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Application of the Sp(2,*R*) model to the nuclear breathing mode

J. Broeckhove and P. Van Leuven
Dienst Teoretische en Wiskundige Natuurkunde, Ryksttniversitair Centrum Antwerpen, Universiteit Antwerpen, Antwerp, Belgium
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We apply the symplectic shell model to the breathing mode in closed-shell nuclei. The group theory of Sp(2,*R*) is used to embed the description of the collective monopole excitation in the framework of the oscillator shell model. This provides a microscopic description of the breathing mode by way of a diagonalization of the effective interaction in an appropriate collective subspace. We apply this model to closed-shell nuclei ranging from ^{56}Ni to ^{208}Pb using various Skyrme-type forces. The results confirm that a force with a nuclear matter incompressibility of about 200 MeV reproduces the experimental breathing mode energies. An evaluation of the calculated wave functions using the formalism of the Sp(2,*R*) boson picture indicates the importance of two-boson correlations that particularly affect the compression modulus. As an element in our analysis we compare Sp(2,*R*) model and random phase approximation results. This reveals the strengths and limitations of the symplectic model.

[NUCLEAR STRUCTURE Sp(2,*R*) symplectic shell model applied to breathing mode in medium and heavy closed shell nuclei.]

I. INTRODUCTION

In recent years much effort has been devoted to the study, both experimental and theoretical, of nuclear giant resonances. Among these the breathing mode has gained special attention because of the connection between its excitation energy and the incompressibility of the nucleus. The breathing mode has been established experimentally in nuclei beyond mass $A \sim 60$. For a discussion on methods of measurement and analysis used in giant resonance experiments, we refer to the review paper (Ref. 1).

Several models, of varying degree of sophistication, have been applied to the description of the giant monopole resonance (for an overview see Refs. 2 and 3). In the macroscopic collective theories, the breathing mode is pictured as a compressional vibration of the nuclear density. The dynamics of this motion is described by treating the nucleus, e.g., as a fluid[4] or as an elastic medium.[5] In contrast, the random phase approximation (RPA) and related theories[3] provide a microscopic treatment in which the many-particle structure of the nucleus is respected. There the breathing mode is to be identified among the excited nuclear states. The signature of its collectivity is a coherent superposition of particle-hole excitations with strong monopole transition from the ground state.

In this paper we intend to apply the symplectic model to the breathing mode. Symplectic models have been used previously in the study of isoscalar giant resonances in the lighter nuclei.[6–8] These models involve all A particles and rigorously respect the Pauli principle, but assume that the nucleus is subject only to collective excitations. This assumption leads to the construction of a space of many-particle configurations that all have the collective signature. Subsequently the nuclear interaction is diagonalized in that space, yielding the collective excited states. Thus the symplectic model occupies an intermediate position in the general scheme of theories: it is a microscopic collective theory.

The implementation of the symplectic model requires essentially the construction of the appropriate model space. This task is accomplished using algebraic and group theoretic techniques involving the symplectic group Sp(2,*R*). The basic principles and mathematical technology are discussed in Sec. II. In Sec. III we present the results for the breathing mode energy obtained with the Sp(2,*R*) model for the medium and heavy closed-shell nuclei. We compare these results to the available experimental data, and also evaluate their dependence on the characteristics of the effective interaction. In this respect we want to stress that we do not employ potentials simulated by group generators but Skyrme-type effective interactions that are commonly used for the heavy spherical nuclei. In Sec. IV we consider the Sp(2,*R*) boson picture, in which the breathing mode is viewed in terms of a bosonic excitation. The relevance of this concept is demonstrated by an analysis of our results. Section V concerns the comparison of breathing mode energies and incompressibilities between the Sp(2,*R*) model and the Tamm-Dancoff approximations and the random-phase approximation theories. The main conclusions to be drawn from the work in this paper are presented in Sec. VI.

