken U_{opt} and this enhances the coupling to breakup; however, an increase of E on the LHS of (2.26) tends to reduce h_L. Moreover, as E increases, the centrifugal cutoff of $\bar{\epsilon}_L$ must move to higher L values. Overall, an increase of E probably leaves the breakup contribution per partial wave nearly unchanged, but it extends this effect to more partial waves.

We note here that a recent paper by Thompson and Nagarajan[15] introduces a related closure method to treat the continuum in a three-body model of ^{7}Li breakup.

Note added in proof. We also note a formal similarity between the present paper and a paper by S. Mukherjee, S. Ray, and S. K. Samaddar, Prog. Theor. Phys. 55, 482 (1976). Their Eq. (2.40) resembles our Eq. (2.26). However, the two papers have very different points of view.

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