Transitions to stretched states in the deformed limit

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The rate for the excitation of stretched states in a deformed model can be considerably smaller than in the single particle model. In one specific case, and in an approximation in which rotational currents are neglected, the two rates differ by a factor of $(2J+1)/2$, where J is the angular momentum of the stretched states. Thus, as has been noted by Amusa and Lawson, nuclear structure effects can explain all or a large part of the observed quenching.

[NUCLEAR STRUCTURE The calculation of the transition rates to stretched states, e.g., 6^- state in ^{28}Si, using a deformed model and comparing this with the single particle model.]

Stretched states are particle-hole states in which the particle and hole have the largest angular momenta within their respective major shells, and which furthermore couple to the largest possible total angular momentum. For example, in the $1s$-$0d$ shell, the stretched state is $(f_{7/2}d_{5/2}^{-1})^{6^-}$; in the $1p$-$0f$ shell it is $(g_{9/2}f_{7/2}^{-1})^{8^-}$.

It has been argued that these states should be rather pure, since no other one-particle–one-hole states could couple to such a large total angular momentum $J_{\rm str}=(j_{\rm p})_{\rm max}+(j_{\rm h})_{\rm max}$. It was therefore surprising to find that states with the stretched total angular momentum and parity quantum number were excited with transition rates much less than the single particle model predicted, e.g., in some cases by about a factor of 3.

Very recently it was noted by Amusa and Lawson[1] that this could be explained as a nuclear structure effect. Referring to the 6^- states in ^{28}Si, the authors noted that if, instead of taking a simple $0d_{5/2}^{12}$ configuration, one allowed the particles to be both in the $0d_{5/2}$ and $1s_{1/2}$ shells then there would be considerable fragmentation of the 6^- strength, and the hindrance of the strong transitions could be partly understood.

In this work, we wish to support this point of view by considering a deformed picture. This picture, while too extreme, will show that certain features of the experiments on stretched states can be explained—e.g., the fact that the quenching of the excitation strength in inelastic scattering is greater than what one would expect from the measured spectroscopic factors in single nucleon transfer reactions.

We take ^{28}Si as an example. If one assumes that ^{28}Si is a closed $d_{5/2}$ shell, and if one limits the particles to the $0f$-$1p$ shell, then the $J=6^-$ "stretched" states have a unique configuration $[f_{7/2}d_{5/2}^{-1}]^{6^-T}$.

In the deformed picture, we first consider the $K=6^-$ band in which the intrinsic state is $f_{7/2,K_{\rm p}=7/2}(d_{5/2}^{-1})_{K_{\rm h}=5/2}$. Values for other values of K can be expressed in terms of this "reference" case.

In the rotational model, we write the $M(6)$ operator for exciting the 6^- state as $M(6)=M(6)_{\rm INT}+M(6)_R$ where $M(6)_R$ is the term analogous to $g_R R$ for $M1$'s and arises from the rotational (convection) current. Note that the rotational term is isoscalar and hence will not contribute to the excitation of the $T=1$ states in ^{28}Si. This may be one reason for the difference in results noted by Petrovich *et al.*,[2] for the pp′ strength to $T=0$ and $T=1$ states.

In what follows we will neglect the rotational term and consider only $M(6)_{\rm INT}$. With this approximation, the expression in the deformed model for the $B(M6)$ rate can be related to the single particle value,

$$B(M6)_{K=6}\approx 2|\langle\tilde{0}[f_{7/2}d_{5/2}^{-1}]^{J=6}_{K=6}M(6)_{\mu=6}\tilde{0}\rangle|^2 ,$$

where $|\tilde{0}\rangle$ is the deformed ground state of ^{28}Si. The expression for the single particle value is

$$B(M6)_{\rm sp}=\frac{1}{2J_i+1}\sum_{M_f M_i}|\langle 0[f_{7/2}d_{5/2}^{-1}]^{J=6}_{M_F}M(6)_{\mu}0\rangle|^2$$

$$=\frac{(2J_f+1)}{(2J_i+1)}|\langle 0[f_{7/2}d_{5/2}^{-1}]^{6}_{M}M(6)_{M}0\rangle|^2 ,$$

where M can be any allowed value. We thus find

$$B(M6)_{K=6}=\frac{2}{(2J_f+1)}B(M6)_{\rm sp}=\tfrac{2}{13}B(M6)_{\rm sp} .$$

Thus, we see that whereas in the extreme single particle picture, the stretched states are unique, one easily obtains large fragmentations in the deformed picture. Amusingly, the larger the angular momentum J_f the greater is the deviation from the single particle model.

