Axial asymmetry and the determination of effective γ values in the interacting boson approximation

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It is shown that the interacting boson approximation naturally incorporates an effective dynamical axial asymmetry which stems from softness in the corresponding classical potential in the γ direction. An explicit relationship is obtained between the effective γ deformation variable and the one parameter, χ, characterizing interacting boson approximation calculations in an SU(3) $\rightarrow$ O(6) region.

The application of the interacting boson approximation (IBA) to deformed nuclei has led to active interest in its relation to geometrical models. Several studies[1–5] of the classical limit of the IBA have been carried out to establish the correspondence with geometrical models in terms of the familiar (β,γ) shape variables, in particular, for the three limiting IBA symmetries. It has also been shown that the IBA-1 Hamiltonian in its usual form, with no terms of higher order than two body, contains *no triaxial* solutions. While a mapping is possible[1], in principle, between any IBA solution and a geometrical framework, little has been done in determining this relationship in practical cases away from the limiting symmetries. Thus, while it is known, in approximate terms, that the transition between SU(3) and O(6) involves a gradual reduction in the effective β deformation parameter, it will be shown here that this cannot be a complete specification and that, in fact, this transition corresponds to the gradual introduction of an effective axial asymmetry resulting from a softening of the potential in the γ degree of freedom, albeit with the minimum remaining at 0°. Indeed, this is plausible since the SU(3) limit corresponds to an axial rotor and the O(6) limit to a γ-unstable rotor, which is approximately simulated by a triaxial configuration with asymmetry parameter $\gamma \approx 30°$. The results here will go further than this, however, by deducing an explicit relation between IBA calculations and values for γ_{eff}.

The essential idea is illustrated schematically in Fig. 1, where the changing shape of the IBA potential[1] in the γ direction is shown as a function of the parameter χ which, as will be seen later, uniquely specifies the transition between the SU(3) ($\chi=-2.958$) and O(6) ($\chi=0$) limits of the model. While there is a small dependence on the β variable which has been ignored in Fig. 1, for a given boson number the shape is principally determined by a term proportional to $\chi\cos3\gamma$, according to the analysis of Ref. 1. It is evident that, while the minimum in the potential is always found at $\gamma=0°$, so that there is no triaxial minimum, its depth and steepness are finite and gradually decrease as the O(6) limit approaches until, in this limit, the potential becomes totally independent of γ. This leads to finite and gradually increasing excursions in γ so that one can speak of an effective average value of γ that increases from $\approx 0°$ to $\approx 30°$ through the transition region. Indeed, this correspondence between effective γ values and the transition from SU(3) to O(6) is also suggested by the $\Delta K=2$ matrix elements that mix SU(3) states when this limit is broken.[6] The resulting K impurities, in turn, generate large variations in a number of $E2$ branching ratios as χ approaches 0, whereas, in the pure harmonic axial deformed geometrical model, these ratios are constants, independent of β, and are given by squares of Clebsch-Gordan coeffi-

