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Proton-oxygen bremsstrahlung calculation

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The proton-oxygen bremsstrahlung cross section is calculated near the 2.66-MeV elastic scattering resonance by using two model-independent approximations, the Feshbach-Yennie approximation and the soft-photon approximation. The soft-photon approximation predicts a typical smooth spectrum, while the Feshbach-Yennie approximation predicts structure in the resonance region. Both the principal term and the correction term are included in our Feshbach-Yennie calculation, but the contribution from the correction term is found to be insignificant. The structure predicted in the Feshbach-Yennie approximation has various forms, from a peak to a dip, depending upon the proton scattering angle. This interesting structure, if verified experimentally, can be used to study the resonance effects, to test the Feshbach-Yennie approximation, and to extract the nuclear time delay. Measurement of the proton-oxygen bremsstrahlung cross section near 2.66 MeV for various proton scattering angles is strongly suggested.

[NUCLEAR REACTIONS p ^{16}O bremsstrahlung near 2.66 MeV, calculate bremsstrahlung cross section.]

I. INTRODUCTION

The effects of nuclear resonances on nuclear bremsstrahlung emission have attracted much attention recently. A process which was studied rather thoroughly during the last several years was the proton-carbon bremsstrahlung process near both the 1.7-MeV double resonance[1–5] and the 461-keV resonance.[6] While these studies have established that the Feshbach-Yennie approximation[7,2] can be used to describe the resonance structure observed in the bremsstrahlung spectra near 1.7 MeV and that (using this approximation) the nuclear time delay associated with the formation of a compound state can be extracted from the measured bremsstrahlung cross sections, the difficulty in describing the spectra observed near 461 keV (Ref. 6) indicates that many problems remain unsolved. Obviously, these problems require further theoretical and experimental study, and above all, further testing of the Feshbach-Yennie approximation.

The main purpose of this paper is to report some important results of our theoretical study of the proton-oxygen bremsstrahlung process (p ^{16}Oγ) near the 2.66-MeV elastic scattering resonance, especially to point out that the measurement of the p ^{16}Oγ cross section at both the forward and backward angles will provide a very sensitive test of the Feshbach-Yennie approximation. The study of the p ^{16}Oγ process near the 2.66-MeV resonance was originally suggested by Barshay and Yao in 1968.[8] However, their suggestion attracted little attention and the process has never been carefully studied either theoretically or experimentally. Our interest in this study was stimulated very recently by discussions with Galonsky.[9] The results which we wish to report here were based upon our calculation of the p ^{16}Oγ cross sections using both the soft-photon approximation and the Feshbach-Yennie approximation, which includes the principal term and the correction term.

It is well known that the p ^{16}O system near the 2.66-MeV resonance has two interesting features:[10] (i) the 2.66-MeV resonance is a well-isolated excited state in ^{17}F ($J^{\pi}=\frac{1}{2}^{-}$); and (ii) the p ^{16}O elastic scattering cross sections exhibit structure in the form of a peak or a dip depending upon the proton scattering angle. In addition to these features, we have found from our Feshbach-Yennie calculation that the predicted p ^{16}Oγ spectra show structure in various forms depending also upon the scattering angle. Perhaps the most attractive feature of the Feshbach-Yennie calculation is the prediction of the dip structure in the p ^{16}Oγ spectrum for scattering angles greater than 110°. Because of these important features and the fact that the contribution from the correction term of the Feshbach-Yennie approximation is small, the p ^{16}Oγ process near 2.66 MeV can be considered as an ideal case for studying the resonance effects, testing the Feshbach-Yennie approximation, and extracting nuclear time delay.[11]

II. BREMSSTRAHLUNG AMPLITUDE AND CROSS SECTION

Since the p ^{16}O system is a typical spin-$\frac{1}{2}$-spin-0 system, the bremsstrahlung calculation for this system is very similar to the p ^{12}C bremsstrahlung calculation.[2] Therefore, with some modification, the formulae obtained in Ref. 2 for the p ^{12}C bremsstrahlung calculation can be used for the p ^{16}O bremsstrahlung calculation. Briefly, the bremsstrahlung amplitude, M_μ, in the Feshbach-Yennie approximation for the p ^{16}Oγ process,

$$\mathrm{p}(q_i^\mu)+{}^{16}\mathrm{O}(p_i^\mu)\rightarrow \mathrm{p}(q_f^\mu)+{}^{16}\mathrm{O}(p_f^\mu)+\gamma(K^\mu)\ ,$$

