Properties of elastic alpha scattering on target nuclei in the mass region $A=14$–48

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Elastic α scattering on the target nuclei ^{14}C, ^{28}Si, and ^{32}S has been studied microscopically using the framework of the generator coordinate method. Combined with the results of previous microscopic investigations of elastic α scattering on the oxygen and calcium isotopes the present studies allow to set up some systematics which may lead to useful predictions concerning the general behavior of elastic α scattering on target nuclei in the mass range $A\simeq 14$–48. It is pointed out that the pure elastic phase shifts (correspondingly, the real part of the optical potential) cannot serve as an explanation for the different cross section behavior at backangles observed for elastic α scattering in this mass region. It is qualitatively shown that these differences are due to absorptive effects.

[NUCLEAR REACTIONS Generator coordinate method, study of α-^{14}C, α-^{28}Si, and α-^{32}S, comparison of elastic α scattering on target nuclei in the mass region $A=14$–48 at low energies, backangle anomaly, molecular resonances.]

I. INTRODUCTION

In the present paper we report on microscopic studies of the α-^{14}C, α-^{28}Si, and α-^{32}S scattering systems using the framework of the generator coordinate method (GCM). These first microscopic investigations on α scattering from nuclei far aside from a main shell closure may finally fill up the gap between the study of oxygen[1] and the calcium isotopes.[2] Together with these previous results they now give a rather complete overview on the elastic α-scattering processes in the mass range $A=14$–48 and may be used to set up some valuable systematics concerning the general behavior of α scattering in this mass region. The strong irregularities observed in the experimental cross sections of elastic α scattering, when passing from one system to its direct neighbor,[3–5] have mainly motivated these rather extensive theoretical studies, as the different explanations given in the past revealed a number of controversies.[6,7] The present investigations and the results from previous studies[1,2] put all α-nucleus systems into one line, saying that none of the investigated systems shows any specific target dependent structure effects. The scattering behavior is a smooth function of the target mass and the occurrence of molecularlike resonance formations seem to be a common property of all systems, thus leaving details of absorption processes as the only reasonable explanation for the experimentally observed strong mass dependencies. In fact, by a slightly more refined target dependent absorption formula these mass dependencies can be at least qualitatively understood. It is, however, stated that for the future only the inclusion of absorption processes into microscopic theories may help to give more insight into the details of scattering processes.

II. THEORETICAL BACKGROUND

The many body wave function Ψ for the elastic channel is expanded in terms of Slater determinants $\phi(\xi,\vec{r})$, describing the system at a given mean distance $\vec{r}$ of the fragment nuclei

$$\Psi(\xi)=\int \phi(\xi,\vec{r})f(\vec{r})d^3\vec{r}\ , \tag{2.1}$$

where ξ stands for all single particle coordinates of the nucleons, including spin and isospin. The expansion coefficients $f(\vec{r})$ are determined by solving the Schrödinger equation in the frozen density approximation

$$\langle\phi(\vec{r}')\,|\,H-E\,|\,\Psi\rangle=0 \text{ for all } \vec{r}'\ . \tag{2.2}$$

Substituting (2.2) into (2.1) one obtains a system of integral equations[8]

$$\int\langle\phi(\vec{r}')\,|\,H-E\,|\,\phi(\vec{r})\rangle f(\vec{r})d^3\vec{r}=0 \text{ for all } \vec{r}'\ . \tag{2.3}$$

For nuclei with total angular momentum $J=0$ like α, ^{14}C, ^{28}Si, and ^{32}S, a partial wave decomposition of Eq. (2.3) leads to the following set of uncoupled integral equations

$$\int_0^\infty dr\, r^2[H_l(r,r')-EN_l(r,r')]f_l(r)=0 \text{ for all } r'\ , \tag{2.4}$$

which can be solved numerically imposing scattering boundary conditions on $f(r)$.[9,10] The integral kernels in Eq. (2.4) are defined as

$$H_l(r,r')=\langle\phi_l(r)\,|\,H\,|\,\phi_l(r')\rangle\ , \tag{2.5a}$$

$$N_l(r,r')=\langle\phi_l(r)\,|\,\phi_l(r')\rangle\ . \tag{2.5b}$$

In the present work, the fragment nuclei are described by their (jj-coupled) oscillator shell mudel ground states $\phi_A(\xi_A,\vec{r}_A)$ and $\phi_B(\xi_B,\vec{r}_B)$ centered around points $\vec{r}_A$ and $\vec{r}_B$. Here the index A refers to the ^{14}C, ^{28}Si, and ^{32}S nuclei, respectively, and B denotes the α particle. The functions $\phi(\xi,\vec{r})$ are then taken as the antisymmetrized products

$$\phi(\xi,\vec{r})=A[\phi_A(\xi_A,\vec{r}_A)\phi_B(\xi_B,\vec{r}_B)] . \tag{2.6}$$

Hence, the actual GCM basis functions run as follows:

$$\phi_1(\xi,\vec{r})=A[\{(0s)^4(0p_{3/2})^8(0p_{1/2},t_z=\tfrac{1}{2})^2,\tfrac{2}{9}\vec{r}\}\{(0s)^4,-\tfrac{7}{9}\vec{r}\}] , \tag{2.7a}$$

$$\phi_2(\xi,\vec{r})=A[\{(0s)^4(0p)^{12}(0d_{5/2})^{12},\tfrac{1}{8}\vec{r}\}\{(0s)^4,-\tfrac{7}{8}\vec{r}\}] , \tag{2.7b}$$

$$\phi_3(\xi,\vec{r})=A[\{(0s)^4(0p)^{12}(0d_{5/2})^{12}(1s_{1/2})^4,\tfrac{1}{9}\vec{r}\}\{(0s)^4,-\tfrac{8}{9}\vec{r}\}] , \tag{2.7c}$$

where the index $\phi_i(\xi,\vec{r})$ ($i=1$, 2, and 3) refers to the α-^{14}C, α-^{28}Si, and α-^{32}S systems, respectively.

