Sum rule calculations for moments of photoeffect of the alpha particle using hyperspherical harmonics

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We use sum rules to calculate the zeroth moment of photoeffect for a central nucleon-nucleon potential with Serber exchange. We explore the consequence of using the full potential on this moment, and hence on the variation of cross section with the incident photon energy. The results are compared with recent experimental observations.

[NUCLEAR REACTIONS Alpha particle, hyperspherical harmonics, moments of photoeffect, sum rule.]

Over the last few years there have been important developments as well as continuing interest in applying the formalism of hyperspherical harmonics (HH) to the study of the alpha particle photoeffect. Levinger[1] and Elminyawi and Levinger[2] use the HH technique to obtain the alpha particle wave function. They employ sum rules to find the moments of photoeffect, and invert them to get the cross section. Dzhibuti[3] also uses the HH technique to tackle the problem of alpha particle photodisintegration, using a square-well potential. In the present work we recalculate the zeroth moment of photoeffect (σ_0) for the Volkov potential with a Serber exchange. This same potential was used in the work of Levinger and Elminyawi. However, we use all the potential multipoles, as distinct from the approach of Refs. 1 and 2, which consider only the first multipole. Hence the present work clearly demonstrates the effect of the full potential on σ_0. We use our value of σ_0 and that of σ_{-1} of Ref. 2 (rechecked by us) to get the cross section $[\sigma(E_\gamma)]$ on the incident photon energy (E_γ). Lastly we compare the results with experimental observations.

In the HH technique the ground state wave function $|i\rangle$ is expanded in the complete orthonormal basis set of HH as[4]

$$|i\rangle=\sum_k U_k(r)H_k(\Omega)\chi_k(s,t) \ . \tag{1}$$

$H_k(\Omega)$ are the HH; the hyper-radial functions $U_k(r)$ are the expansion coefficients. $\chi_k(s,t)$ are some suitable spin-isospin functions. In the present work we restrict the above expansion to the first term alone. Thus,

$$|i\rangle=U_0(r)H_0(\Omega)\chi_0(s,t) \ . \tag{1a}$$

We note that both $U_0(r)$ and $H_0(\Omega)$ are completely symmetric with respect to exchange of identical particles. For the wave function to be antisymmetric $\chi_0(s,t)$ has to be a normalized, antisymmetric function, e.g., a (4×4) Slater determinant of the individual spins and isospins of the four nucleons. $U_0(r)$ is obtained as a solution of the equation:

$$\left[-\frac{\hbar^2}{m}\left(\frac{d^2}{dr^2}-\frac{12}{r^2}\right)+(H_0(\Omega)|V|H_0(\Omega))\right]U_0(r)=EU_0(r) \ . \tag{2}$$

V is the total potential of the system. Let V be given by

$$V=\sum_{i<j}V(r_{ij})(1-x+xP_{ij}) \ . \tag{3}$$

Here $(1-x)$ is the Wigner exchange fraction and "x" is the Majorana fraction. P_{ij} is the Majorana exchange operator. We see

$$P_{ij}|i\rangle=|i\rangle \ . \tag{4}$$

The moment σ_0 is given by[5]

$$\sigma_0=\frac{2\pi^2}{\hbar c}(i|[D,[H,D]]|i) \ . \tag{5}$$

In the above expression D is the dipole operator and H is the Hamiltonian of the system. Owing to the presence of the Majorana exchange operator the potential does not commute with D. It can be shown using (4) that, for the wave function (1a), σ_0 is

$$\sigma_0=-\tfrac{2}{3}\pi^2\frac{e^2}{\hbar c}x\left(i\left|\sum_n\sum_p V(r_{\rm np})r_{\rm np}{}^2\right|i\right)+59.7\text{ MeV mb} \ . \tag{6}$$

The summation symbols "n" and "p" stand for neutrons and protons, respectively. The numerical factor 59.7 MeV mb comes from the kinetic energy part of the commutator $[H,D]$. Expressing the right-hand side of (6) in hyperspherical coordinates we obtain

$$\sigma_0=-\tfrac{8}{3}\pi^2\frac{e^2}{\hbar c}x(i|V_{12}(r\cos\theta)r^2\cos^2\theta|i)+59.7\text{ MeV mb} \ . \tag{7}$$

$V_{12}(r\cos\theta)$ is the potential between particles "1" and "2." Since the wave function $|i\rangle$ given by (1a) is completely antisymmetric, the potential between all particle pairs will be equal. We now expand $V_{12}(r\cos\theta)$ in the symmetric HH as

$$V_{12}(r\cos\theta)=\sum_k V_{2k}(r)H_{2k}(\Omega) \ . \tag{8}$$

Applying the triangular inequality obeyed by the HH to the integral over "θ" in (7) we find that only the first two terms of the expansion (8) contribute to σ_0. The major point of difference between our work and that of Ref. 2 is that in the latter only the first term of the potential expansion is taken into account. Hence they do not consider the full potential. Using the Volkov[6] potential we get

$$\sigma_0=59.7-x(-119.4+40.2)\text{ MeV mb} \ .$$

TABLE I. The energy parameter D and the expansion coefficients λ_n of the present work and Ref. 2.