II. THE Sp(2,*R*) MODEL

The symplectic algebra sp(6,*R*) is spanned by the following 21 operators (μ,ν indicate spatial directions, j stands for the particle index, and ρ and π stand for position and momentum)

$$Q_{\mu\nu}=\sum_j \rho_j(\mu)\rho_j(\nu) \quad \text{(monopole and quadrupole tensor)}\ , \tag{1a}$$

$$K_{\mu\nu}=\sum_j \pi_j(\mu)\pi_j(\nu) \quad \text{(kinetic energy tensor)}\ , \tag{1b}$$

$$L_{\mu\nu}=\sum_j \rho_j(\mu)\pi(\nu)-\pi_j(\mu)\rho_j(\nu) \quad \text{(angular momenta)}\ , \tag{1c}$$

$$S_{\mu\nu}=\sum_j \rho_j(\mu)\pi_j(\nu)+\pi_j(\mu)\rho_j(\nu) \quad \text{(vibrational momenta)}\ . \tag{1d}$$

The algebra includes all the essential observables for monopole and quadrupole collective vibrations, as well as for rigid and irrotational flow rotations or any combination of these nuclear motions. Thus the use of sp(6,R) allows one to derive microscopic realizations of the familiar collective models.[9]

The set of operators (1) can be transformed into an equivalent one by introducing oscillator creation and annihilation operators $\alpha_j^\dagger(\nu),\alpha_j(\nu)$

$$\begin{aligned} A_{\mu\nu}^\dagger&=\tfrac{1}{2}\sum_j \alpha_j^\dagger(\mu)\alpha_j^\dagger(\nu)\ ,\\ A_{\mu\nu}&=\tfrac{1}{2}\sum_j \alpha_j(\mu)\alpha_j(\nu)\ ,\\ C_{\mu\nu}&=\tfrac{1}{4}\sum_j \alpha_j^\dagger(\mu)\alpha_j(\nu)+\alpha_j(\nu)\alpha_j^\dagger(\mu)\ . \end{aligned} \tag{2}$$

This form of the symplectic algebra indicates that the description of collective motion can be embedded into the oscillator shell model. Action of the operators on the ground state establishes the shell-model configurations necessary for the development of collective vibrational and rotational motions.[9] Hence the symplectic model can be viewed as a means of generating the appropriate basis states for the diagonalization of the Hamiltonian.

In order not to have spurious center-of-mass components in the symplectic basis, one uses relative position and momentum operators in (1)

$$\begin{aligned} \vec{\rho}_j&=\vec{\mathrm{x}}_j-\vec{\mathrm{R}},\quad \vec{\mathrm{R}}=\frac{1}{A}\sum \vec{\mathrm{x}}_j,\\ \vec{\pi}_j&=\vec{\mathrm{p}}_j-\frac{1}{A}\vec{\mathrm{P}},\quad \vec{\mathrm{P}}=\sum \vec{\mathrm{p}}_j, \end{aligned} \tag{3}$$

and similarly defined creation and annihilation operators in (2). Thus if an oscillator shell-model wave function is not spurious,[10] i.e.,

$$\Psi(\vec{\mathrm{x}}_1,\vec{\mathrm{x}}_2,\ldots,)=\chi(\vec{\mathrm{R}})\Phi(\vec{\rho}_1,\vec{\rho}_2,\ldots,)\ , \tag{4}$$

the action of the symplectic operators on it will affect only the internal part and the resulting wave function will also be nonspurious. Because any state in the lowest oscillator shell configuration is nonspurious,[10] the symplectic basis that is built on it will not have spurious center-of-mass components.

At present we are not concerned with the full range of collective motions, but we focus our attention on the breathing mode. The breathing mode or isoscalar giant monopole resonance is an isotropic compressional vibration of the nucleus. It is evident from geometrical considerations that in spherical nuclei this motion is decoupled from the quadrupolar vibrations and the rotations. The only relevant variable is the nuclear radius which expands and contracts according to the dynamics regulated by the compressibility. Therefore the sp(2,R) subalgebra, consisting of the monopole operators

$$M=\tfrac{1}{2}\sum_\mu Q_{\mu\mu} \quad \text{(isoscalar monopole operator)}\ , \tag{5a}$$

$$K=\tfrac{1}{2}\sum_\mu K_{\mu\mu} \quad \text{(kinetic energy)}\ , \tag{5b}$$

$$D=\tfrac{1}{2}\sum_\mu S_{\mu\mu} \quad \text{(scaling operator)}\ , \tag{5c}$$

engenders a genuine physical submodel for the isoscalar monopole collective excitation. In this model one diagonalizes the effective interaction in the basis generated by the sp(2,R) operators only. To be precise, one chooses as a model space the Sp(2,R) subspace of the Sp(6,R) model that is related to the monopole vibrations. In the Sp(2,R) model space one finds the scaled states

$$\begin{aligned} &\exp(i\theta D)\Psi_0(x_1,x_2,\ldots,)\\ &\qquad=\chi(R)[e^{-(3/2)(A-1)\theta}\Phi_0(e^{-\theta}\vec{\rho}_1,e^{-\theta}\vec{\rho}_2,\ldots,)] \end{aligned} \tag{6}$$

in which the shell-model ground state Ψ_0 is compressed ($\theta<0$) or expanded ($\theta>0$). These states embody the breathing of the nucleus in a microscopic wave function.