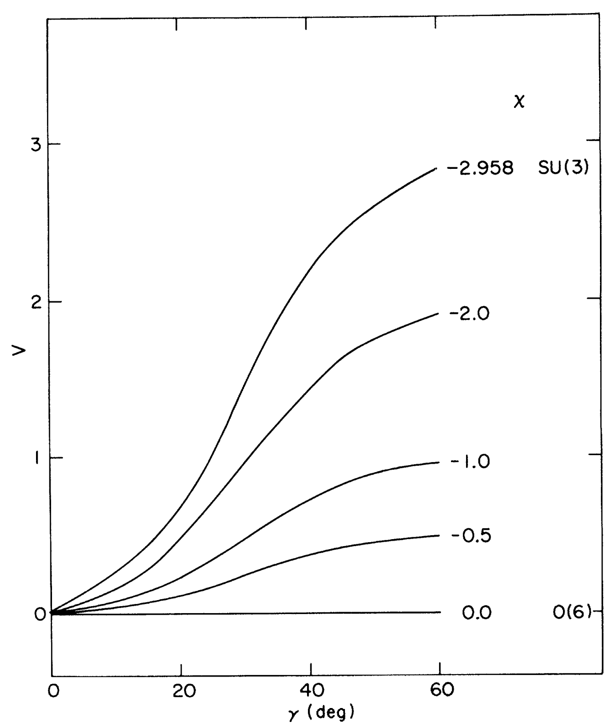

FIG. 1. Schematic illustration of the gradual softening of the potential (Ref. 1) corresponding to different IBA calculations from SU(3) to O(6). For clarity the potentials have been set to the same value at $\gamma=0°$.

cients. These features are exactly analogous to the consequences of a finite rigid asymmetry γ, in the asymmetric rotor model.[7-10] This is not surprising since it is well known[7] that the effects of a nonzero rms γ value on many nuclear properties are similar to those resulting from the assumption of a finite (rigid) asymmetry.

It is the purpose of this Communication to exploit the analogy with the asymmetric rotor model to test the above ideas by determining if, despite the fact that no rigid triaxial minimum exists in IBA-1, it is possible to assign an effective γ value to a given IBA calculation, at least for nuclei in the $SU(3) \rightarrow O(6)$ region. This will be done by calculating different observables in the consistent Q formalism[11] of the IBA. The extreme simplicity of this formalism, with only one consequential parameter χ, then allows a relation to be deduced between this parameter and the asymmetry γ of the asymmetric rotor model[7-10] of Davydov and co-workers by assigning to each χ a value for a γ_{eff} that gives the same result for a specific observable. It will be shown that, in fact, a consistent picture does emerge since very similar γ_{eff} values are deduced for different observables. The effects of the finite boson number on the $\gamma_{\text{eff}} \leftrightarrow \chi$ relationship will also be discussed. The association of γ_{eff} and χ values, coupled with the concept of a decreasing β in the $SU(3) \rightarrow O(6)$ transition, is valuable in providing a first order physical image of the nuclear shape for a given IBA calculation, for understanding in a simple way the dependence of various observables on χ, and for providing a benchmark for analytical models relating the IBA to the classical limit. Of course, it must not be construed that the resultant associated geometrical image of an IBA calculation will yield all the same predictions as the IBA since some excitations in the latter (e.g., β vibrations) are inherently different from their geometrical namesakes.

In the consistent Q formalism[11] an IBA Hamiltonian can be written

$$H = -\kappa Q \cdot Q - \kappa' L \cdot L \ , \tag{1}$$

where L is a boson angular momentum operator and Q is the quadrupole operator

$$Q = (s^\dagger \tilde{d} + d^\dagger s)^{(2)} + (\chi/\sqrt{5})(d^\dagger \tilde{d})^{(2)} \ .$$

Additional terms in H can always be added to fine tune specific calculations but will not significantly alter any of the conclusions below. The term in L can be neglected since it is diagonal. Then the Hamiltonian contains only one term. Since κ acts only as an energy scale factor, the one parameter of significance is χ in the Q operator. It takes on the values -2.958 and 0 in the SU(3) and O(6) limits, respectively, and the crucial characteristics of the whole range of spectra from SU(3) through transitional nuclei to O(6) are generated[11] simply by allowing χ to vary from -2.958 to 0. The same value of χ is used in the $E2$ transition operator $T(E2) = \alpha Q(\chi)$. The IBA calculations were performed with the codes[12] PHINT and FBEM. From these calculations the energy ratio $E_{2_2^+}/E_{2_1^+}$ was obtained, the coefficients κ and κ' in Eq. (1) being chosen to give the triaxial value of 2.0 for $\gamma = 30°$ when $\chi = 0$. The $E2$ branching ratios