In order to get a simple factorization of the c.m. motion equal oscillator widths for both fragment nuclei have been chosen. In the present calculations they are the weighted means of the b values which minimize the internal binding energy of each individual fragment nucleus. The Brink-Boeker force $B1$ was used as an effective nucleon-nucleon interaction.[11] The Coulomb interaction was treated exactly including the exchange parts by use of a Gaussian representation of the $1/r$ term.[12] Only for the α-^{14}C system a spin-orbit (so) interaction was included which turned out to be short-ranged compared to the central part of the interaction. Thus, in the present studies inclusion of so forces mainly affects bound state energies, but has only little influence on the behavior of the phase shifts. Since it has been shown also for other scattering systems consisting of spin-zero nuclei[13,14] that the inclusion of a spin-orbit interaction has a neglectable impact on the phase shifts, we omitted the spin-orbit interaction within the calculations of the α-^{28}Si and α-^{32}S systems.

Furthermore, it has to be noted that the present descriptions of ^{14}C, ^{28}Si, and ^{32}S are rather poor approximations of the realistic ground state wave functions.[15–17] As it has been shown explicitly in studies of the α-^{18}O system, different, but reasonable assumptions of the internal structure of the fragments only affects the prediction of bound states and quasibound state energies.[1,18] Consequently all results sensitively depending on the intrinsic structure of the nuclei will only be discussed qualitatively. However, attention will be drawn on an energy range where the influence of the intrinsic structure of both nuclei is small. This turns out to be at energies around the barrier in the respective partial waves. These energy regimes primarily are of interest for an understanding of the experimentally observed differences in the backangle cross sections of various target nuclei. It should be mentioned that in these energy regimes elastic scattering is found to be mainly sensitive to the tail region of the nucleus-nucleus potential,[19] for which the neglect of a spin-orbit interaction[13,14] as well as the adoption of common oscillator parameters for both fragment nuclei (i.e., see Ref. 20) are justified. Furthermore, resonances predicted for these energy regimes in recent microscopic studies which were based on the same approximations as used in the present investigation (i.e., GCM studies of the ^{16}O-^{28}Si, ^{16}O-^{40}Ca, and ^{28}Si-^{28}Si systems[21–23]) are in fair agreement with resonant structures deduced from the experimental data. Consequently, despite the approximations incorporated into our calculations we expect our results to be a meaningful tool to study the experimentally observed irregularities of elastic α scattering as outlined in the Introduction.

III. MICROSCOPIC STUDIES OF THE α-^{14}C, α-^{28}Si, AND α-^{32}S SYSTEMS

A. The α-^{14}C system

The α-^{14}C system serves as an example to study possible effects especially arising from target nuclei with a closed neutron shell, and a 2h-proton configuration. Within this calculation the oscillator parameter was fixed at $b=1.7$ fm. As already mentioned the microscopic Hamiltonian includes a nuclear spin-orbit interaction whose strength was fitted to reproduce the splitting of the $p_{3/2}$ and $p_{1/2}$ levels in ^{13}C. The effect of the spin-orbit interaction on the α-^{14}C bound states and phase shifts is discussed in Ref. 14.

First indications of the properties of the α-^{14}C system can be found in the energy surfaces

$$E_l(r)=\frac{\langle\phi_l(r)\,|\,H\,|\,\phi_l(r)\rangle}{\langle\phi_l(r)\,|\,\phi_l(r)\rangle} , \tag{3.1}$$

which are shown for the partial waves $l=0$–6 in Fig. 1. It turns out that the energy surfaces for even $l<4$ exhibit two minima at small and large separations, respectively, in contrast to the occurrence of only one minimum in other scattering systems (Sec. IV). Although minima of energy surfaces cannot be directly interpreted as states of the combined system,[24] there is at least a close correspondence to bound and quasibound states as is confirmed by solving[25]

$$\det[H_l(r,r')-E_lN_l(r,r')]=0 , \tag{3.2}$$

where E_l is now the energy of the bound and quasibound state in the lth partial wave. The energy levels calculated by (3.2) turn out to be lowered in energy by 1–2 MeV compared to the energies of the corresponding minima of the energy surfaces. However, from Fig. 1 it can be concluded that the minima at large distances ($r\approx4.5$ fm) correspond to cluster states of the system, while the minima near the shell model limit configuration ($r\lesssim1$ fm) contain a dominating amount of shell model components. Due to the strong restriction of our model space one does not ex-

