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Binding energies of hypernuclei and three-body ΛNN forces

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Binding energies of hypernuclei are calculated with use of accurate variational methods for $A=3$, 5, with effective interaction calculations for $A=5$, 9, 13, ∞, and with Fermi hypernetted-chain variational calculations for $A=\infty$. Effective interactions and variational results agree within a few percent. For ΛN forces consistent with Λp scattering, agreement with the experimental energies is obtained with strongly repulsive Wigner type ΛNN forces, and for $A>5$ also with a weakened p-state ΛN strength about half that of the s-state strength.

A long-standing problem of hypernuclei is that of overbinding: The ground state Λ separation energies B_Λ calculated with ΛN potentials consistent with low-energy Λp scattering are about twice the experimental values for ${}^5_\Lambda$He and heavier hypernuclei.[1-3] We strengthen this result, and obtain a consistent description with use of strongly repulsive Wigner-type ΛNN forces and, for $A>5$, also with a weakend p-state ΛN interaction consistent with Λp scattering.

For our ΛN potentials we use a central Urbana-type ΛN two-π-exchange potential[4] with the same form for the singlet and triplet potentials (r in fm):

$$V_{\Lambda N}=V_0(r)[1-\epsilon+\epsilon P_x] \ ,$$

$$V_0=V_C(r)-V_{2\pi}(r) \ ,$$

$$V_{2\pi}(r)=VT_\pi^2 \ ,$$

$$T_\pi=(1+3/x+3/x^2)(e^{-x}/x)(1-e^{-2r^2})^2, \quad x=0.7r \ ,$$

$$V_C(r)=W_C[1+\exp(r-R)/a]^{-1} \ .$$

Here V denotes the singlet and triplet strengths ${}^1V$, ${}^3V$ as appropriate and $V_\sigma={}^1V-{}^3V$ denotes the spin dependence. P_x is the space-exchange operator. For the s-shell hypernuclei ($A<5$), only the s-state interaction, i.e., $V_0(r)$, is effective.[5] For $A>5$ the p-state interaction $V_1=(1-2\epsilon)\ V_0$ becomes significant. The scattering data are consistent with the above form of $V_{\Lambda N}$ for $\epsilon\approx0.25$, in agreement with previous calculations.[1,2,5] T_π is the one-π-exchange tensor potential shape modified with a cutoff, and V_C is a Woods-Saxon repulsive core with $W_C=2137$ MeV, $R=0.5$ fm, $a=0.2$ fm, appropriate to the spin, isospin independent core of the NN potential of Ref. 4. $V_{2\pi}$ corresponds to a two-π-exchange mechanism due to the one-π transition potentials ($\Lambda N\rightarrow\Sigma N$, $\Sigma\Delta$) dominated by their tensor components. The intrinsic range of $V_{\Lambda N}$ is 2.0 fm, very close to that of the hard core potential used in Ref. 2. (The intrinsic range is the effective range when V is just large enough to give a bound state.) For ${}^3_\Lambda\mathrm{H}(S=\frac{1}{2})$ the appropriate spin combination is $V\equiv V^{(3)}=\frac{3}{4}{}^1V+\frac{1}{4}{}^3V$; for ${}^5_\Lambda$He it is $V\equiv\bar{V}=\frac{1}{4}{}^1V+\frac{3}{4}{}^3V$. All these strengths are n, p charge symmetric averages. Λp strengths are subscripted appropriately.