$$B(E2{:}2_2^+ \rightarrow 2_1^+)/B(E2{:}2_2^+ \rightarrow 0_1^+)$$

and

$$B(E2{:}3_1^+ \rightarrow 4_1^+)/B(E2{:}3_1^+ \rightarrow 2_1^+) \ ,$$

which are independent of κ and κ', were calculated with a consistent $T(E2)$ transition operator. Using the Davydov model, the same quantities were calculated as a function of γ. Examples of the results are compared in Fig. 2. It is immediately evident that the overall behavior of the various quantities versus χ, or versus γ, is remarkably similar, even to the extent that the ratio

$$B(E2{:}2_2^+ \rightarrow 0_1^+)/B(E2{:}2_1^+ \rightarrow 0_1^+)$$

maximizes at the same value of ≈ 0.07 in the two formalisms. There are, of course, evident differences in the *detailed* shapes of corresponding curves in the two models: this will be reflected below in the result that while each observable leads to a well defined $\gamma_{\text{eff}} \leftrightarrow \chi$ correspondence, it is nonetheless nonlinear. It is also not immediately clear that each observable will lead to the *same* $\gamma_{\text{eff}} \leftrightarrow \chi$ correspondence. As alluded to above, though, a consistent correspondence does result, within an accuracy of a couple of degrees, lending credence to the concept of a valid effective asymmetry for an IBA-1 calculation. The relationship between γ_{eff} and χ is displayed in Fig. 3.

An interesting aspect of this figure is that γ_{eff} does not $\rightarrow 0$ as $\chi \rightarrow -2.958$. This is a finite boson number effect. For example, the ratio $E_{2_2^+}/E_{2_1^+}$ is infinite in the Davydov model for $\gamma = 0°$ and finite for finite γ. In the SU(3) limit of the IBA, it is given by $[\kappa(2N-1)/(0.75\kappa - \kappa')] + 1$ in terms of the coefficients in Eq. (1) and the boson number N. Clearly, as $N \rightarrow \infty$ so will this ratio, and the corresponding γ_{eff} value is 0° as expected. For finite N, however, the SU(3) limit gives a finite ratio and therefore the associated γ_{eff} will be finite. In any case, for small γ, differences of a few degrees correspond to miniscule wave function differences. Thus, for example, in the Davydov model, for $\gamma = 10°$ the $K = 2$ admixture[10] in the 2_1^+ ground band state is only 0.008. The potential curves in Fig. 1 are for a given N, and become steeper as N increases, effectively localizing γ near 0°. Indeed, in the limit $N \rightarrow \infty, \gamma_{\text{eff}} = 30°$ only for $\chi = 0$ and drops increasingly rapidly to 0° for finite χ.

A further point of interest concerns the deformation vari-

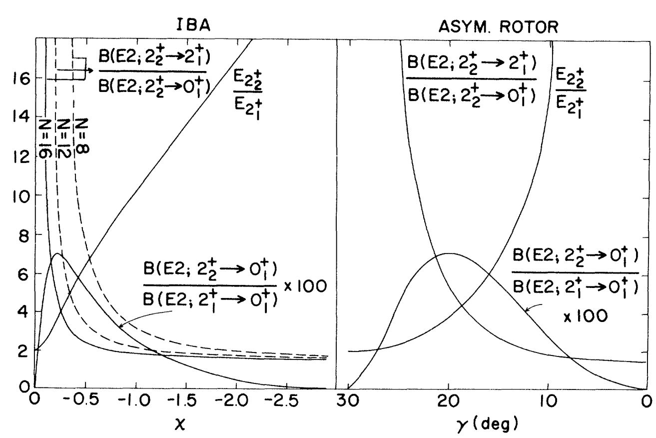

FIG. 2. Energy and $B(E2)$ ratios in the IBA and the asymmetric rotor model (Refs. 9–12).

