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Quasiparticle properties of neutron matter

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The effective mass and pole strength for protons and neutrons are evaluated in neutron matter for different densities. The source for the N-N interaction is the nonstatic one-boson-exchange model, and the many-body problem is treated self-consistently by looking simultaneously at the renormalization of the nucleons and the bosons in the medium. The π and ρ meson exchanges in the exchange channel are shown to play an important role in the energy and momentum dependence of the nucleon self-energy. The renormalization of the meson properties in the medium, especially the pion properties, stresses the energy dependence of the interaction, making retardation effects important at high densities. Some astrophysical implications of our results are also discussed.

[NUCLEAR STRUCTURE Neutron matter. Effective mass, strength factor. Boson exchange model used. Self-consistent treatment.]

I. INTRODUCTION

In this paper we want to study the nuclear properties of pure neutron matter, which are of particular relevance in neutron stars where there is only a small fraction of protons to provide the equilibrium against neutron β decay.

Earlier studies of neutron matter have been carried out in the framework of the Brueckner-Bethe theory[1,2] or by means of hypernetted chain expansions.[3,4] A common feature of these expansions is the use of a static N-N interaction, which neglects the finiteness of the velocity of propagation of the interaction. While this approximation is very reasonable for the short range pieces of the N-N force, it is more questionable when dealing with the long range part, i.e., the one pion exchange part of the interaction. It becomes even more questionable at large nuclear densities, where the pion becomes progressively softer, and consequently the interaction due to one pion exchange of longer range.

In a previous paper[5] the authors have developed a many-body scheme in order to study the properties of symmetric nuclear matter, starting from a nonstatic source for the N-N interaction, which is the one-boson-exchange model. The problem is solved self-consistently in the sense that both the nucleons and the mesons are allowed to interact with the medium. Thus the same N-N interaction now becomes a function of the nuclear density. The pion and, to a lesser extent, the ρ meson are shown to play a major role as the factors responsible for the quasiparticle properties of the nucleons, the effective mass and the pole strength. At normal nuclear matter density the results are comparable to the empirical data extrapolated from finite nuclei and also comparable to other results based on the Brueckner-Bethe expansion. The scheme provides a method to study the properties of nuclear matter at higher densities, where other approaches, based on static potentials and without the self-consistency requirement, are expected to give inaccurate results. On the other hand, in this self-consistent scheme the pions are allowed to interact with a Fermi sea of interacting nucleons, in contrast to standard calculations which use an uncorrelated Fermi sea. This feature has some important consequences with respect to the problem of pion condensation, raising the threshold for condensation four times above nuclear matter density, with standard values for the spin-spin correlation parameter g'. The purpose of this paper is to use the same scheme to study the quasiparticle properties of pure neutron matter.

II. MODEL FOR THE NUCLEON SELF-ENERGY

As shown in Ref. 5, the scalar and vector mesons σ and ω of the one-boson-exchange model[6,7] contribute only a small fraction of the energy and momentum dependence of the nucleon self-energy. Hence only π and ρ will be taken into account to construct the nucleon self-energy. The starting point will be the effective Lagrangians, coupling the mesons to the nucleons, which in the nonrelativistic reduction for the nucleons can be written as[8]

$$\begin{aligned} \delta\mathscr{L}_{\pi\mathrm{NN}}(x)&=\frac{f}{m_\pi}\psi^\dagger(x)\sigma_i\nabla_i\tau_\lambda\phi^\lambda(x)\psi(x)\ ,\\ \delta\mathscr{L}_{\rho\mathrm{NN}}(x)&=\frac{f_\rho}{m_\rho}\psi^\dagger(x)[\vec{\sigma}\times\vec{\nabla}]_i\tau_\lambda\rho_i^\lambda(x)\psi(x)\ , \end{aligned} \tag{1}$$

where $\psi(x)$, $\phi_\lambda(x)$, and $\rho_i^\lambda(x)$ are the nucleon, π, and ρ meson fields, respectively.

The first step in calculating the nucleon self-energy would be to consider the Hartree terms, which in this case do not contribute since we have a spin saturated neutron Fermi sea. Thus the first term to contribute would be the Fock term shown in Fig. 1. The nonstatic effects of the

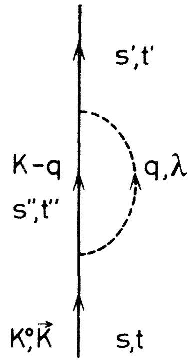

FIG. 1. Fock term in the nucleon self-energy from pion exchange. The variables k,q are quadrimomenta and s,t refer to the spin and isospin indices, respectively.

nucleon-nucleon interaction will be already present in this Feynman diagram since there is an integration over the internal pion four momentum, and thus the pion propagator will appear with its energy dependence. This is in contrast to the static potential picture where the energy variable is set to zero. Another interesting feature at this lowest order is that the nucleon self-energy will already have an energy dependence, i.e., it will have a "dynamical" character in contrast to the standard Hartree-Fock approximation with a potential, which is known to be static, i.e., there is no dependence of the nucleon self-energy on the nucleon external energy.