(the four-momentum of each particle is given in parentheses) can be written as[2]

$$M_\mu=\bar{u}(q_f,\nu_f)\left[a_{f\mu}T(s_i,t)-a_{i\mu}T(s_f,t)+b_{f\mu}\frac{\partial T(s_i,t)}{\partial t}-b_{i\mu}\frac{\partial T(s_f,t)}{\partial t}\right]u(q_i,\nu_i)\ , \tag{1}$$

where

$$a_{f\mu}=\frac{q_{f\mu}}{q_f\cdot K}+\frac{Zp_{f\mu}}{p_f\cdot K}-(1+Z)\frac{(q_f+p_f)_\mu}{(q_f+p_f)\cdot K}\ ,$$

$$a_{i\mu}=\frac{q_{i\mu}}{q_i\cdot K}+\frac{Zp_{i\mu}}{p_i\cdot K}-(1+Z)\frac{(q_i+p_i)_\mu}{(q_i+p_i)\cdot K}\ ,$$

$$\begin{aligned}b_{f\mu}=&\frac{q_{f\mu}}{q_f\cdot K}[-2(\bar{p}_f-p_i)\cdot(R+K)+(R+K)^2]+\frac{Zp_{f\mu}}{p_f\cdot K}[2(\bar{q}_f-q_i)\cdot R+R^2]\\&-[-2(\bar{p}_f-p_i)\cdot N_R\bar{p}_{f\mu}-2(\bar{p}_f-p_i)_\mu+(N_R\cdot K)\bar{p}_{f\mu}+(N_R\cdot N_R)(\bar{p}_f\cdot K)\bar{p}_{f\mu}+(\bar{p}_f\cdot K)N_{R\mu}]\\&-Z[2(\bar{q}_f-q_i)\cdot N_R\bar{p}_{f\mu}+(N_R\cdot N_R)(\bar{p}_f\cdot K)\bar{p}_{f\mu}]\ ,\end{aligned}$$

$$\begin{aligned}b_{i\mu}=&\frac{q_{i\mu}}{q_i\cdot K}[-2(\bar{p}_f-p_i)\cdot(R+K)+(R+K)^2]+\frac{Zp_{i\mu}}{p_i\cdot K}[2(\bar{q}_f-q_i)\cdot R+R^2]\\&-[-2(\bar{p}_f-p_i)\cdot N_R\bar{p}_{f\mu}-2(\bar{p}_f-p_i)_\mu+(N_R\cdot K)\bar{p}_{f\mu}+(N_R\cdot N_R)(\bar{p}_f\cdot K)\bar{p}_{f\mu}+(\bar{p}_f\cdot K)N_{R\mu}]\\&-Z[2(\bar{q}_f-q_i)\cdot N_R\bar{p}_{f\mu}+(N_R\cdot N_R)(\bar{p}_f\cdot K)\bar{p}_{f\mu}]\ ,\end{aligned}$$

$$N_{R\mu}=[m^2p_{i\mu}-(p_i\cdot\bar{q}_f)\bar{q}_{f\mu}]/[(p_i\cdot\bar{q}_f)(\bar{p}_f\cdot\bar{q}_f)-m^2(p_i\cdot\bar{p}_f)]\ ,$$

$$R_\mu=(\bar{p}_f\cdot K)N_{R\mu}\ ,$$

$$\bar{q}_{f\mu}=\lim_{K\to 0}q_{f\mu},\quad \bar{p}_{f\mu}=\lim_{K\to 0}p_{f\mu}\ ,$$

$$s_i=(q_i+p_i)^2\ ,$$

$$s_f=(q_f+p_f)^2\ ,$$

$$t=(\bar{p}_f-p_i)^2=(\bar{q}_f-q_i)^2\ ,$$

Z is the atomic number of oxygen, u is the Dirac spinor, and T is the T matrix evaluated at two different energies, s_i or s_f. In terms of the bremsstrahlung amplitude M_μ, the bremsstrahlung cross section for the p ^{16}Oγ process has the form

$$\sigma_\gamma=\frac{d^3\sigma}{d\Omega_q d\Omega_\gamma dK}=GJK\left[\frac{1}{2}\sum_{\text{spins}}(-M_\mu^\dagger M^\mu)\right]F\ . \tag{2}$$