For the ΛNN potential we consider a Wigner-type potential[6,7]

$$V_{\Lambda NN}=WT_\pi^2(r_{1\Lambda})T_\pi^2(r_{2\Lambda}) \ .$$

This is phenomenological, but its form is suggested by dispersive modifications of the two-body ΛN potential $V_{2\pi}$ due to medium modifications of the intermediate-state potentials for the Σ, N, Δ.[1,8] Thus the interaction of the intermediate state Σ, N, Δ with a nucleon of the medium will again be predominantly through a $V_{2\pi}$ potential, proportional to T_π^2. A closure approximation will then give the above form for $V_{\Lambda NN}$.[9]

Scattering data and ${}^3_\Lambda H$. Variational calculations for ${}^3_\Lambda$H were made with a product function $f_{\Lambda N}(r_{1\Lambda})f_{\Lambda N}(r_{2\Lambda})f_{NN}(r_{12})$. The correlation functions $f_{\Lambda N}$, f_{NN} are parametrized and determined as in Ref. 7. The integrations were done numerically and not using a Monte Carlo (MC) procedure. For V_{NN}, the Urbana V_{14} potential[4] was used. This has a tensor component and $f_{\Lambda N}$ thus includes a tensor correlation function. For $B_\Lambda=0.13\pm0.05$ MeV this calculation then determines a relation between $V^{(3)}$ and W: $V^{(3)}=6.27+5.0W\pm0.02$. All strengths V, W are in MeV. To relate the charge symmetric ΛN potential to the Λp potential and to Λp scattering we have made a phenomenological correction for charge symmetry breaking (CSB) effects by taking these to be spin independent. This is consistent with the ground and excited state energies of the $A=4$ hypernuclei.[10] Estimates, based on variational Monte Carlo calculations for the $A=4$ hypernuclei, of the CSB effects, then give $V_{\Lambda p}^{(3)}\approx V^{(3)}+0.05$, $\bar{V}_{\Lambda p}\approx\bar{V}+0.05$. For any assumed value of $V_{\Lambda p}^{(3)}$ the experimental low-energy Λp total cross section $\sigma_{tot}=\frac{1}{4}{}^1\sigma+\frac{3}{4}{}^3\sigma$ then determines ${}^1V_{\Lambda p}$, ${}^3V_{\Lambda p}$, and hence $\bar{V}_{\Lambda p}$ and $V_\sigma(=V_{\sigma\Lambda P})$ in terms of $V_{\Lambda p}^{(3)}$. The $V^{(3)}-W$ relation together with the CSB corrections then in turn give $\bar{V}$, V_σ as functions of W.

An important conclusion is that $\bar{V}$—the effective s-state two-body ΛN potential relevant for ${}^5_\Lambda$He and also for the $A>5$ hypernuclei we consider below — is rather well determined by just σ_{tot}, and is thus effectively independent of $V^{(3)}$ and hence of W. Thus we obtain $\bar{V}=6.15\pm0.05$ ($\bar{V}_{\Lambda p}=6.2\pm0.05$) for $-0.01\lesssim W\lesssim0.02$. ($W=0.02$ corresponds to a quite repulsive $V_{\Lambda NN}$.) The errors correspond to a quite generous allowance for the errors in σ_{tot}. The corresponding values of V_σ are $0 \lesssim V_\sigma \lesssim 0.4 \pm 0.1$, with errors about twice those of $\bar{V}$ for a given value of W. Since V_σ also increases quite rapidly with W it is therefore quite sensitive to assumptions about ΛNN forces in ${}^3_\Lambda H$ in contrast to $\bar{V}$. Also, V_σ is quite sensitive to assumptions about the CSB interaction.[2]