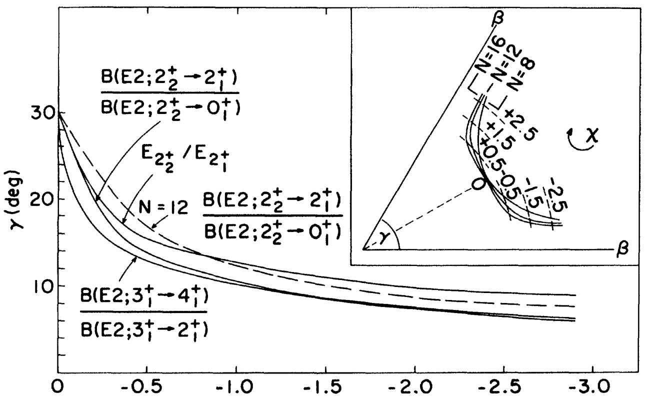

FIG. 3. Asymmetry parameter γ vs χ deduced from several energy and $B(E2)$ ratios for $N=16$ at $N=12$ (dashed line). Insert: "pie" plot for $N=8$, 12, and 16: the schematic dependence on β is obtained with Eq. (2).

able β of the geometrical framework. As shown by Ginnochio and Kirson,[1] the β corresponding to a given IBA solution depends, in general, on all of the parameters in the IBA Hamiltonian, and cannot therefore be simply related to χ across the full transitional region. Nevertheless, the intrinsic state approach, in its simplest form, which assumes no K mixing and large N values, yields[1] a relationship between β and χ for the Hamiltonian $H=-\kappa Q\cdot Q$, given by

$$\beta_{\rm IBA}=\tfrac{1}{2}\{-(\tfrac{2}{35})^{1/2}\chi\pm[(\tfrac{2}{35})\chi^2+4]^{1/2}\} \ , \qquad (2)$$

which gives $\beta_{\rm IBA}=\sqrt{2}=1.414$ in SU(3) ($\chi=-2.958$). As $\chi\rightarrow 0$, $\beta_{\rm IBA}$ decreases, reaching the value $\beta_{\rm IBA}=1.0$ in the O(6) limit ($\chi=0$). One can regain contact with geometrical models with the relation[1] $\beta_{\rm geom}\approx 1.18(2N/A)\beta_{\rm IBA}$, where N is the boson number deduced from the number of valence nucleons.

The insert to Fig. 3 shows the approximate $(\beta,\gamma)\leftrightarrow\chi$ correspondence deduced here, for several N values, in terms of the familiar $\beta-\gamma$ "pie" plot which here includes positive χ values from 0 to $+2.958$ corresponding to the O(6) to oblate SU(3) limit, and therefore to γ values from near 30° to near 60°. In Fig. 2, the results for one of the $B(E2)$ ratios were shown for several N values and, for $N=12$, the $\gamma_{\rm eff}\leftrightarrow\chi$ correspondence is shown in Fig. 3 as deduced from this branching ratio. Clearly, it corresponds to slightly larger γ values. From the N dependence of the $\gamma_{\rm eff}\leftrightarrow\chi$ relationship, it is clear that, given the near constancy[11] of χ for well deformed nuclei, the IBA will predict a cup-shaped systematics for γ, minimizing at midshell. This is in agreement with known[6,10] empirical trends and is another example of an effectively microscopic aspect of the phenomenological IBA-1.

To summarize, it has been shown that the IBA-1 automatically contains an effective asymmetry. Values for $\gamma_{\rm eff}$ have been deduced in terms of the parameter χ characterizing IBA calculations in the consistent Q formalism. These results have the practical benefit of providing a prescription for an IBA treatment of a given nucleus: once a $\gamma_{\rm eff}$ value has been deduced, the $\gamma_{\rm eff}\leftrightarrow\chi$ association determines the required input for an IBA calculation. The strong boson number dependence of extracted γ values was noted. It is interesting to note that in the IBA-1, the γ *softness* necessarily increases with the mean $\gamma_{\rm eff}$ (see Fig. 1) so that, for example, large $\gamma_{\rm eff}$ values imply large γ softness, corresponding to γ unstable, rather than triaxial, spectra. The addition of cubic terms[13] in the IBA–1 Hamiltonian introduces a component in the potential with a minimum at $\gamma=30°$ and therefore allows more flexibility both in the mean γ and in the associated γ softness. It is an important open question whether the spectra of asymmetric nuclei will require that added complexity.

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