Here,

$$G=e^2/(16\pi^3)\ ,$$

$$J=m^2/\{16\pi^2[(q_i\cdot p_i)^2-m^2M^2]^{1/2}\}\ ,$$

$$F=\frac{[(p_i\cdot q_f)^2-m^2M^2]^{3/2}}{M^2[(p_i\cdot q_f)(p_f\cdot q_f)-m^2(p_i\cdot p_f)]}\ ,$$

e is the proton charge, m (M) is the proton (oxygen) mass, and $d\Omega_q$ and $d\Omega_\gamma$ are the proton and photon solid angles, respectively.

In addition to the absolute bremsstrahlung cross section σ_γ, we calculate the relative bremsstrahlung cross section,

$$\sigma_{\text{rel}}=\sigma_\gamma/(d\sigma_{\text{el}}/d\Omega_q)\ , \tag{3}$$

where $d\sigma_{\text{el}}/d\Omega_q$ is the elastic scattering cross section evaluated at the bombarding energy.

Since the structure predicted in the Feshbach-Yennie approximation can be seen clearly when it is compared with the typical soft-photon spectrum, we have also used the leading term of the soft-photon approximation[12] to calculate the p ^{16}Oγ cross section. The leading term of the bremsstrahlung amplitude in this approximation can be simply obtained from Eq. (1) by setting $T(s_f,t)\to T(s_i,t)$, $q_f^\mu\to\bar{q}_f^\mu$, $p_f^\mu\to\bar{p}_f^\mu$, and $b_{f\mu}=b_{i\mu}=0$. (See Ref. 12 for the detailed derivation.) The bremsstrahlung cross section calculated from this amplitude can be obtained from Eq. (2) by replacing F by $\bar{F}$,

$$\bar{F}=\lim_{K\to 0}F=\frac{[(p_i\cdot\bar{q}_f)^2-m^2M^2]^{3/2}}{M^2[(p_i\cdot\bar{q}_f)(\bar{p}_f\cdot\bar{q}_f)-m^2(p_i\cdot\bar{p}_f)]}\ .$$

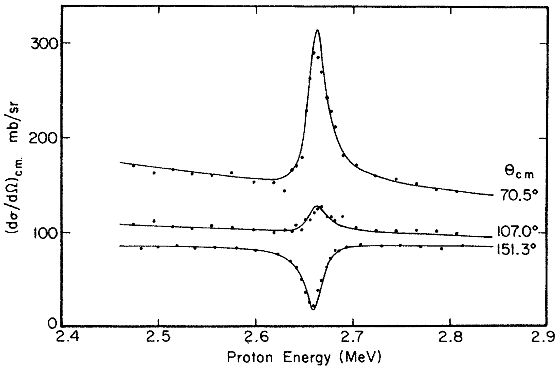

FIG. 1. Elastic scattering cross sections. Experimental results which are shown as points are from Ref. 10. The solid curves represent the results of our calculation.

III. RESULTS AND DISCUSSION

We have calculated the p ^{16}Oγ cross sections, σ_γ and σ_{rel}, as a function of the photon energy in both the soft-photon approximation and the Feshbach-Yennie approximation. In all these calculations, we have assumed that the photon is emitted vertically out of the scattering plane. The geometry assumed here is similar to the experimental arrangement used by the Bologna group,[1] the Brooklyn group,[3] and the Tokyo group.[5] In order to make a thorough investigation of the resonance structure predicted in the Feshbach-Yennie approximation, we have calculated σ_γ and σ_{rel} at several incident energies, both above and below the 2.66-MeV resonance energy, and at various proton scattering angles.