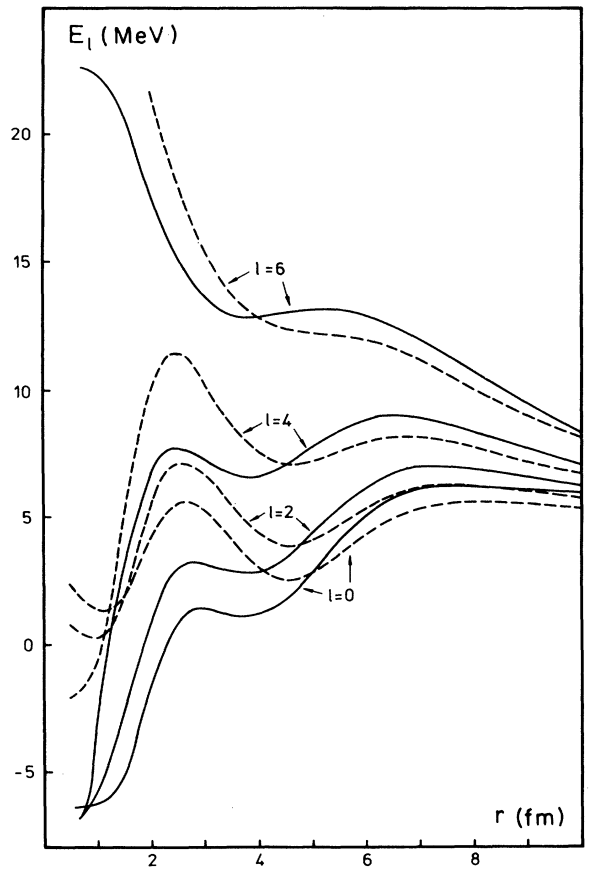

FIG. 1. α-^{14}C energy surfaces for the even partial waves $l=0$–6 calculated with the Brink-Boeker interaction $B1$ (dashed line) and the Volkov interaction $V2$ (solid line).

pect the energy positions of low-lying ^{18}O states being reproduced. By using a less repulsive nucleon-nucleon interaction the agreement of the cluster states to experimental data can be improved, as this is indicated by Fig. 1 using the Volkov force $V2$ (Ref. 26) ($b=1.57$ fm) for comparison.[27] Although the quantitative agreement to the experimental values is rather bad for all used effective interactions we suppose that the physics behind the present qualitative results do not change considerably using a larger model space or a more realistic interaction: At small distances the $2p$-shell model components, which correspond to a break of the inert α or ^{14}C structure, are favored in energy. Considering Pauli repulsion as well as absorption these $2p$ configurations should not play an important role in elastic α scattering in ^{14}C. Therefore α-^{14}C scattering states will be more dominated by α-2h-cluster configurations.[28]

Due to angular momentum restrictions the shell model limit configuration for the $l=6$ partial wave is a 4p-2h excited state. A shell model minimum does not appear in the $l=6$ energy surface and the only minimum corresponds to the α-2h configuration which then gets favored due to the energy gain by clustering.

Figure 2 shows the α-^{14}C phase shifts as calculated by the Volkov force $V2$. The sharp resonances in the even partial waves correspond to the cluster states mentioned above. The resonances in the odd partial waves represent states which contain both, clusters and shell model components, due to the fact that shell model and cluster states within our calculations turn out to be nearly degenerate for negative parity partial waves.

B. The α-^{28}Si and α-^{32}S systems

Since the results from the calculations of elastic α scattering on ^{28}Si and ^{32}S turn out to be rather similar to one another we discuss the results of these calculations in the light of α-^{28}Si. Typical examples of α-^{32}S phase shifts are included within the discussion of Sec. V. The Brink-Boeker force $B1$ has been used as an effective nucleon-nucleon interaction. The oscillator widths have been set to $b=1.88$ fm for the α-^{28}Si system and to 1.91 fm for the

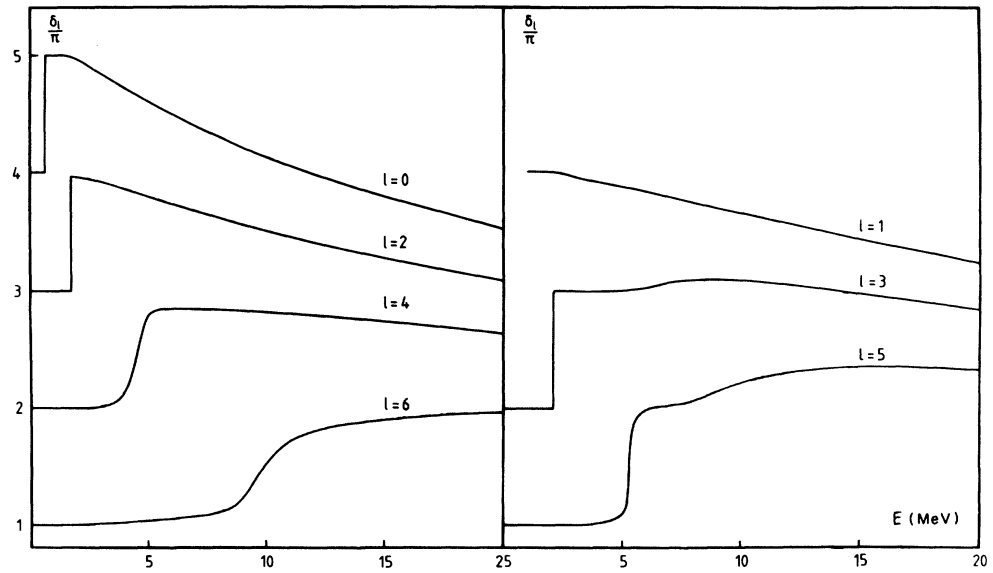

FIG. 2. Phase shifts for the α-^{14}C system calculated with the Volkov interaction $V2$.

α-^{32}S system. A spin-orbit interaction has been neglected in both calculations.