The construction of the Sp(2,R) model space, which in group-theoretic terms is an irreducible representation space, proceeds along familiar lines.[11] Within the algebra one has a raising, lowering, and weight operator, that are, respectively,

$$A^\dagger=\sum_\mu A_{\mu\mu}^\dagger\ ,\quad A=\sum_\mu A_{\mu\mu}\ , \tag{7}$$

and

$$C=\sum_\mu C_{\mu\mu}\ .$$

They are combinations of the M, K, and D in (5). A lowest weight state is an eigenstate of C that is also annihilated by A, i.e.,

$$C\,|0\rangle=k\,|0\rangle \quad \text{and} \quad A\,|0\rangle=0\ . \tag{8}$$

Any such state supports an irreducible representation whose basis is generated by the action of $A^\dagger$ on the lowest weight

$$|n\rangle=\frac{1}{n!}\begin{bmatrix} n+2k-1\\ n\end{bmatrix}^{-1/2}(A^\dagger)^n|0\rangle\ . \tag{9}$$

The eigenvalue k is the label for the representation. The action of the algebra operators in this basis is determined by

$$A^\dagger |n\rangle = [(2k+n)(n+1)]^{1/2} |n+1\rangle ,$$
$$A |n\rangle = [(2k+n-1)n]^{1/2} |n-1\rangle , \quad (10)$$
$$C |n\rangle = (k+n) |n\rangle .$$

Because (7) can be expressed in oscillator creation-annihilation operators and C is the oscillator Hamiltonian, the above construction is immediately applicable within the oscillator shell model. It is evident that any state in the lowest configuration is a lowest weight and that the symplectic label k is the oscillator energy minus the center-of-mass contribution. For closed shell nuclei the ground state configuration is nondegenerate, and hence the representation is uniquely determined (cf. Table I). Thus we have in these nuclei a well-defined, highly structured model space suitable for our purposes.

To complete our model we have to choose a nuclear Hamiltonian. In the calculations presented here, involving doubly closed-shell nuclei ranging from calcium to lead, we use a number of Skyrme interactions

$$H = \frac{1}{2m} \sum_j \overline{\pi}_j^2 + V_{\text{Skyrme}} + V_{\text{Coulomb}} . \quad (11)$$

All of these forces reproduce the ground state properties accurately for the closed nuclei and therefore we shall dispense with a discussion of these properties, focusing instead on the breathing mode. As we expect the nuclear matter incompressibility K_{nm} of the interaction to be a critical parameter, we have chosen a set of interactions with K_{nm} ranging from 200 to 365 MeV, namely SII,[12] SIII to SVI,[13] MDI,[14] Sk_a,[15] and SkM.[16] For the computation the Hamiltonian matrix elements $\langle m | H | n \rangle$ we have used the generating function method discussed in Ref. 17. The method is based in the following relation:

$$(\cosh\theta)^{2k} \exp(i\theta D) |0\rangle = \sum_n \begin{bmatrix} n+2k-1 \\ n \end{bmatrix} \alpha^n |n\rangle = |\alpha\rangle , \quad (12)$$

where $\alpha = \tanh\theta$. It shows that the scaled state $|\alpha\rangle$ is a superposition of all symplectic basis states $|n\rangle$ with an analytical parameter dependence. Consequently, the matrix element $\langle \alpha | H | \beta \rangle$ is a generating function for the $\langle m | H | n \rangle$, i.e., the latter can be found by taking the appropriate derivatives with respect to α and β of $\langle \alpha | H | \beta \rangle$. Previous calculations on light nuclei with the Brink-Boeker and Skyrme interactions have shown that the method provides enough flexibility to be used with various effective interactions.[8] For the technical details we refer to Ref. 18. The need for using this somewhat unfamiliar method can be clearly felt on contemplating formula (9). It reveals that the symplectic states become extremely complex owing to the large number of excited configurations involved. Thus one needs to avoid using these wave functions explicitly and instead evaluate the matrix element directly as we do with the generating function method.

In the calculations reported in this paper we have used no approximations other than those involved in the very definition of the model, i.e., the choice of model space and the choice of effective interaction. In particular, the Coulomb repulsion has been included and taken into account exactly, as have the effects owing to the center-of-mass in the calculation of the kinetic energy. The latter is possible because the kinetic energy is a sp(2,R) operator and its matrix elements between symplectic states can be determined from formulae (10).