${}^5_\Lambda He$. B_Λ was calculated in two ways. (1) Variational MC calculations were made with a product function analogous to that for ${}^3_\Lambda H$. The NN potentials were those of Mafliet and Tjon.[11] These are central and give a satisfactory energy and rms point radius for ${}^4He$.[6] (2) Effective interaction (EI) calculations were made with effective interactions $\tilde{V}_{\Lambda N}$ and $\tilde{V}_{\Lambda NN}$ obtained from $V_{\Lambda N}$ and $V_{\Lambda NN}$ as described below for the calculation of the well depth D. These are then folded into a Gaussian point nucleon distribution to give an $\alpha\Lambda$ potential $V_{\alpha\Lambda} = V^{(2)}_{\alpha\Lambda} + V^{(3)}_{\alpha\Lambda}$ with contributions from $V_{\Lambda N}$ and $V_{\Lambda NN}$.[12] B_Λ is then calculated from the $\alpha\Lambda$ Schrödinger equation. The folding procedure requires the use of effective interactions because of the strongly repulsive core in the ΛN potential. EI calculations are required for heavier finite hypernuclei ($A > 5$), since realistic A-body variational MC calculations are then not practicable. The MC and EI results for B_Λ agree to within the statistical MC error of ± 0.15 MeV. The EI results may thus be considered as reliable to within this error. This agreement also indicates that the effect of any ${}^4He$ core distortion in the EI approach is small, since core distortion is included in the MC calculations but not in the EI calculations. The experimental value (3.12 ± 0.02 MeV) then determines a relation between $\bar{V}$ and W: $\bar{V} = 5.99 + 11.3\,W \pm 0.001$. For $V_{\Lambda\Lambda N} = 0$ one obtains a much too large $B_\Lambda = 5.9 \pm 1.2$ MeV (MC) for the values of $\bar{V} = 6.15 \pm 0.05$ consistent with scattering.

$A \leq 5$. The $\bar{V}$, W relation for ${}^5_\Lambda He$ together with the effectively unique value of $\bar{V}$ obtained from σ_{tot} then gives $W = 0.014 \pm 0.004$ with $\bar{V} = 6.15 \pm 0.05$. This value of W together with the $V^{(3)}$, W relation for ${}^3_\Lambda H$ then implies $V^{(3)} = 6.35 \pm 0.05$ and $V_\sigma = 0.4 \pm 0.1$. The corresponding potential energies (MeV) for ${}^5_\Lambda He$ are $\langle \tilde{V}_{\Lambda N} \rangle = 11.25 \pm 0.3$, $\langle V_{\Lambda NN} \rangle = 1.95 \pm 0.4$, appropriate to a large repulsive ΛNN contribution. The spin dependence is large because the same $V_{\Lambda NN}$ is taken for ${}^3_\Lambda H$ as for ${}^5_\Lambda He$. This, however, may not be justified. Thus in particular the dispersive ΛNN forces may effectively be very different for ${}^3_\Lambda H$ than for ${}^5_\Lambda He$ and $A > 5$, as discussed below. To allow for such uncertainties, we consider ${}^3_\Lambda H$ with $V_{\Lambda NN} = 0$. This leaves $\bar{V}$ and W unchanged but now gives $V_\sigma = 0.25 \pm 0.1$, appropriate to $V^{(3)} = 6.27$. This corresponds to a weaker spin dependence than obtained above; however, V_σ is still quite large because of the spin independence assumed for the CSB interaction. (With $\bar{V} = \bar{V}_{\Lambda p} = 6.2 \pm 0.05$ we obtain $V_\sigma = 0.25 \pm 0.1$ if $V_{\Lambda NN}$ is the same for ${}^3_\Lambda H$ and ${}^5_\Lambda He$, and 0.15 ± 0.1 if $V_{\Lambda NN} = 0$ for ${}^3_\Lambda H$.) The case for strongly repulsive ΛNN forces is thus quite strong, whereas the ΛN spin dependence is poorly determined.

The excitation energy ΔE of the 1^+ state of the $A = 4$ hypernuclei is (for spin independent ΛNN forces) given by

$$\Delta E \approx (dB^{(4)}/dV^{(4)})\, V_\sigma/3 \simeq 3.2\, V_\sigma \ .$$

Thus the experimental value of $\Delta E \approx 1$ MeV implies $V_\sigma \approx 0.3$, consistent (for $\bar{V} = 6.15 \pm 0.05$) with the values of V_σ obtained with both assumptions about $V_{\Lambda NN}$ for ${}^3_\Lambda H$. (However, $\bar{V} = V_{\Lambda p} = 6.2 \pm 0.05$ together with $V_{\Lambda NN} = 0$ for ${}^3_\Lambda H$ would give a too small value $\Delta E \approx 0.5 \pm 0.3$.)