One of the aims of this many-body scheme is to treat both nucleons and mesons on the same footing. Thus we want to calculate the meson self-energy and use, in the Feynman diagram of Fig. 1, the full meson propagators in the medium, in the same way as we have done for the nucleons.

The propagation of pions in a nuclear medium has received a great deal of attention so far and there is a widely accepted microscopic picture consisting of the excitation of p-h or Δ-h for the p-wave part,[8] plus an s-wave part which can be conveniently represented, on a phenomenological level, by means of the zero-range effective Hamiltonian of Koltun and Reitan[9]

$$\delta H_s = 4\pi\left[\frac{\lambda_1(t)}{m_\pi}\bar{\psi}\vec{\phi}\vec{\phi}\psi + \frac{\lambda_2(t)}{m_\pi^2}\bar{\psi}\vec{\tau}(\vec{\phi}\times\vec{\pi})\psi\right]. \qquad (2)$$

In this expression t is the pion four momentum transfer, and $\vec{\pi}$ is the momentum canonically conjugate to $\vec{\phi}$ and λ_1, λ_2, two parameters that can be related, on shell, to simple linear combinations of the empirical πN s-wave scattering lengths.

With all these ingredients the resulting diagram for the nucleon self-energy is shown in Fig. 2, where the excited p-h or Δ-h states are seen to propagate in the medium via the corresponding p-h or Δ-h interaction which is shown there by the wavy lines. By recalling that originally we exchanged pions or ρ mesons it is clear that we do not need the full p-h or Δ-h interaction, but only the part of it that carries the appropriate quantum numbers corresponding to the pion or ρ meson.

A model is made in Ref. 10, where the corresponding p-h, Δ-h interaction is constructed from the bare one pion plus one ρ-meson exchange modulated by a correlation function which is generated by the simultaneous multiple exchange of ω mesons, the responsible agent for short range repulsion in our model. The corresponding Δ-h interaction is given by

$$W(\vec{q},w) = [V_\pi(\vec{q},w)\vec{S}_1\cdot\hat{q}\vec{S}_2^\dagger\cdot\hat{q} + V_\rho(\vec{q},w)(\vec{S}_1\times\hat{q})(\vec{S}_2^\dagger\times\hat{q})$$
$$+g'(\vec{q},w)\vec{S}_1\cdot\vec{S}_2^\dagger + h'(\vec{q},w)S_{12}^*(\hat{q})]\vec{T}_1\cdot\vec{T}_2^\dagger, \qquad (3)$$

where

$$V_i(\vec{q},w) = \frac{f_i^*(q^2)}{m_i^2}\frac{\vec{q}^{\,2}}{w^2-\vec{q}^{\,2}-m_i^2+i\delta} \qquad (4)$$

and

$$S_{12}^* = 3\vec{S}_1\cdot\hat{q}\vec{S}_2^\dagger\cdot\hat{q} - \vec{S}_1\vec{S}_2^\dagger .$$

$\vec{S}$ and $\vec{T}$ are the transition spin and isospin operators and $f_i^*(q^2)$ the corresponding coupling constants which incorporate a monopole form factor consistent with empirical determinations from pion absorption in the deuteron,[11] the np and $\bar{p}p$ charge exchange scattering[12] and dispersion-theoretical analysis[13] (the last two for the πNN vertex). For the p-h interaction a similar form holds with $\vec{\sigma}$ and $\vec{\tau}$ replacing $\vec{S}$ and $\vec{T}$ and different coupling constants $F_i(q^2)$. The main effect of the short range interaction here is the introduction of the g' term, which has a smooth dependence on the energy and momentum variables[10] and can be very well approximated by a constant g', the usual parameter to account for the Lorentz-Lorenz effect. The tensor correlation $h'(\vec{q},w)$ has only influence at large momenta. Explicit form of this interaction in terms of the π and ρ coupling constants can be seen in Ref. 10. The correlated pieces associated with the ρ exchange give the largest contribution to the g' parameter in this model; hence changes in g' are implemented here by changing the ρ coupling constant, which is not determined too precisely.[14]

The actual calculation of $\Sigma(k^0,\vec{k})$, the nucleon self-energy, is more complicated than the one for symmetric

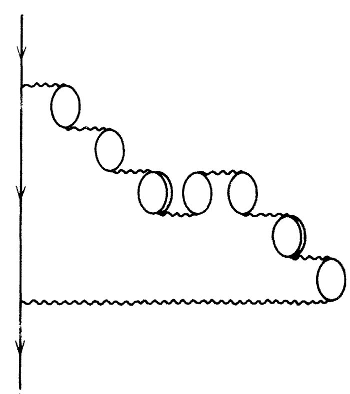

FIG. 2. Model for the nucleon self-energy. The wavy lines refer to π and ρ exchange plus additional short range correlations. The π and ρ are allowed to interact with the medium via p-h or Δ-h excitation.

nuclear matter, in the sense that one has to calculate explicitly the contribution from the exchange of every charged state and also carry out the calculation separately for neutrons and protons.