III. RESULTS FOR THE BREATHING MODE ENERGY

For each nucleus and force, the oscillator width parameter b_0 is fixed by taking the value that minimizes $\langle 0 | H | 0 \rangle$ as a function of b. Having determined b_0, we diagonalize the Hamiltonian in the basis $|n\rangle$, $n = 1, \ldots, N$, for increasing N until convergence is attained. We find that in all cases the energies, monopole transitions, incompressibilities, etc., reach stable values at N about 5 to 10. Unless stated otherwise, results quoted hereafter will be those of the $N = 10$ calculation.

As we explained in the previous sections, the model requires a diagonalization of the effective interaction in the Sp(2,R) basis and interprets the first excited state of the ensuing spectrum as a microscopic collective state for the breathing mode or monopole resonance. The corresponding excitation energy is the breathing mode energy. It is presented in Table II for all nuclei and forces considered in this paper.

The experimental data on the monopole resonance are obtained mainly in inelastic scattering of hadrons.[1] A rather exhaustive compilation of these data for the doubly closed shell nuclei can be found in Table III. One observes that the isotope ^{58}Ni appears in the table instead of ^{56}Ni. The doubly closed shell nucleus ^{56}Ni which we include in our calculations is β unstable which makes it impossible to use as a scattering target. However, giant reso-

TABLE I. Ground state configuration and symplectic label of doubly-closed shell nuclei.

Nucleus	Highest occupied p-orbital	Highest occupied n-orbital	Symplectic label k
$^{40}_{20}\text{Ca}_{20}$	$(0d)_{3/2}$	$(0d)_{3/2}$	237/4
$^{56}_{28}\text{Ni}_{28}$	$(0f)_{7/2}$	$(0f)_{7/2}$	381/4
$^{90}_{40}\text{Zr}_{50}$	$(1p)_{1/2}$	$(0g)_{9/2}$	707/4
$^{140}_{58}\text{Ce}_{82}$	$(0g)_{7/2}$	$(2s)_{1/2}$	1281/4
$^{208}_{82}\text{Pb}_{126}$	$(2s)_{1/2}$	$(2p)_{1/2}$	2169/4

TABLE II. Calculated breathing mode energies in MeV.

	^{40}Ca	^{56}Ni	^{90}Zr	^{140}Ce	^{208}Pb
MDI	21.8	20.0	17.5	15.2	13.2
SkM	21.7	20.6	17.8	15.5	13.5
Sk_a	23.5	22.0	19.2	16.7	14.6
SV	25.3	23.5	20.7	17.9	15.6
SIV	26.0	24.4	21.3	18.5	16.2
SII	26.5	24.7	21.7	18.9	16.5
SIII	27.0	25.7	22.2	19.5	17.0
SVI	27.3	26.0	22.5	19.8	17.3

TABLE III. Compilation of giant monopole resonance (GMR) data from inelastic hadron scattering. Peak energy E_x, full width at half maximum Γ, and monopole transition strength in % EWSR (energy weighted sum rule).

	E_x (MeV)	Γ (MeV)	% EWSR	Ref.
^{208}Pb	13.2	2.5		20
	13.5±0.3	2.8±0.2		21
	13.2±0.3	2.8±0.3	92±12	22
	13.7±0.4	3.0±0.5	105±20	23
	13.9±0.3	2.5±0.6	110±25	24,25
	13.9±0.4	3.2±0.4	100±20	26
	13.7±0.4	3.0±0.5	90±20	27
	13.4±0.5	3.0±0.5	90±25	28
^{140}Ce	14.8±0.2	3.0±0.2	53±10	22
^{90}Zr	17	4		20
	17.2±0.5	4.3±0.3		21
	16.4±0.3	3.6±0.3	60	29
	17.5±0.5	3.0±0.5	60±25	28
	16.2±0.5	3.5±0.3	90±25	30
	17.0±0.5	2.9±0.5	80±20	26
^{58}Ni	19.8±0.5	3.5±0.5	30±10	28
	17.1±0.2	2.5±0.2	10±2	31
	16.5±0.5	6		21
	20.0±0.5	3.0±0.5	40±10	26
^{40}Ca	20	3.5		20
	20.6		17	32
	21		12	33,34
		Evidence for monopole strength		21,28
		of order 10% around 20 MeV		31,35

nance properties vary slowly with mass number[1] and therefore it makes sense to compare calculations on the monopole resonance in ^{56}Ni with data for ^{58}Ni.