$A > 5$ hypernuclei. We consider ${}^9_\Lambda Be$, ${}^{13}_\Lambda C$, and the well depth D for a Λ in nuclear matter, since here we focus on the overbinding problem and do not consider spin dependent effects in any detail. We use values $\bar{V}$, W obtained from the $A \leq 5$ analysis. These always fit B_Λ (${}^5_\Lambda He$), *except* for $W = 0$, i.e., in the absence of ΛNN forces when B_Λ is much too large.

${}^9_\Lambda Be$. Since ${}^8Be$ is unbound and a realistic $8N + \Lambda$ calculation is not practicable, we use a $2\alpha + \Lambda$ model[13] with the same type of s-state wave function and procedure as for ${}^3_\Lambda H$. The results we show are for the $\alpha\alpha$ potentials $V_{\alpha\alpha}$ of Ref. 14. These were obtained by fitting to accurate elastic $\alpha\alpha$ scattering data[14] and have an attractive part consistent with a microscopic folding calculation. The $\alpha\Lambda$ potentials $V_{\alpha V}$ are those obtained by the EI calculations for ${}^5_\Lambda He$ and reproduce $B_\Lambda({}^5_\Lambda He)$ except for $W = 0$. However, in addition to the ΛNN contribution $V^{(3)}_{\alpha\Lambda}$ to $V_{\alpha\Lambda}$ arising from the 6 pairs of nucleons in each α, there is also a ΛNN contribution from the 16 pairs with nucleons in a different α. By use of folding with $\tilde{V}_{\Lambda NN}$ this contribution gives an $\alpha\alpha\Lambda$ potential $V_{\alpha\alpha\Lambda}$. This is proportional to W and thus completely determined for a given $V_{\alpha\Lambda}$. The results are shown in Table I. The principal error ± 0.2 MeV is from the uncertainty in $V_{\alpha\alpha}$.[15] The rms $\alpha\alpha$ separation is 3.82 fm, considerably greater than twice the rms α radius, supporting the internal consistency of the model.

With no ΛNN forces, ${}^9_\Lambda Be$ is also grossly overbound, which is mostly a reflection of the overbinding of ${}^5_\Lambda He$ for $W = 0$. With ΛNN forces, B_Λ is close to the experimental value (6.71 ± 0.04 MeV). A weakened p-state interaction, corresponding to $\epsilon \approx 0.25$ and also consistent with $B_\Lambda({}^{13}_\Lambda C)$ and D, would reduce B_Λ by about 0.4 MeV or less. The resulting values of B_Λ are slightly low, but quite consistent with the experimental value. It is essential that $V_{\alpha\alpha\Lambda}$ be included, as otherwise B_Λ would be larger by $\langle V_{\alpha\alpha\Lambda} \rangle \approx 1$

TABLE I. Calculated energies (MeV) for ${}^9_\Lambda Be$ and ${}^{13}_\Lambda C$.

			${}^9_\Lambda Be$			${}^{13}_\Lambda C$	
$\bar{V}$	W	B_Λ	$\langle -V_{\alpha\Lambda} \rangle$	$\langle V_{\alpha\alpha\Lambda} \rangle$	B_Λ	$\langle -\tilde{V}_{\Lambda N} \rangle$	$\langle \tilde{V}_{\Lambda NN} \rangle$
6.15	0	11.6	21.0	0	22.2	34.8	0
6.12	0.011	7.1	15.9	0.7	14.1	28.6	5.2
6.15	0.014	6.9	15.8	0.85	13.4	28.4	6.1
6.2	0.018	6.7	15.8	1.0	13.1	29.3	7.6

MeV, and thus significantly too large, even though $V_{\alpha\Lambda}$ reproduces $B_\Lambda({}^5_\Lambda\text{He})$. This strongly supports predominantly Wigner-type ΛNN forces. Thus if $V_{\Lambda NN}$ were multiplied by a spin, isospin factor $(\vec{\tau}_1\cdot\vec{\tau}_2)\,(\vec{\sigma}_1\cdot\vec{\sigma}_2)$, the considerations for $A\leq 5$ inclusive of $V_{\alpha\Lambda}$ would be unchanged,[12] but for ${}^9_\Lambda\text{Be}$ there would now be no contributions from nucleons in different αs, i.e., $V_{\alpha\alpha\Lambda}\equiv 0$, and B_Λ would thus be significantly too large.