By use of Eq. (3.2) bound states have been calculated for the partial waves $l \leq 5$ and $l=7$. As indicated by the respective energy surfaces (see also the discussion in Sec. IV) the bound states for the even partial waves seem to be clearly dominated by shell-model components. Contrary for odd partial waves a large amount of cluster contributions are present.

The 0^+ ground state of α-^{28}Si is calculated at -11.7 MeV compared to the α threshold. The discrepancy of about 5 MeV compared to the experimental value is due to the assumption of identical width parameters for both fragments as well as to the sudden approximation. If one minimizes the energy of the α particle and the ^{28}Si and ^{32}S nuclei independently by a variation of the respective b values, the energy splitting between the ^{32}S shell model limit configuration ($r \to 0$) [approximated by the limit of ansatz (2.7c)] and the asymptotic $\alpha + ^{28}$Si energy ($r \to \infty$) is reduced by ~ 4.5 MeV. In previous calculations of elastic α scattering on the oxygen and calcium isotopes the assumption of identical width parameters and the sudden approximation have only little influence on the results for the following reasons: In the case of the oxygen isotopes the width parameters of the individual nuclei do not differ strongly[29] when using the Volkov force $V1$ and in the case of the calcium isotopes the resonance structures depend only weakly on the internal nuclear region due to strong Pauli repulsion.[2]

The relative excitation energies of the 2^+ and 4^+ levels in $\alpha + ^{28}$Si calculated at 0.12 and 0.8 MeV deviate considerably from the experimental data. In this case this discrepancy is mainly due to the neglect of a spin-orbit interaction. Assuming that the ^{32}S ground state is a rather pure $(1s_{1/2})^4$ configuration outside the ^{28}Si core and the 2^+ and 4^+ levels contain $(0d_{3/2})^1$ and $(0d_{3/2})^2$ configurations as suggested by shell model calculations,[17] the so interaction would be considerably repulsive and further separate the 2^+ and 4^+ levels. By considering a reasonable spin-orbit contribution of ≈ 1.9 MeV to the $(d_{5/2})$ single particle energies the calculated 2^+ and 4^+ levels would be shifted to almost the experimental values. Unfortunately a computational check of these assumptions was not possible within reasonable computer time.

The energy positions of the first observed 1^- and 3^- states in ^{32}S at -1.1 and -1.95 MeV (relatively to the α threshold) are of the same order as the calculated 1^- and 3^- levels at -2.4 and -1.85 MeV; however, a direct correspondence between states must be regarded with reservation due to the restrictions of our calculations.

The scattering phase shifts for the α-^{28}Si system (Fig. 3) show the same overall behavior as it is known from elastic α scattering on other target nuclei (cf. Sec. IV). The present calculation gives evidence for the existence of long-lived α resonances, most pronounced for the even partial waves. A comparison of these predicted resonances to experimental data is, however, not possible, since elastic α-^{28}Si scattering has not yet been studied in

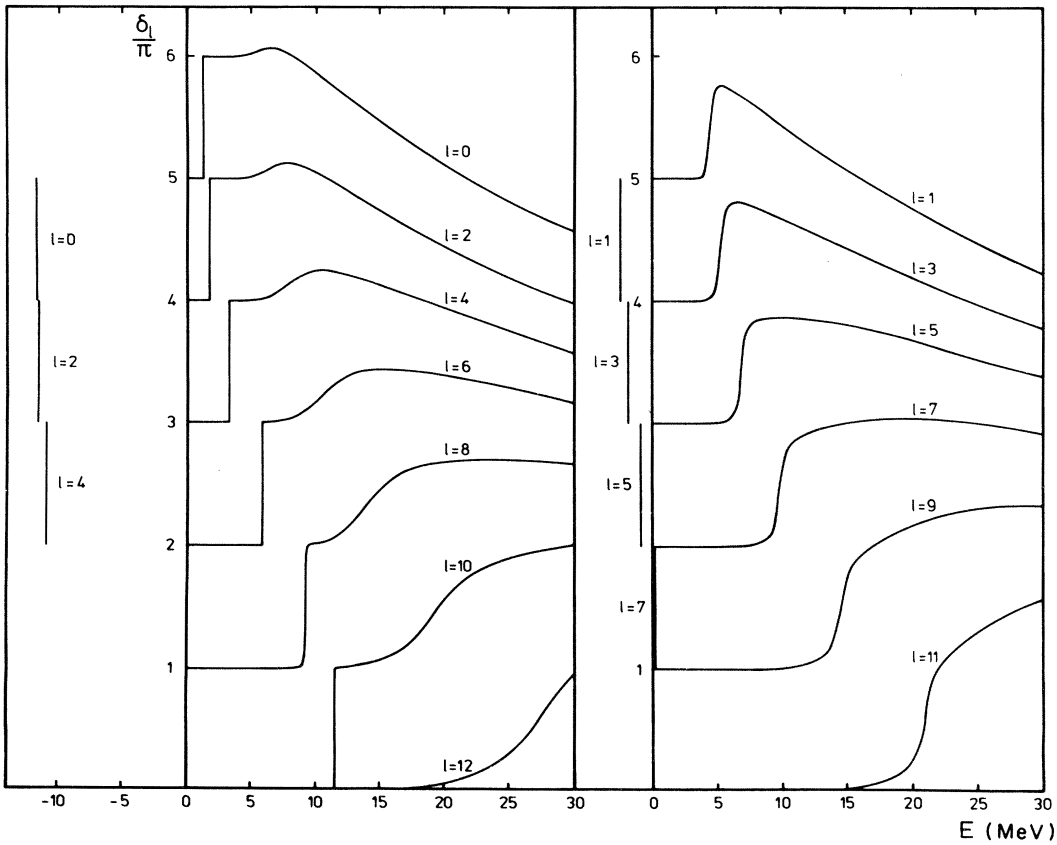

FIG. 3. Phase shifts for elastic α scattering on ^{28}Si. The bars at negative energies mark the energy position of the bound states calculated by Eq. (3.2).

detail to energies below the Coulomb barrier.