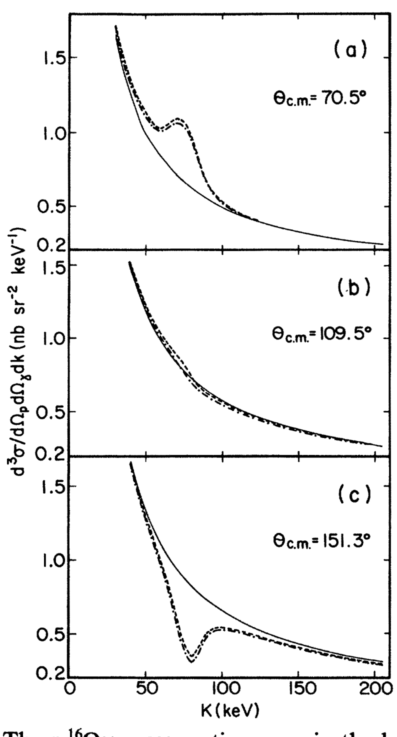

FIG. 2. The p ^{16}Oγ cross sections, σ_γ, in the laboratory system as a function of photon energy. Three sets of the calculated cross sections are plotted together in order to show the change of the resonance structure with the proton scattering angle. The solid curves represent the calculation using the leading term of the soft-photon approximation. The dash-dotted curves represent the calculation using the principal term of the Feshbach-Yennie approximation and the dashed curves represent the calculation using the same approximation but including also the correction term.

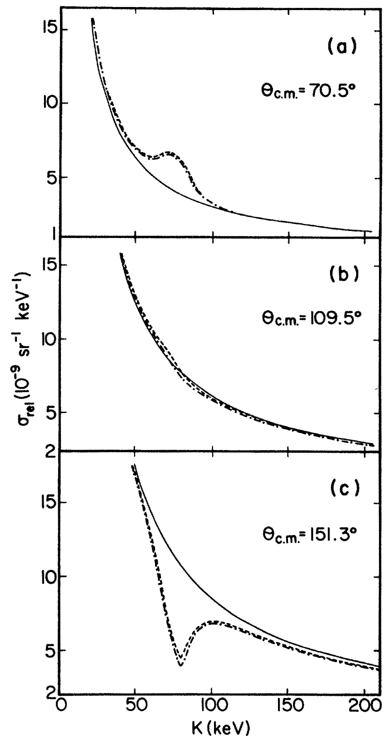

FIG. 3. Same as Fig. 2 but for the relative bremsstrahlung cross section, σ_{rel}.

The input for our calculations is the phase shifts determined from the elastic scattering data. Phase shift analyses were performed in the past by several authors.[10,13,14] We took the phase shifts from Ref. 10 and modified them slightly to give a better fit to the experimental data. A comparison of the calculated and measured elastic scattering cross sections is shown in Fig. 1. The agreement is very good.

In general, at an incident energy E ($E=E_R+\epsilon$, E_R = the resonance energy), our soft-photon calculation shows no structure on the entire bremsstrahlung spectrum, as expected, but our Feshbach-Yennie calculation predicts interesting structure near the photon energy $K=\epsilon$, depending upon the proton scattering angle. (We wish to point out here that the precise shape of the bremsstrahlung structure cannot be predicted from the structure observed in the elastic scattering cross section without actually performing a detailed bremsstrahlung calculation.) Some results of our calculation at an incident energy of 2.74 MeV, which is 80 keV (ϵ = 80 keV) above the resonance energy, are shown in Figs. 2 and 3. From these figures, we can see that the contribution from the correction term is not significant; it has the effect of increasing the cross section slightly in the region of the resonance. Hence the spectra calculated in the Feshbach-Yennie approximation with or without the correction term are almost the same. The small contribution from the correction term (together with those interesting features discussed in the Introduction) makes the p ^{16}Oγ process near the 2.66-MeV resonance an ideal case for testing the Feshbach-Yennie approximation and extracting the nuclear time delay. The reasons are the following: (i) As we know, the contribution from the correction term is very important for the p ^{12}Cγ process near both the 1.7-MeV resonance and the 461-keV resonance. In that case the contribution from higher order terms in the expansion of the bremsstrahlung cross section (or amplitude) may also be important. A small contribution from the correction term (which is the second term of the expansion) is always a good sign, for it means less ambiguity in the theoretical prediction. (ii) It is difficult to extract the nuclear time delay when the correction term must be included in the calculation.[15] As pointed out in Ref. 15, the correction term cannot be determined directly from the elastic data but has to be calculated theoretically from the elastic scattering amplitude determined from the elastic data. The error introduced by this hybrid method for the determination of the time delay will be small if the contribution from the correction term is negligible.