	D (MeV)	λ_0 (mb)	λ_1 (mb)
Present work	0.509	2.780	−3.407
Ref. 2	0.746	1.895	−2.321

For $x=\frac{1}{2}$

$$\sigma_0 = 99.3 \text{ MeV mb} . \tag{9}$$

For the sake of completeness we outline the calculation for σ_{-1}. For the alpha particle, σ_{-1} is given by[5]

$$\sigma_{-1} = \frac{16}{9}\pi^2 \frac{e^2}{\hbar c} (i|R_c^2|i) . \tag{10}$$

Here $(i|R_c^2|i)$ is the expectation value of the mean square charge radius. It should be noted that σ_{-1} does not depend upon the potential explicitly. Using (1a) we find

$$\sigma_{-1} = \frac{2}{9}\pi^2 \frac{e^2}{\hbar c} (U_0(r)|r^2|U_0(r)) ,$$
$$\sigma_{-1} = 2.83 \text{ mb} . \tag{11}$$

We now invert the two moments to get $\sigma(E_\gamma)$ as a function of E_γ. We express $\sigma(E_\gamma)$ as

$$\frac{\sigma(E_\gamma)}{E_\gamma} = w^{1/2}\exp(-w^{1/2}) \sum_n \lambda_n S_n(w) . \tag{12}$$

Here $w=(E_\gamma - B)/D$. B is the threshold of two-body break up; $B=19.8$ MeV. D is an adjustable parameter with energy units, such that

$$\sum_n \lambda_n S_n(0) = 0 . \tag{13}$$

$S_n(w)$ are orthonormal polynomials[2] with weight functions as $w^{1/2}\exp(-w^{1/2})$.

In Table I we present the values of λ_n and D of our work and those of Ref. 2. Figure 1 shows the respective cross-section curves as well as the experimental results of Gibson[7] (circles) and Gorbunov[8] (dots). The experimental error bars are not shown because of large discrepancy between the results of various groups for the (γ,n) reaction.

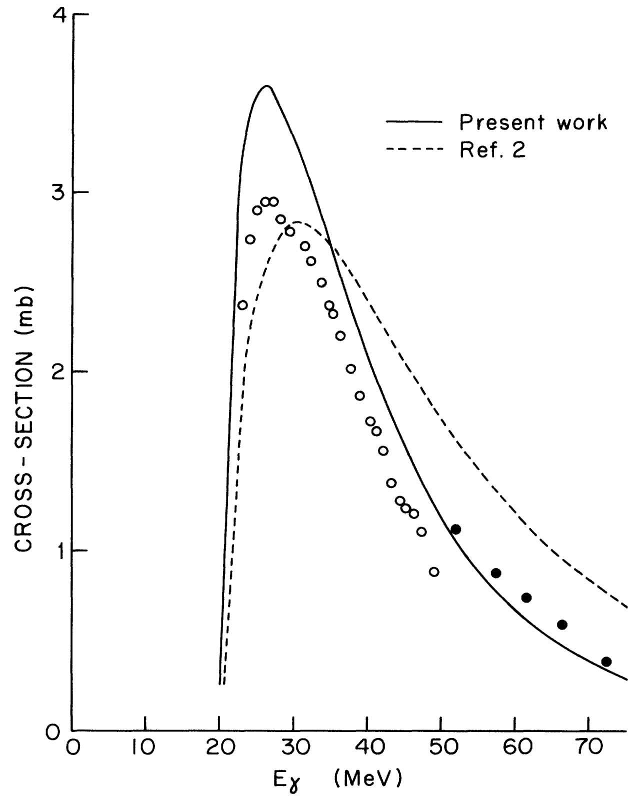

FIG. 1. Photoeffect cross section of alpha using the values of D and λ_n of Table I. Present work—solid line. Reference 2—dashed line. Experimental points—dots (Ref. 8) and circles (Ref. 7).

To conclude, we see that the integrated cross section (σ_0) of the present work [Eq. (9)] is closer to the experimental result of 103 MeV mb.[8] Elminyawi and Levinger[2] obtain 119.4 MeV mb. We also note that the position of the cross-section peak is reproduced more accurately, as also is the higher energy part. We would expect the theoretical results to improve still further if the full potential were used to calculate the higher moments of photoeffect.

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[1] J. S. Levinger, Phys. Rev. C 19, 2083 (1979).
[2] I. Elminyawi and J. S. Levinger, Phys. Rev. C 28, 82 (1983).
[3] R. I. Dzhibuti, R Ya. Kezerashvili, and K. I. Signa, Yad. Fiz. 29, 65 (1979) [Sov. J. Nucl. Phys. 29, 31 (1979)].
[4] J. L. Ballot and M. Fabre de la Ripelle, Ann. Phys. (N.Y.) 127, 62 (1980).
[5] J. S. Levinger, *Nuclear Photodisintegration* (Oxford Univ., London, 1960).
[6] A. B. Volkov, Nucl. Phys. 74, 33 (1965).
[7] B. F. Gibson, in *Proceedings of the International Conference on Few Body Problems, Eugene, Oregon, 1980,* edited by F. S. Levin (North-Holland, New York, 1981).
[8] B. C. Berman, R. Bergere, and P. Carlos, Phys. Rev. C 26, 304 (1982).