The present restrictions, i.e., identical b values for the fragments, and neglect of a spin orbit interaction, are in fact severe when calculating bound states or other quantities which are sensitive to the internal nuclear region, but they are supposed to have only little influence on the positions of the high-lying α resonances.

We would like to note that the present investigation also serves as an instructive example to demonstrate the equivalence of bound and redundant states with what their influence on nuclear phase shifts concerns. As it can be seen from Table I, the number of redundant states n_l is not a monotonously decreasing function of l as it is for two doubly magic nuclei. These discontinuities of n_l appearing around $l=4$ and 7 are due to angular momentum restrictions and reflect the open-shell character of the target nuclei. On the other hand, the sum (m_l+n_l) of bound and redundant states decreases monotonously with l, and according to Swan's generalization of Levinson's theorem[30] leads to a continuous behavior of the respective phase shifts.

IV. COMPARISON OF ELASTIC α SCATTERING ON *sd* AND *pf*-SHELL NUCLEI

The microscopic study of α scattering from composite target nuclei is now covering rather completely the mass range up to $A=48$ involving ^{16}O, ^{18}O, ^{28}Si, ^{32}S, ^{40}Ca, ^{42}Ca, ^{44}Ca, and ^{48}Ca as typical examples. At this stage it therefore seems to be worthwhile commenting on the basic results of these calculations and setting up some systematics which may lead to useful predictions concerning the general trend of α scattering in the *sd* and *pf* shell.

The behavior of the energy surfaces for various α-nucleus systems can be studied by the $l=0$ and 1 partial waves serving as typical examples (Fig. 4). It turns out that if the target is ^{16}O or an open *sd*-shell nucleus the minima of the $l=0$ partial wave are located near the shell model limit, while the minima of the $l=1$ partial waves lie around $r=3$–4 fm, resulting in a strong parity dependence of the energy surfaces for these systems. The parity dependence almost completely disappears when passing to the Ca isotopes, where the minima of the $l=0$ and 1 partial waves both are located around $r=5$–6 fm.

These overall properties can be easily understood by the effect of the Pauli principle and will be discussed for the $l=0$ and 1 partial waves in the following. While in the shell model limit the Pauli principle requires for the nuclei ^{16}O, ^{18}O, ^{28}Si, and ^{32}S, an excitation of the α particle to *sd*-shell states (corresponding to an excitation energy of $8\hbar\omega$) for the Ca isotope an excitation to *pf*-shell configurations ($12\hbar\omega$) is necessary. Correspondingly the effect of the Pauli principle is much more repulsive for the Ca isotopes than for the other nuclei discussed here and explains why in the Ca case the minima exhibit at large separation distances (the corresponding resonances are of cluster type). For the other nuclei the minima are located at rather small separations of the fragments corresponding to states of the combined system with a larger amount of shell model components.

The different parity dependence can be understood similarly. Due to parity conservation the shell model limit configuration for the negative parity (odd) partial waves correspond to $1\hbar\omega$ excited states compared to the positive parity (even) partial waves. Consequently, systems, for which at least the minima of the even partial waves corre-

TABLE I. Number of Pauli forbidden states for different α-nucleus systems. In all partial waves $l \geq 14$ the number of Pauli forbidden states is zero for the discussed systems.

l	0	2	4	6	8	10	12
^{14}C	3	2	1	1	0	0	0
^{16}O, ^{18}O	4	3	2	1	0	0	0
^{28}Si, ^{32}S	4	3	2	2	1	0	0
^{40}Ca, ^{42}Ca, ^{44}Ca	6	5	4	3	2	1	0
^{48}Ca	6	5	4	3	2	1	1
l	1	3	5	7	9	11	13
^{14}C	3	2	1	0	0	0	0
^{16}O, ^{18}O	4	3	2	1	0	0	0
^{28}Si, ^{32}S	4	3	2	1	1	0	0
^{40}Ca, ^{46}Ca, ^{44}Ca, ^{48}Ca	6	5	4	3	2	1	0

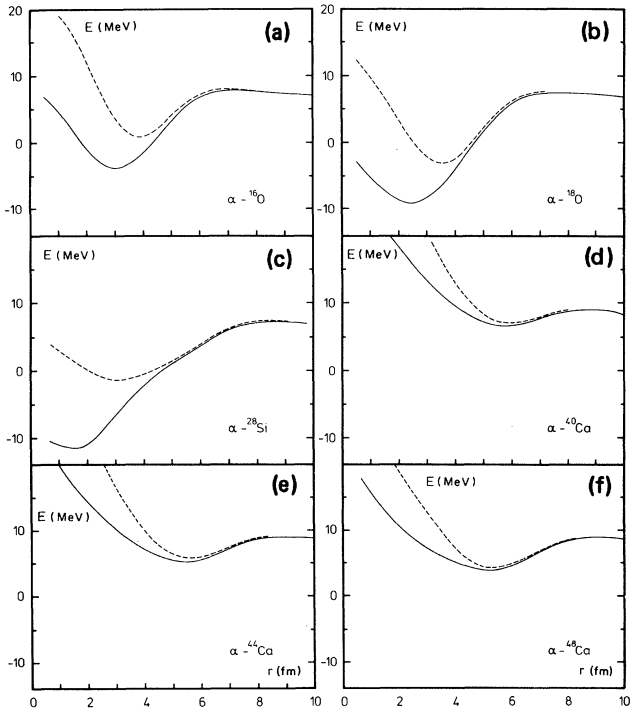

FIG. 4. Energy surfaces for the systems (a) α-^{16}O; (b) α-^{18}O; (c) α-^{28}Si; (d) α-^{40}Ca; (e) α-^{44}Ca; and (f) α-^{48}Ca exemplified for the partial waves $l=0$ (solid curves) and $l=1$ (dashed curves).