A first comparison of the above tables points to the following. One knows from experiment, especially from the data on width and sum rule strength, that continuum effects play a major part in the monopole resonance. As in most models other than the continuum RPA, we have neglected the continuum and given a bound state description for the collective degree of freedom associated with the breathing mode. This abstraction of the physical phenomenon implies that only certain quantities, e.g., energy or compression modulus, can rightfully be considered in the model. Other quantities, e.g., width or transition strength, simply are not defined or cannot be sensibly calculated because they have a dominant continuum contribution. We will therefore restrict our discussion to the breathing mode energy and to the incompressibility.

If one compares the experimental and calculated values for the energy, one finds that the results for the interactions MDI and SkM are in good agreement with experiment whereas the other forces give energies that are significantly above the experimental value. This conclusion holds for all nuclei, regardless of mass number A. The energy varies smoothly with A and for heavy nuclei all forces yield an $E_x \sim A^{-1/3}$ dependence. Experimental data indicate that $E_x \sim 80A^{-1/3}$ for heavy nuclei,[1] a result that is reproduced by the interactions SkM and MDI.

In Table II we have ordered the effective interactions with increasing nuclear matter incompressibility K_{nm}. One notices a pronounced correlation between K_{nm} and the breathing mode energy. This is to be expected, for K_{nm} is a measure of the energy required to perform an infinitesimal compression of nuclear matter. We can analyze the relation between K_{nm}, the nuclear matter incompressibility of the interaction, and E_x, the breathing mode energy calculated with the interaction in a quantitative way as in Fig. 1. One finds that for each nucleus E_x is a smooth almost linear function of K_{nm} with a slope that depends slightly on the nucleus. This allows us to draw the following conclusion. The forces that we have used differ from one another in many respects, cf. the respective references.[12–16] Nevertheless only the variation in K_{nm} apparently determines the variation in E_x for each nucleus. This implies that in order to fit the breathing mode energies to the experimental values, it suffices to adjust the nuclear matter incompressibility of the force. Moreover by comparing the calculated E_x with the experimental data in Table III, we conclude that K_{nm} should be 215±25 MeV. This is in perfect agreement with the value obtained in Refs. 2, 19, and 32. Of the various forces that we have used, only the MDI and SkM fall in this range.

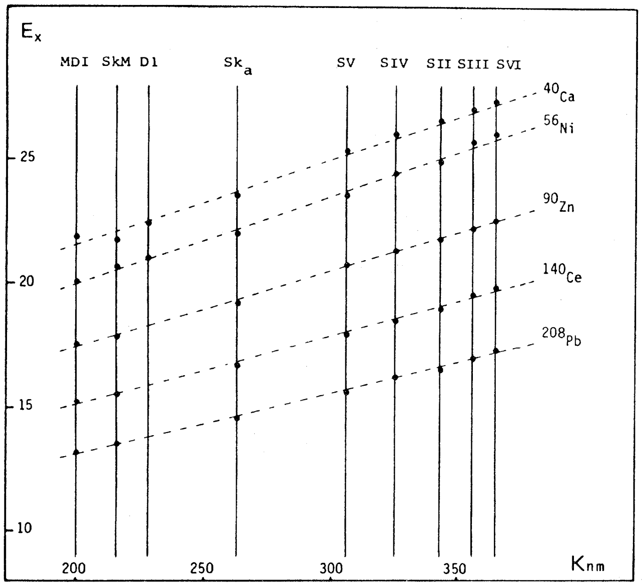

FIG. 1. Calculated breathing mode energy versus nuclear matter incompressibility, $K_{\rm nm}$, both in MeV.

IV. THE Sp(2,*R*) BOSON PICTURE

From the exposition of the model, one knows that the symplectic basis is a band of excited states, generated by the action of the raising operator in the monopole algebra. Within each $2n\hbar\omega$ oscillator shell lies precisely one member $|n\rangle$ of the band, and the contributions of the various configurations to $|n\rangle$ are weighted in a way that ensures its collective, monopole nature. The interrelation between the members of the band can be elucidated by introducing the boson picture of the sp(2,*R*) algebra.[36] One does this through a Holstein-Primakoff construction, showing essentially that the sp(2,*R*) operators can be rewritten as follows:

$$A^\dagger = S^\dagger(2k+S^\dagger S)^{1/2} ,\quad A=(2k+S^\dagger S)^{1/2}S ,\quad C=k+S^\dagger S , \tag{13}$$

where $S^\dagger, S$ are boson creation and annihilation operators, $[S^\dagger,S]=1$.