${}^{13}_\Lambda C$. We use the EI procedure: $\tilde{V}_{\Lambda N}, \tilde{V}_{\Lambda NN}$ are folded into the point nucleon density distribution ρ of ${}^{12}\text{C}$ to give a ${}^{12}\text{C}$-Λ potential from which B_Λ is calculated; ρ is obtained from the charge distribution of Ref. 16. Uncertainties in ρ give an error ± 0.1 MeV. Distortion of the ${}^{12}\text{C}$ core is estimated to give $\Delta B_\Lambda \approx +0.3$ MeV.[12] Results are shown in Table I. Again, for $W=0$, B_Λ is much too large. With ΛNN forces, B_Λ is now only 1.5–2 MeV larger than the experimental value (11.7 ± 0.1 MeV). The remaining difference is in good agreement with the reduction due to a reasonably weakened p-state potential, $\epsilon\approx 0.25$; interactions in p states contributing about 5% of the ΛN potential energy.[12]

Λ well depth D. D is calculated variationally with the Fermi hypernetted chain method as described by one of us.[3] The correlation function $f_{\Lambda N}$ is of the Pandharipande and Bethe (PB) type.[17] Results are shown in Table II. $\langle \tilde{V}_{\Lambda N}\rangle_l$ are the contributions from ΛN states of relative angular momentum l. $D^{(0)}$ is the result for the lowest order in the cluster expansion. The higher-order, many-body contributions $D-D^{(0)}$ are seen to be quite small, only a few percent. An effective interaction calculation, corresponding to $D^{(0)}$, is therefore an excellent approximation. The effective interactions are

$$\tilde{V}_{\Lambda N}=f^2_{\Lambda N}\left[V_{\Lambda N}-\frac{\hbar^2}{4}\left(\frac{M_N+M_\Lambda}{M_N M_\Lambda}\right)\nabla^2 \ln f_{\Lambda N}\right] ,$$

$$\tilde{V}_{\Lambda NN}=W(f^2_{\Lambda N}T^2_\pi)(f^2_{\Lambda N}T^2_\pi) ,$$

with $f_{\Lambda N}$ determined by nuclear matter calculations. These effective interactions are then used, as was done above for ${}^5_\Lambda\text{He}$ and ${}^{13}_\Lambda\text{C}$, in a first order calculation made with the appropriate single-particle wave functions. The second term in $\tilde{V}_{\Lambda N}$ is due to the correlation kinetic energy.

Again with $V_{\Lambda NN}=0$, D is much too large. With ΛNN forces, D is about 10–15 MeV larger than the empirical value (30 ± 3 MeV). This difference can again be accounted for by a weakened p-state interaction, $\epsilon\approx 0.25$, in agreement with the reduction needed for ${}^{13}_\Lambda\text{C}$. The results for ${}^{13}_\Lambda\text{C}$ and D again strongly support a predominantly Wigner-type ΛNN force. Thus a factor $(\vec{\tau}_1\cdot\vec{\tau}_2)(\vec{\sigma}_1\cdot\vec{\sigma}_2)$ multiplying $V_{\Lambda NN}$ would reduce $\langle V_{\Lambda NN}\rangle$ to about $\frac{1}{3}$ and $\frac{1}{5}$, for ${}^{13}_\Lambda\text{C}$ and D, respectively, of their values for a Wigner-type force.[12] The corresponding values (for $\epsilon=0$) of $B_\Lambda({}^{13}_\Lambda\text{C})\approx 17$ MeV and $D\approx 65$ MeV are then much too large, and would require an (unacceptable) strongly repulsive p-state interaction.