The calculated values of σ_γ and σ_{rel} for the proton scattering angle of 70.5° are shown in Figs. 2(a) and 3(a), respectively. As we can see from these figures, the soft-photon approximation gives the expected spectrum with $1/K$ dependence, while the Feshbach-Yennie approximation predicts resonance structure in the form of a peak in the energy region of the resonance, near $K=80$ keV. Our calculation indicates that the peak reduces as the proton scattering angle increases. At an angle of 109.5°, the structure has almost disappeared, as shown in Figs. 2(b) and 3(b). For a scattering angle which is greater than 109.5°, the resonance structure becomes a dip. An example of this structure for a scattering angle of 151.3° is shown in Figs. 2(c) and 3(c).

The prediction of bremsstrahlung structure in various forms depending upon the proton scattering angles is very interesting, and the most interesting structure predicted in the Feshbach-Yennie approximation is the dip-bump structure. Such structure has never before been studied. Obviously, the existence of the resonance structure must be verified experimentally and the experimental result is needed for a complete understanding of this structure. It is our hope that our theoretical study will stimulate experimentalists to measure the p ^{16}Oγ cross section at various scattering angles.

In conclusion, we have applied the soft-photon approximation and the Feshbach-Yennie approximation to study the p ^{16}Oγ process near the 2.66-MeV resonance. At an energy far from the resonance, both approximations predict very similar spectra which have a typical $1/K$ dependence. In the vicinity of the resonance, however, the Feshbach-Yennie approximation predicts structure in various forms depending upon the proton scattering angle. Since the contribution from the correction term is not significant, the structure predicted by the calculation without the correction term is very similar to the one predicted by the calculation with the correction term. This interesting structure, if verified experimentally, can be used to study the resonance effects, especially to test the Feshbach-Yennie approximation and to extract nuclear time delay.

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[1]C. Maroni, I. Massa, and G. Vannini, Nucl. Phys. A273, 429 (1976); Phys. Lett. 60B, 344 (1976); A. Cristallini, C. Maroni, I. Massa, and G. Vannini, *ibid.* 56B, 245 (1975).

[2]M. K. Liou, C. K. Liu, P. M. S. Lesser, and C. C. Trail, Phys. Rev. C 21, 518 (1980).

[3]C. C. Trail, P. M. S. Lesser, A. H. Bond, Jr., M. K. Liou, and C. K. Liu, Phys. Rev. C 21, 2131 (1980); C. C. Trail, P. M. S. Lesser, A. H. Bond, Jr., and M. K. Liou, IEEE Trans. Nucl. Sci. NS-26, 1174 (1979).

[4]C. C. Perng, G. J. Jan, and M. K. Liou, Phys. Rev. C 23, 2357 (1981); G. J. Jan, C. C. Perng, and M. K. Liou, Phys. Lett. 85B, 25 (1979).

[5]H. Taketani, M. Adachi, N. Endo, and T. Suzuki, Phys. Lett. 113B, 11 (1982); H. Taketani, N. Endo, G. Ishikawa, T. Suzuki, M. Adachi, T. Kohno, A. Makishima, and T. Ikuta, Nucl. Instrum. Methods 196, 283 (1982).

[6]P. M. S. Lesser, C. C. Trail, C. C. Perng, and M. K. Liou, Phys. Rev. Lett. 48, 308 (1982).

[7]H. Feshbach and D. R. Yennie, Nucl. Phys. 37, 150 (1962).

[8]S. Barshay and T. Yao, Phys. Rev. 171, 1708 (1968).

[9]A. Galonsky, private communication.

[10]V. Gomes, R. A. Douglas, T. Polga, and O. Sala, Nucl. Phys. 68, 417 (1965), and references therein.

[11]R. M. Eisberg, D. R. Yennie, and D. H. Wilkinson, Nucl. Phys. 18, 338 (1960).

[12]F. E. Low, Phys. Rev. 110, 974 (1958). For the formulae used in this calculation, see M. K. Liou and W. T. Nutt, Nuovo Cimento 46A, 365 (1978).

[13]S. R. Salisbury and H. T. Richards, Phys. Rev. 126, 2147 (1962).

[14]R. A. Blue and W. Haeberli, Phys. Rev. 137, B284 (1965).

[15]C. K. Liu, M. K. Liou, C. C. Trail, and P. M. S. Lesser, Phys. Rev. C 26, 723 (1982).