From these expressions and matrix elements (10) one deduces that S and $S^\dagger$ act on the symplectic states in the canonical fashion

$$S^\dagger|n\rangle=(n+1)^{1/2}|n+1\rangle ,\quad S|n\rangle=(n)^{1/2}|n-1\rangle . \tag{14}$$

As a consequence one has for the symplectic states that

$$S|0\rangle=0,\quad |n\rangle=(n!)^{-1/2}(S)^n|0\rangle , \tag{15}$$

i.e., the lowest weight $|0\rangle$ is the boson vacuum and $|n\rangle$ are the n-boson states. Thus, the band of symplectic states constitutes a state space for a bosonic excitation that is the elementary, unperturbed quantum for the breathing mode.

At present we aim to analyze, using the boson picture, the wave functions $|\Phi_0\rangle$ and $|\Phi_1\rangle$ of ground state and breathing modes that we have calculated. In the simplest approximation, where we truncate the symplectic model space at $N=1$, one obtains $|\Phi_0\rangle=|0\rangle$ and $|\Phi_1\rangle=|1\rangle$. Indeed, our choice of oscillator width parameter b implies that

$$\left[\left[\frac{\partial}{\partial b}\right]\langle 0|H|0\rangle\right]_{b=b_0}=2(2k)^{1/2}\langle 0|H|1\rangle=0 , \tag{16}$$

i.e., the Hamiltonian does not couple the vacuum and one-boson state. When the basis is extended beyond $N=1$, other boson components are mixed into $|\Phi_0\rangle$ and $|\Phi_1\rangle$. In Table V we give the overlap of $|\Phi_1\rangle$ with the one-boson state, in the case of the interaction SIII. The wave function is clearly dominated by its one-boson component, a feature that turns out to be common to all forces: admixture of other boson states in $|\Phi_1\rangle$ is always less than three percent. For the ground state a similar analysis reveals that admixtures other than the vacuum are always less than one percent. Thus the unperturbed picture in which the ground state is the vacuum, and the breathing mode is the one-boson state, is a very good approximation.

The correlations introduced by the various other boson states evidently represent but small fractions of the wave functions of $|\Phi_0\rangle$ and $|\Phi_1\rangle$. Nevertheless these components can have disproportionately large effects owing to the sensitivity of certain operators to their presence. We consider the nuclear compression modulus, more precisely the scaling incompressibility defined by

$$K=\left[\left[\frac{d}{d\theta}\right]^2 E(\theta)/A\right]_{\theta=0} , \tag{17}$$

where

$$E(\theta)=\langle \exp(i\theta D)\Phi_0|H|\exp(i\theta D)\Phi_0\rangle \tag{18}$$

is the energy calculated with a scaled ground-state wave function. The incompressibility is a measure of the stiffness of the nucleus against an isotropic compression or expansion of its volume. In Table IV we present the K calculated in the basis $|n\rangle$, $n=0,1,\ldots,N$, for increasing N, showing that the inclusion of the two-boson state in $|\Phi_0\rangle$ is critical in determining the incompressibility. We may

TABLE IV. Scaling incompressibility K (MeV) and factors $\bar{C}$ and $\bar{E}$ (MeV) of the symplectic formula (23) for K.

N	1	2	3	6	10
	Nucleus: ^{208}Pb			Force: Sk$_a$	
K	153	165	165	165	165
$\bar{C}$	10.4	11.3	11.3	11.3	11.3
$\bar{E}$	14.6	14.6	14.6	14.6	14.6
	Nucleus: ^{208}Pb			Force: SIII	
K	183	226	226	227	228
$\bar{C}$	10.4	13.2	13.2	13.3	13.3
$\bar{E}$	17.6	17.1	17.1	17.1	17.1

understand this as follows. A direct calculation, starting from (17), leads to an expression

$$K=\frac{2}{A}\langle D\Phi_0 \mid H-E_0 \mid D\Phi_0\rangle \tag{19}$$

that suggests we decompose K in two factors

$$K=\bar{C}\times\bar{E}\begin{cases}\bar{C}=\frac{2}{A}\langle\Phi_0 \mid D^2 \mid \Phi_0\rangle,\\ \bar{E}=\frac{\langle D\Phi_0 \mid H-E_0 \mid D\Phi_0\rangle}{\langle D\Phi_0 \mid D\Phi_0\rangle}\ .\end{cases} \tag{20}$$