For a band with $K\neq 6$ for which the particle and hole have values $K_{\rm p}$ and $K_{\rm h}$ ($K=K_{\rm p}+K_{\rm h}$), we have

$$B(M6)_{K_{\rm p},K_{\rm h}}$$

$$=|C^{\alpha}(f_{7/2})_{K_{\rm p}}|^2|C^{B}(d_{5/2})_{K_{\rm h}}|^2\begin{pmatrix}\frac{7}{2} & \frac{5}{2} & 6\\ K_{\rm p} & K_{\rm h} & K\end{pmatrix}^2 B(M6)_{K=6} ,$$

where $|C^{\alpha}(f_{7/2})_{K_{\rm p}}|^2$ is the probability that in the Nilsson orbit α the particle is in an $f_{7/2}$ state with projection $K_{\rm p}$, etc.

We can now sum the strength. We are allowed to do the unrestricted sum

$$\sum_{\alpha}|C^{\alpha}(f_{7/2})_{K_{\rm p}}|^2=1 .$$

However, the sum $\sum_{\beta}|C^{\beta}(d_{5/2})_{K_{\rm h}}|^2$ is restricted to occupied states. Of course in the spherical limit this sum would be

unity. Because of deformations this sum will be less than one. For example, with a deformation $\delta=-0.2$ the above sum is 0.72.

This means that 26% of the single particle $M(6)$ strength in this model *disappears* because of deformation effects. The remaining 74% gets *fragmented.*

As we approach the spherical limit, i.e., replace $\sum_\beta$ by 1, we note that

$$\sum_{K_p K_h}\begin{pmatrix}\frac{7}{2} & \frac{5}{2} & 6\\ K_p & K_h & K\end{pmatrix}^2=1 \ .$$

Hence for each $K\neq 0$ we get $2/(2J_f+1)$ of the single particle strength. For $K=0$ we get $1/(2J_f+1)$ of the strength. Summing over K, then, this model gives us back, in the zero deformation limit, all of the single particle strength.

The summed strength can be written as

$$\sum B(M6)=\tfrac{1}{3}B_{\text{s.p.}}\sum_{\substack{K_h,\beta\\ \text{occupied}}}|C^\beta(d_{5/2})_{K_h}|^2 \ .$$

For K values less than 6, we will limit ourselves to the case where $K_h=\frac{5}{2}$. The reason for this is that for ^{27}Al $K_h=\frac{5}{2}$ for the $J=\frac{5}{2}^+$ ground state. If K_h is not equal to $\frac{5}{2}$ in the 6^- states, the spectroscopic strength for proton transfer to these states would be zero. The value of the square of the Clebsch-Gordan coefficients

$$\begin{pmatrix}\frac{7}{2} & \frac{5}{2} & 6\\ K_p & K_h & K\end{pmatrix}^2$$

are (for $K_h=\frac{5}{2}$)

$$\begin{aligned}K_p&=\tfrac{1}{2}\ \tfrac{7}{44}\\ &=\tfrac{3}{2}\ \tfrac{7}{22}\\ &=\tfrac{5}{2}\ \tfrac{7}{12}\\ &=\tfrac{7}{2}\ 1 \ .\end{aligned}$$

(Note that the coefficients themselves are all positive.) Thus, for a $K=5$ band, we would have (with $K_h=\frac{5}{2}$)

$$B(M6)_{K=5}=|C^\alpha(f_{7/2})_{K=5/2}|^2\tfrac{7}{12}\tfrac{2}{13}B(M6)_{\text{sp}} \ ,$$

etc.