spond to states with a large contribution of shell model components, show a strong parity splitting. The minima of the odd partial waves are pushed to greater separation distances by the stronger Pauli repulsion and their energy positions are shifted up by some MeV compared to the positive parity states. In contrast, for the Ca isotopes the stronger Pauli repulsion hinders both fragments to penetrate each other noticeably even for the positive parity states. The shell model components to the α-Ca resonances are therefore rather small which is reflected by the weakness of the parity splitting.

For a discussion of the angular momentum dependence see, for example, Sec. III and Refs. 24 and 31.

In Fig. 5 phase shifts for all microscopically investigated α-nucleus systems with target mass $A > 16$ are summarized, where partial waves $l=0,1$ and 8,9 have been chosen since they turn out to be typical examples. For sake of clearness the phase shifts for the system α-^{42}Ca and α-^{44}Ca have been omitted, as they fit very well into the general behavior of ^{40}Ca and ^{48}Ca.[2] The phase shifts have been normalized to $(n_l+m_l)\pi$ at $E=0$ according to Swan's generalization of Levinson's theorem. Bound states of the systems calculated by Eq. (3.2) are indicated by bars in Fig. 5 (except the $l=0$ bound state of the α-^{32}S system which is calculated at far too low energy due to the neglect of a spin-orbit interaction). The numbers of redundant states are given in Table I.

The general behavior of the phase shifts can be summarized as follows:

(1) The phase shifts exhibit two different series of rotational states: narrow resonances with widths of less than 100 keV in the lower partial waves and broad barrier resonances with widths up to some MeV. The barrier resonances are supposed to dominate the cross section at energies above the Coulomb barrier and to be responsible for the backangle anomaly in elastic α scattering.[2,32,33] A recent investigation[34] of the narrow resonances in terms of a microscopic potential model clearly supports the interpretation of them as quasimolecular states. The barrier resonances are also of molecular structure (two cluster rotator states), but due to the difference in lifetimes only the nar-