The factor $\bar{C}$ depends only on the structure of the ground state wave function, whereas $\bar{E}$ is to be interpreted as an excitation energy. From the results in Table IV it is obvious that $\bar{E}$ is insensitive to the details in $\mid\Phi_0\rangle$ and K depends on the two-boson correlation solely through $\bar{C}$. From the previous paragraph we know that

$$\mid\Phi_0\rangle\cong\mid 0\rangle+\sum_{n\geq 1}\epsilon_n \mid n\rangle \tag{21}$$

is a correct representation of $\mid\Phi_0\rangle$, with all ϵ_n an order of magnitude smaller than 1. If one inserts this in the expression for $\bar{C}$, and uses the matrix elements for the symplectic operators, one obtains to first order

$$\bar{C}=\frac{4k}{A}(1-2\sqrt{2}\epsilon_2)=\frac{4k}{A}(1-2.83\epsilon_2)\ . \tag{22}$$

Thus even a small two-boson admixture $\mid\langle\Phi_0 \mid 2\rangle\mid^2 \simeq 0.01$ yields a 30% change in $\bar{C}$ and hence in the incompressibility. In our calculations we do indeed find effects of this order: changes are smallest for the interaction SkM (of the order of a few percent) and largest for the interaction SVI (of order 30%), irrespective of the nucleus.

The existence of correlations decisively influences not only certain quantities such as the compression modulus, but also tends to obscure the picture that we evoked earlier in this section, i.e., of $\mid\Phi_1\rangle$ as a one-boson state built on a vacuum $\mid\Phi_0\rangle$. The doorway construction[37] allows an alternative interpretation of the relation between ground state and breathing mode. For the breathing mode, the doorway $\mid\Phi_0\rangle$ is defined as the state which takes up all the isoscalar monopole transition strength. It is constructed by the action of the monopole operator M on the ground state

$$\mid\Phi_D\rangle=\frac{1}{\sqrt{\alpha}}M'\mid\Phi_0\rangle\ , \tag{23}$$

where

$$M'=M-\langle\Phi_0 \mid M \mid \Phi_0\rangle,$$
$$\alpha=\langle\Phi_0 \mid M'^2 \mid \Phi_0\rangle. \tag{24}$$

In our calculations the breathing mode almost completely exhausts the monopole sum rules (see also the next section). From this, one deduces that $\mid\Phi_1\rangle$ should be equal to the doorway state. The overlaps in Table V demonstrate that this is indeed so. Moreover they again indicate the effect of correlations. If the ground state were to be the unperturbed vacuum $\mid 0\rangle$, then $\mid\Phi_D\rangle$ would be equal to the one-boson state. The enhancement of $\mid\langle\Phi_1 \mid \Phi_D\rangle\mid^2$ over $\mid\langle\Phi_1 \mid 1\rangle\mid^2$ is owing to the correlation of $\mid\Phi_0\rangle$ that are propagated into the excited state by the action of the operator in (23). The doorway construction incorporates the correlation and thus provides an improved description of the breathing mode.

TABLE V. Overlaps of breathing mode with the one boson and with the doorway state. Interaction SIII.

	^{40}Ca	^{56}Ni	^{90}Zr	^{140}Ce	^{208}Pb
$\mid\langle\Phi_1 \mid 1\rangle\mid^2$	0.970	0.974	0.973	0.976	0.979
$\mid\langle\Phi_1 \mid \Phi_D\rangle\mid^2$	0.992	0.995	0.997	0.998	0.999

V. COMPARISON WITH THE RPA

The Tamm-Dancoff approximation (TDA) and the random phase approximation (RPA) are considered among the most complete and detailed theories applicable to nuclear collective motion. It therefore is evident that we should measure our results against those obtained with these methods even if the Sp(2,R) model does not purport to have the same depth of description. Indeed, the simplicity of the model ensues from restrictive assumptions concerning the nature of the breathing mode. These concern mainly the use of oscillator one-particle orbitals and the built in collectivity of the mode of excitation, i.e., the excited configurations allowed in the model space. Our aim is to show in what respect and to what extent one still obtains a valid description of the breathing mode.

We consider first the isoscalar monopole transition strength. From the previous section we know that Φ_1 and Φ_D are essentially the same. Thus we may conclude that in the symplectic model the breathing mode completely exhausts the monopole sum rule. This is in fact corroborated by the explicit computation of the transition matrix elements. In the TDA and RPA calculations[38] on the monopole resonance, the situation is slightly different. There one obtains a cluster of excited states, of which one takes up almost all of the monopole transition strength, while the total strength is concentrated in an interval of a few MeV. However, neither result is particularly meaningful because continuum effects, essential for a proper description of the transition strength, have been neglected in both calculations.