Other contributions to B_Λ, although clearly in need of further study, are not likely to change our conclusions. Thus reasonable ΛN tensor forces, due to kaon exchange, give only a small reduction ($\lesssim 4$ MeV) in D and (≈ 0.5 MeV) in $B_\Lambda({}^5_\Lambda\text{He})$,[18] although especially for ${}^5_\Lambda\text{He}$ better estimates are required. NN tensor forces were included in our calculations of ${}^3_\Lambda\text{H}$; they will give only a small contribution to D since they occur only through higher order terms in the Fermi hypernetted-chain calculation; also preliminary calculations for $A=4$ indicate a contribution from NN tensor forces of $\approx +0.2\pm 0.1$ MeV to B_Λ, and for ${}^5_\Lambda\text{He}$ a similar contribution may be expected. Our assumption of spin independent CSB effects is conservative in that a spin dependent CSB interaction more in accord with particle mixing theories[10] will give even more repulsive ΛNN forces. Two-pion exchange ΛNN forces[19] $V^{2\pi}_{\Lambda NN}$ gave fairly small contributions for $A\lesssim 5$ in the calculations of Ref. 2, and gave moderate contributions for D.[19] However, further calculations which also include ΛNN correlations are required for these forces.

Our values of W are comparable with those (≈ 0.003) obtained for ordinary nuclei,[7] allowing for a factor of 3 from symmetrization of V_{NNN}. Also, our corresponding values of $\langle V_{\Lambda NN}\rangle\approx 20$ MeV for D are consistent with estimates of a reduction of about 15 MeV, arising from suppression of the ΣN channel, obtained in lowest order of Brueckner-Bethe coupled channel ($\Lambda N, \Sigma N$) reaction matrix calculations.[8] Finally we remark that such suppression effects, i.e., dispersive ΛNN forces, are in fact expected to be spin dependent. Thus if $\Lambda N \leftrightarrow \Sigma N$ 1π couplings are predominant then suppression will occur mostly in the ${}^3S_1$ ΛN state.[8,20] Assuming that suppression is entirely in the triplet state and that the intermediate state potential modifications due to the medium nucleon are spin independent gives a spin dependent ΛNN potential $V_{\Lambda NN}[1+\frac{2}{3}\vec{s}_\Lambda\cdot(\vec{s}_1+\vec{s}_2)]$. *This does not change any of our considerations for $A\geq 5$ since then the spin dependent term does not contribute.* However, for ${}^3_\Lambda\text{H}$ the spin dependence reduces the effective strength to $\frac{1}{3}W$, i.e., close to the case $V_{\Lambda NN}=0$ we considered. For $A=4$ the above spin dependent ΛNN force will give a positive contribution (≈ 0.4 MeV) to the excitation energy of the 1^+ state, requiring a correspondingly reduced $V_\sigma\approx 0.2$. We are making a more detailed study, especially for $A\leq 5$, which includes both the above spin dependent ΛNN force and also contributions from $V^{2\pi}_{\Lambda NN}$.

TABLE II. Results (MeV) for the well depth.

$\bar{V}$	W	D	$D^{(0)}$	$\langle -\tilde{V}_{\Lambda N}\rangle$	$\langle -\tilde{V}_{\Lambda N}\rangle_0$	$\langle -\tilde{V}_{\Lambda N}\rangle_1$	$\langle -\tilde{V}_{\Lambda N}\rangle_2$	$\langle \tilde{V}_{\Lambda NN}\rangle$
6.15	0	71.4	70.8	70.8	48.7	20.0	2.0	0
6.12	0.011	48.4	47.0	65.5	43.8	19.5	2.0	18.5
6.15	0.014	45.0	43.3	66.2	44.3	19.7	2.0	22.6
6.20	0.018	41.5	39.7	68.4	46.1	20.1	2.0	29.3

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