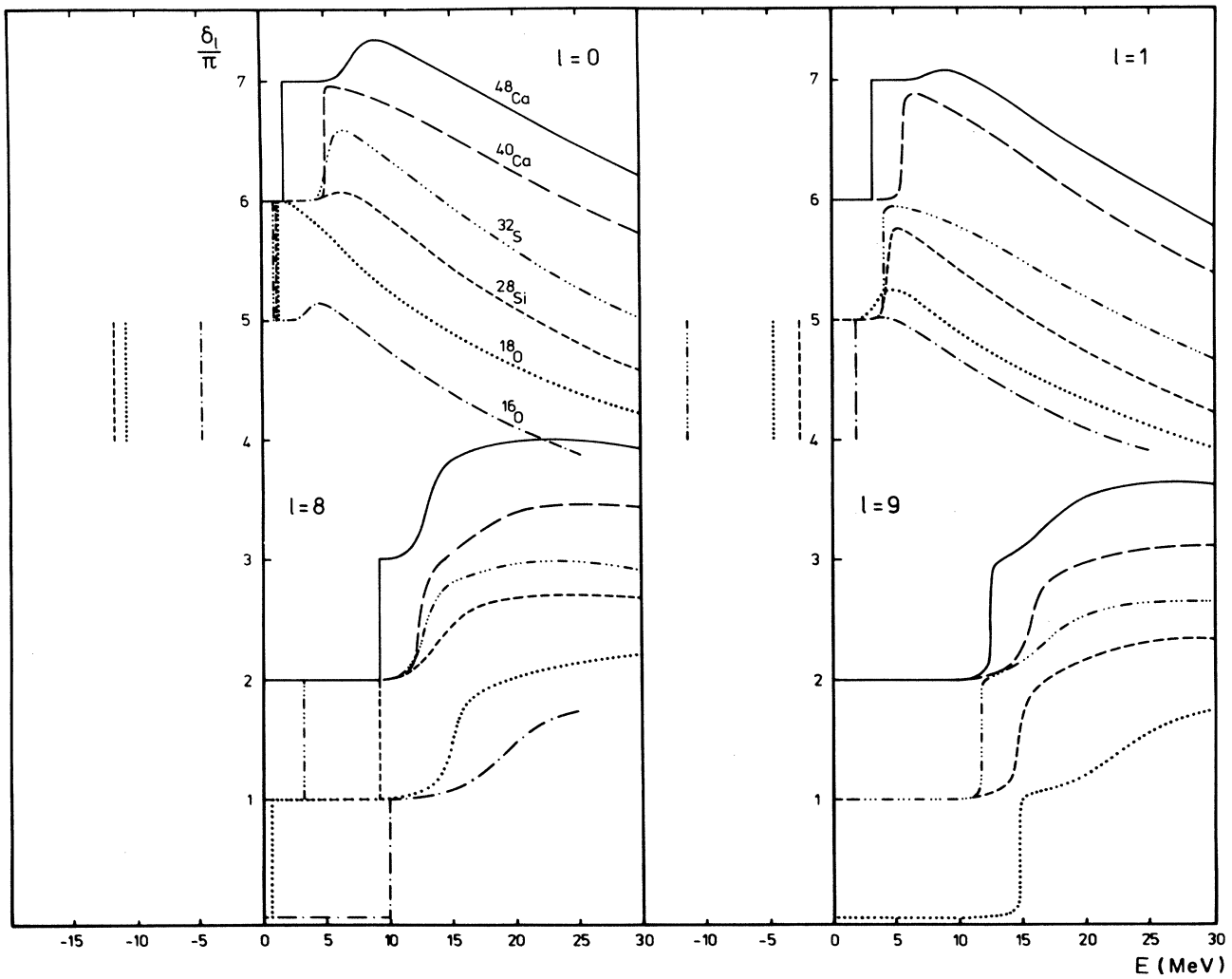

FIG. 5. Phase shifts for the systems α-^{16}O (long dashed-dotted), α-^{18}O (dotted), α-^{28}Si (short dashed), α-^{32}S (short dashed-dotted), α-^{40}Ca (long dashed), and α-^{48}Ca (full) exemplified for the partial waves $l=0,1$ and 8,9. The α-^{16}O phase shifts are taken from Ref. 29. The bars mark energy positions of bound states calculated by Eq. (3.2).

row resonances may be interpreted as quasimolecular states following the criteria given by Stokstad.[35] It should be stressed that the existence of quasimolecular resonances in the lower partial waves is predicted for all investigated α-nucleus systems and may therefore be assumed as a general property of the pure elastic α-nucleus potential. The actual observation of these states, however, depends strongly on target-dependent absorptive effects.[36] Until now, the existence of quasimolecular states in elastic α scattering has only been confirmed for the α-^{40}Ca system.[36]

(2) At sufficiently high energies the phase shifts can be parametrized by

$$\delta_l(E) \approx C_1 \cdot E + C_2 \ ,$$

where C_1 turns out to be nearly independent of the respective scattering system. This result shows higher energy elastic scattering to be rather insensitive to the internal structure of the nuclei involved (except for absorptive effects). A similar parametrization has been found by Lee *et al.*,[37] except that the parameter of C_1 has been given as a positive number. The negative sign of C_1 within our calculations is, however, simply a matter of the generalized Levinson's theorem.

(3) Comparing the phase shifts of the different scattering systems the respective partial wave phase shifts show rather similar overall behavior. However, they increase with increasing mass numbers of the target nuclei. Arguing in terms of phase-equivalent local potentials the potential shapes have to be rather similar for the different systems and the potential depth will increase with increasing mass numbers. Both effects have been confirmed within recent calculations of microscopic α-nucleus potentials,[34] which have been determined from the GCM energy surfaces by use of Friedrich's method.[24,38]

However, as the main conclusion from Fig. 5 we claim that at energies sufficiently above the Coulomb barrier the phase shifts follow a general trend monotonously depending on the mass numbers. Consequently, the pure elastic calculations cannot give any explanation for the different cross section behavior observed in elastic α scattering at backangles. One may therefore conclude that this effect is due to a different absorptive behavior of the various composite nuclei as this is also indicated by experimental results.[39] Unfortunately this assumption cannot be proved on a microscopic level due to present day computer facilities, but it has been supported by semimicroscopic investigations which considered absorptive effects by the inclusion of a phenomenological imaginary potential within a microscopic theory.[2,40,41] In the following we want to show a rather simple connection between the relative strength of the backangle cross section and of the absorptive strength, where both quantities are estimated as follows: following Eck *et al.*,[42]

$$\epsilon(E) = \frac{1}{M} \sum_{i=1}^{M} \frac{\sigma(\theta_i)}{\sigma_R(\theta_i)} \tag{4.1}$$

gives a measure for the strength of the backangle anomaly. The quantities θ_i denote such angles, where the backangle cross section (in Rutherford's units) has a maximum. The relative absorptive strength of elastic α scattering on the target nuclei a_i, a_j may now be approximated by

$$Q_{ij} = \frac{|\langle A_i | V | a_i + 4 \rangle|^2}{|\langle A_j | V | a_j + 4 \rangle|^2} \frac{\rho(E_i^*, l_i)}{\rho(E_j^*, l_j)} \ , \qquad E_k^* = E + Q_k(\alpha,\gamma), \quad k = i,j \ , \tag{4.2}$$

assuming absorption proceeds mainly via coupling to the compound nucleus.[43] In Eq. (4.2) $\rho(E^*,l)$ describes the level density in the compound nucleus A, while the matrix element $\langle A | V | a+4 \rangle$ denotes the coupling of the elastic channel $|a+4\rangle$ to the absorptive channels $|A\rangle$. Deviating from Ref. 42, we do not assume that Q_{ij} is determined simply by the ratio of the level densities. Hence, it is known that for energies above the Coulomb barrier, to which the following considerations are restricted, elastic α scattering is dominated by broad overlapping barrier resonances with l values around the grazing angular momentum. At these considerably high energies the barrier resonances in the elastic channel $|a_i+4\rangle$ will most likely couple to configurations of the composite system which are similar to the entrance channel resonance. Since only little is known about the level densities of such particular cluster configurations in nuclei (denoted by $\rho_{\rm cl}$), we assume the quantity $\rho_{\rm cl}$ to be proportional to the strength of α structures within the compound system approximated by the ground state α-spectroscopic factor $S_\alpha(a_i+4 \rightarrow A_i)$ and the level density in the compound system,

$$\rho_{\rm cl}(E^*,l) \sim S_\alpha(a_i+4 \rightarrow A_i)\rho(E^*,l) \ . \tag{4.3}$$