In Table VI we have presented the scaling incompressibilities obtained in the Sp(2,R) model and in the RPA.[38] They are in good agreement with one another, with, however, a tendency for the Sp(2,R) value to be lower by about 5%. This is an indication of the existence of yet different correlations than the one we discussed in the previous section, correlations that are not included in the symplectic model space.

In the RPA and TDA the breathing mode energy is defined as the average excitation energy of the cluster of excited states using their monopole transition strength states as a weight. This actually means that the breathing mode energy is taken to be the excitation energy of the doorway

TABLE VI. Scaling incompressibility in MeV from the Sp(2,*R*) (upper row) and RPA (lower row) calculations.

	^{40}Ca	^{90}Zr	^{208}Pb
Sk$_a$	160	169	165
	165	175	174
SIV	195	208	204
	203	218	217
SIII	215	230	228
	227	242	242

TABLE VII. Comparison of energies (MeV) of the various calculations.

		$\bar{E}_{\text{TDA}}$	Sp(2,*R*)$N=1$	$\bar{E}_{\text{RPA}}$	Sp(2,*R*)
	Sk$_a$	24.1	24.0	23.0	23.5
^{40}Ca	SIV	27.3	27.1	25.3	25.9
	SIII	28.2	28.6	26.3	27.0
	Sk$_a$	19.7	19.5	19.1	19.2
^{90}Zr	SIV	22.2	21.9	21.2	21.3
	SIII	22.9	23.1	22.1	22.2
	Sk$_a$	15.0	14.8	14.7	14.6
^{208}Pb	SIV	16.9	16.5	16.5	16.2
	SIII	17.5	17.6	17.2	17.1

state constructed on the TDA or RPA ground states. The same definition is in effect applicable in the symplectic model for we know that $\Phi_1 \sim \Phi_D$; we can therefore rightfully compare the energies of the various calculations. Of course, one should not expect them to be exactly equal since the ground states differ. In the TDA one has a Hartree-Fock ground state and this should be compared with the $N=1$ restricted Sp(2,*R*) which has the oscillator shell model ground state. In the RPA the ground state is the Hartree-Fock state with particle-hole correlations and this should be compared with the full Sp(2,*R*) which has the shell model ground state plus *n*-boson correlations. We see in Table VII that the systematics associated with the effect of correlations is the same in the calculation using oscillator orbitals [Sp(2,*R*)] and Hartree-Fock orbitals (TDA,RPA). The overall agreement of the results is very good and justifies *a posteriori* the assumptions introduced in the Sp(2,*R*) model inasmuch as one is concerned with determining the excitation energy. It also confirms a basic feature of spectroscopic calculations, namely, that the excitation energy depends more on the relation between ground and excited states than on the detailed structure of each state separately. Because of this the symplectic model, although it is much simpler than the RPA, provides a good description of the breathing mode energy.

VI. CONCLUSION

The Sp(2,*R*) model has been applied to a number of doubly-closed shell nuclei throughout the mass table with a variety of Skyrme-type interactions with nuclear matter incompressibilities ranging from 200 to 370 MeV. We have presented results for the breathing mode wave function and energy and for the incompressibility in these nuclei.

The symplectic model has been implemented in the framework of the shell model by an explicit construction of a collective subspace. It can be introduced by referring to the physical picture of dilated wave functions. The space generated by all dilations of the shell-model ground state proves to be endowed with the mathematical structure of an Sp(2,*R*) representation space. It has been an essential part of this work to exploit the group theoretic properties for the calculation of matrix elements.

Analysis of the results for the breathing mode energy indicates that for all forces the $A^{-1/3}$ dependence is well reproduced. Moreover it is found that the energy depends exclusively on the nuclear matter incompressibility of the interactions. Comparison with the experimental energies restricts the range of incompressibility to 215 ± 25 MeV. The analysis of the wave functions confirms the validity of the one-boson approximation for the breathing mode state. However, the doorway construction provides a better approximation because it incorporates two-boson correlations in both ground and excited states. These correlations are particularly significant for the incompressibility in the finite nuclei, where they account for about twenty percent of the total value.

The agreement between the above results and those of the TDA and RPA is striking. We conclude therefore that the symplectic model is a worthwhile alternative to the RPA as it yields these results with much less computational effort. On the conceptual level, the symplectic model can be considered as an intermediary between the macroscopic collective models and the RPA, as it incorporates the collective description in the microscopic formalism.

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