In the work of Petrovich *et al.*,[2] one finds that the ratio of the experimental to single particle strength in inelastic proton scattering is 0.29 for the 6_1^- $T=1$ state at 14.4 MeV and 0.10 to the 6_1^- $T=0$ state at 11.6 MeV. They conclude that the difference in the ratios implies that the 6_1^- $T=0$ state has a more complicated structure than the $T=1$ state.

In our model, the $T=0$ ratio would not be inconsistent with a $K=5$ which is dominantly $[f_{7/2}d_{5/2}^{-1}]_{K=5}$—we get $\frac{7}{12}\times\frac{2}{13}\simeq 0.09$. The $K=6$ band value of $\frac{2}{13}=0.15$ cannot be ruled out either. The $K=6$ value, which is the largest for any K band, is still smaller than the $T=1$ ratio of 0.29. This indicates that band mixing is important for the $T=1$ states.

We next consider the spectroscopic strength in the reaction ^{27}Al(^{3}He,d)^{28}Si. We refer the reader to the work of Nann[3] and Snover *et al.*[4]

In a stripping reaction, the cross section is proportional to the quantity $M(J_iT_i,J_fT_f)$, which is related to the spectroscopic factor via

$$M(J_iT_i,J_fT_f)=\frac{(2J_f+1)}{(2J_i+1)}C^2S \ .$$

In the above $C^2=\frac{1}{2}$, i.e., a state of isospin 0 or 1 is a proton particle-hole 50% of the time.

As noted by Nann[3] [who defines the stripping strength $G=(2J_f+1)C^2S$], the quantity M obeys the sum rule

$$\sum M=1/(2T_i+1)\langle\text{neutron holes}\rangle_{lj}, \quad \text{for } T_f=T_i+\tfrac{1}{2} \ ,$$

$$\sum M=\langle\text{proton holes}\rangle_{lj}-1/(2T_i+1)\langle\text{neutron holes}\rangle_{lj} \ ,$$

where l_j characterizes the orbit of the transferred particle.

Thus in the single particle limit in which ^{27}Al consists of a $d_{5/2}$ proton hole, and in which the transferred particle is in the $f_{7/2}$ shell, the sum rules become

$$\sum M=4, \quad \text{for } T_f=0$$

$$\sum M=4, \quad \text{for } T_f=1 \ .$$

We thus get the expected result that the entire sum is 8, the number of available states in the $f_{7/2}$ shell.

If we now sum over M only for states of a given J_fT_f, we get the well known $(2J+1)$ rule,

$$M(J_iT_i,J_fT_f)=\left(\sum_{J_f}M\right)\frac{(2J_f+1)}{\sum_1^6(2J+1)}=4\frac{(2J_f+1)}{48} \ .$$

Thus, for $J=6$, $M=\frac{13}{12}$; alternately $S=1$. Evidently the experimental values of S^T are of the order of 0.4 of the sum rule values for both $T=0$ and $T=1$ states.

In the deformed model, we take ^{27}Al to be a proton hole with $K=\frac{5}{2}$. For $K=6$, we must deposit the proton in a $K_p=\frac{7}{2}$ orbit. There is only one way of doing this. Thus, for $K=6$,

$$\frac{(2J_f+1)}{(2J_i+1)}S=1 \ .$$

Including the factor of C^2 yields a strength of 0.5 for $T=0$ and 0.5 for $T=1$.

Note that for $K=6$, the spectroscopic strength is $\frac{6}{13}$ of the sum rule value while the $M(6)$ strength is only $\frac{2}{13}$ of the single particle value. Thus, there is no reason why the $M(6)$ fragmentation should be given by the spectroscopic fragmentation.

For other values of $K=\frac{5}{2}+K_p$,

$$S_K=|C^g_{K_p}|^2\begin{pmatrix}\frac{5}{2} & \frac{7}{2} & 6\\ \frac{5}{2} & K_p & K\end{pmatrix}^2 S_{K=6} \ .$$

Thus

$$S_6=\tfrac{6}{13}\sum S_{\text{sp}} \ \text{(as before)} \ ,$$

$$S_5=|C^g_{5/2}|^2(\tfrac{7}{12})(\tfrac{6}{13})\sum S_{\text{sp}} \ ,$$

$$S_4=|C^g_{3/2}|^2(\tfrac{7}{22})(\tfrac{6}{13})\sum S_{\text{sp}} \ ,$$

$$S_3=|C^g_{1/2}|^2(\tfrac{7}{44})(\tfrac{6}{13})\sum S_{\text{sp}} \ .$$

The spectroscopic factor data is fit best by $K=6$ for both

$T=0$ and $T=1$, although $K=5$ is marginally tolerable. Mixtures of $K=6$ and $K=5$ are certainly acceptable.