Making the further assumption that the coupling matrix element $|\langle a_i+4 | V | A_i \rangle|^2$ is target independent, Eq. (4.2) can be approximated by

$$Q_{ij} \approx \frac{S_\alpha(a_i+4 \rightarrow A_i)}{S_\alpha(a_j+4 \rightarrow A_j)} \frac{\rho(E_i^*,l_i)}{\rho(E_j^*,l_j)} \ , \tag{4.4}$$

where the level density is given by the statistical model.[44] The level densities have been calculated as outlined in Ref. 2. Although the particular approximations (4.2)—(4.4) are rather rude, most of these uncertainties will cancel within the ratio (4.4). Hence, the quantity Q_{ij} may be considered as a reasonable estimate of the relative absorptive strength.

Figure 6 shows the quantities Q_{ij}^{-1} and $\epsilon_{ij} = \epsilon_i/\epsilon_j$ for some target nuclei at the closures of the p and sd shell calculated for $E_{\rm rel} = 24$ MeV (experimental data are taken from Ref. 5) and normalized to the ^{18}O and ^{44}Ca nuclei, respectively. To reduce the uncertainties within the α-spectroscopic factors as much as possible the S_α factors have been taken as the mean values of those experimentally determined ground state spectroscopic factors given in Refs. 45—49. It should be mentioned that by omitting the spectroscopic factors in Eq. (4.4) Q_{ij} is not able to reproduce the relative strength of the backangle cross section enhancement. For example, this assumption would predict the ratio for the Ca isotopes ^{42}Ca and ^{48}Ca contrary to the experimental findings. On the other hand, the ansatz (4.4) fails for the open-shell nuclei ^{32}S and ^{34}S. Nevertheless, Fig. 6 gives strong evidence for the assump-

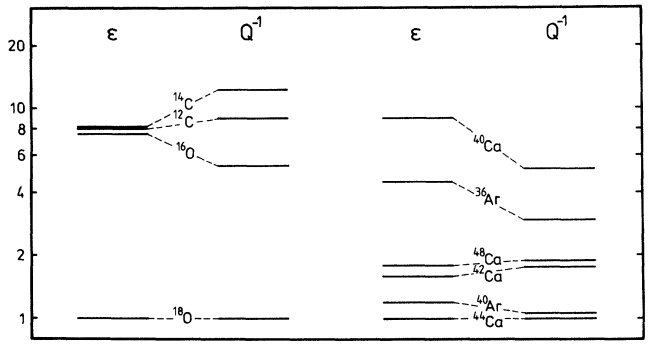

FIG. 6. Comparison between the relative strength of backangle cross section and absorption for elastic α scattering on different target nuclei (normalized to ^{18}O and ^{44}Ca, respectively). For definitions see the text.

tion of the backward angle α scattering being mainly influenced by absorptive effects.

V. CONCLUDING REMARKS

The present paper summarizes the studies of elastic α scattering on some target nuclei in the mass region $A=14$—48 performed in the framework of the generator coordinate method. As the main results these calculations give explanations for most of the features of elastic α scattering in the energy range $E<50$ MeV.

(1) In previous calculations the *backangle anomaly* could be traced back to overlapping barrier resonances whose description is rather insensitive to a correct treatment of the Pauli exchange between the two fragment nuclei. Furthermore, the GCM explains why pure folding potential models are successful in reproducing elastic α scattering at energies above the Coulomb barrier.[2]

(2) At sufficiently high energies ($E \geq 15$ MeV) the phase shifts follow a systematic trend and increase with increasing mass number of the target nucleus. Therefore, the different backangle cross-section behavior in elastic α scattering on nuclei of the mass region $A=14$—48 cannot be caused by dynamical properties of the pure elastic channel or by structure effects of the target nuclei. Especially neutron blocking fails as a possible explanation of the *isotope effect*.

However, there is strong evidence for the isotope effect to be caused by a different absorptive behavior of the scattering systems. Estimating the ratio of the absorptive strength for different scattering systems roughly by assuming absorption proceeds mainly via coupling of the barrier resonances to rather similar clusterlike configurations of the composed system, a reasonable correspondence between absorptive strength and backangle enhancement of the cross section has been established.

(3) Long-lived resonances which can be interpreted as *molecular α-rotator states* are predicted for all investigated α-nucleus systems. The molecular resonances correspond to cluster states in which both fragments overlap each other and which makes a correct treatment of the Pauli principle indispensable for their description. The observation of these molecular states, however, strongly depends on absorption. Until now, their existence has been confirmed only for the α-^{40}Ca system experimentally.[36]

The present paper also indicates the limits of microscopic one-channel calculations. Even in the case that some of the shortcomings of these investigations (the effective nucleon-nucleon interaction, internal wave functions, and adiabatic approximation using individual b values) may be improved, one does not expect that these calculations will give a deeper fundamental insight into the properties of elastic scattering. The future task of microscopic investigations should therefore be extended to the study of absorptive effects by including one or more inelastic or reaction channels within the microscopic theories.

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