We next consider the $M1$ transition from the $J=6$, $T=1$ to the $J=6$, $T=0$ state.

As noted by Amusa and Lawson,[1] and by Snover,[4] the value of the $M1$ rate is much less than the single particle value,

$$B(M1)/B(M1)_{\rm sp}=0.19\pm0.03 \ .$$

For a $K=6$ band, we obtain

$$B(M1)_{K=6}=\begin{pmatrix}6&1&6\\6&0&6\end{pmatrix}^4 B(M1)_{\rm sp}=\left(\frac{J}{J+1}\right)^2 B(M1)_{\rm sp}$$

$$=\tfrac{36}{49}B(M1)_{\rm sp} \ .$$

Thus there is some reduction in the rate, but not nearly enough to explain the experiment.

We now consider a more general case in which the transition from

$$(f_{7/2};K_{{\rm p}_i}d_{5/2,5/2}{}^{-1})_{K_i}\rightarrow(f_{7/2},K_{{\rm p}_f}d_{5/2,5/2}{}^{-1})_{K_f} \ .$$

This is not the most general case because we are assuming $C^{\alpha}(f_{7/2})_{K_{\rm p}}=1$. We now find, with these limitations, that

$$B(M1)_{\rm def}(K_i\rightarrow K_f)$$

$$=\begin{pmatrix}6&1&6\\K_i&\Delta K&K_f\end{pmatrix}^2\begin{pmatrix}6&1&6\\6&0&6\end{pmatrix}^2|f(K_i,K_f)|^2B(M1)_{\rm sp} \ .$$

Selected values of $f(K_i,K_f)$ are

$$f(K,K)=\left[\left(2+\frac{6K_{\rm p}}{7}\right)g_{l_v}+(1+\tfrac{2}{7}K_{\rm p})g_{s_v/2}\right]\Big/\{5g_{l_v}+g_{s_v}\} \ ,$$

with $g_{l_v}=0.5$, $g_{s_v}=4.706$; note that $f(6,6)=1$,

$$f(6,5)=-f(5,6)=(\tfrac{2}{7})^{1/2}(3g_{l_v}+g_{s_v/2})/(5g_{l_v}+g_{s_v}) \ .$$

Defining the ratio $R(K_i,K_f)$ as $B(M1)_{\rm def}/B(M1)_{\rm sp}$ we find $R(6,6)=0.73$, $R(5,5)=0.36$, $R(4,4)=0.16$, $R(3,3)=0.05$, and $R(5,6)=R(6,5)=0.01$. This is to be compared with the above experimental value of 0.19 ± 0.03.

The $M1$ data can be fitted to a low K, i.e., $K=4$ or 5. Alternately, band mixing of $K=5$ and $K=6$ could also explain the data.

Looking at all the data at once, we note again that the deformed model gives predictions which are significantly different from the single particle model and which *qualitatively* go in the right direction. However, the extreme deformed picture, in which no band mixing is allowed, cannot explain all the data at once. The deformed picture, though, provides insight into why there can be greater fragmentation of the excitation strength [(p,p'), (e,e'), or (π,π')] than of the spectroscopic strength.

A possible scenario which could fit all the data, transfer, inelastic, and $M1$, is one in which the $J=6^-$, $T=0$ and $T=1$ states consist of admixtures of $K=5$ and $K=6$ bands. Not to be forgotten is the possibility that the rotational current being isoscalar could be a factor in why the inelastic excitation of $T=0$ states is different than the excitation of $T=1$ states.

We have hoped to convince the reader that the considerations in this work, while somewhat crude, nevertheless help to unravel the mystery of why the stretched state properties deviate so much from the single particle values.

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