We will omit here all intermediate steps leading to the final form of the nucleon self-energy, which can be easily deduced from Ref. 5 with minor modifications. In the actual calculation we will also take the numerical constants that were used in that paper. The expression for Σ now reads

$$\Sigma_{tt'}(k)=i3\delta_{ss'}\delta_{tt'}\sum_r C(\tfrac{1}{2},1,\tfrac{1}{2};t,-r,t-r)^2 \int \frac{d^4q}{(2\pi)^4}\left[G_0^{(t-r)}(k-q)\frac{1}{1-aU_r}\left[3a+\frac{b\vec{q}^{\,2}}{1-aU_r-b\vec{q}^{\,2}U_r}\right] - \frac{1}{k^0-q^0-\epsilon(\vec{k}-\vec{q})+i\delta}(3a+b\vec{q}^{\,2})\right] . \tag{5}$$

where s,t are the nucleon spin and isospin third components, $\epsilon(\vec{k})$ is the nucleon kinetic energy, and $G_0^{(t)}(k)$ is the nucleon propagator for the neutrons or protons in an uncorrelated neutron Fermi sea. In the above equation the nucleon propagator is given by

$$G_0^{(t)}(k)=\frac{1}{k^0-\dfrac{\vec{k}^{\,2}}{2m}+i\delta}+2\pi i n(\vec{k})\delta_{t,-1/2}\delta\left[k^0-\frac{\vec{k}^{\,2}}{2m}\right] , \tag{6}$$

where $n(\vec{k})$ is the occupation number. The medium correction, given by the second term, appears only for neutrons since there is no Fermi sea for protons. With trivial changes in the Clebsch-Gordan coefficient and the substitution of G_0 by the Δ propagator, one can immediately include terms with Δ's in intermediate states. Our calculations include these terms.

The function U_r in (5) is the Lindhard function corresponding to p-h plus Δh excitation by a pion of isospin third component r. Explicit formulas for real and complex values of the energy variable, with different proton and neutron masses, can be seen in the Appendix.

Finally, a and b are given by the following expression:

$$\begin{aligned} a&=\frac{f^2}{m_\pi^2}[\vec{q}^{\,2}D_0^\rho(q)F_\rho^2(q^2)C_\rho-\tfrac{1}{3}q_c^2\widetilde{D}_0^\pi(q)\widetilde{F}_\pi^2(q^2)\\ &\quad -(\vec{q}^{\,2}+\tfrac{2}{3}q_c^2)\widetilde{D}_0^\rho(q)\widetilde{F}_\rho^2(q^2)C_\rho] ,\\ b&=\frac{f^2}{m_\pi^2}[D_0^\pi(q)F_\pi^2(q^2)-D_0^\rho(q)F_\rho^2(q^2)C_\rho\\ &\quad -\widetilde{D}_0^\pi(q)\widetilde{F}_\pi^2(q^2)+\widetilde{D}_0^\rho(q)\widetilde{F}_\rho^2(q^2)C_\rho] , \end{aligned} \tag{7}$$

where $D_0^i(q)$ are the meson propagators

$$D_0^i(q)=(q^{02}-\vec{q}^{\,2}-m_i^2+i\delta)^{-1} , \tag{8}$$

$F_i(q^2)$ are the corresponding form factors

$$F_i(q^2)=\left[\frac{\Lambda_i^2-m_i^2}{\Lambda_i^2-q^2}\right] , \tag{9}$$

and C_ρ is the ratio of ρ meson to pion coupling constant

$$C_\rho=\left[\frac{f_\rho^2}{m_\rho^2}\right]\bigg/\left[\frac{f^2}{m_\pi^2}\right] .$$

The functions $\widetilde{D}_0^i(q)$, $\widetilde{F}_i(q^2)$, are the corresponding meson propagator and form factor obtained by substituting $\vec{q}^{\,2}$ by $\vec{q}^{\,2}+q_c^2$, q_c being a parameter that characterizes the range of the nuclear correlation function and which is taken equal to the ω mass.

The expression for Σ given by (5) clearly exhibits its genuine many-body content, in the sense that it vanishes when the nuclear density, and thus U_r goes to zero.

The s-wave pion self-energy is immediately taken into account by adding it to the pion square mass in the pion propagator $D_0(q)$ in b given by (7). The pion self-energy is given explicitly by

$$\Pi_r^{(s)}(q)=4\pi\left[\frac{2\lambda_1}{m_\pi}(\rho_n+\rho_p)-r\frac{\lambda_2}{m_\pi^2}2q^0(\rho_n-\rho_p)\right] \tag{10}$$

with ρ_n, ρ_p the neutron and proton densities. The parameters λ_1 and λ_2 at $t=0$ can be related to the s-wave π-N scattering lengths in the isospin channels $T=\frac{1}{2}$, $\frac{3}{2}$, a_1, and a_3, by the simple relations

$$\begin{aligned} \lambda_1&=-\frac{1}{6}\frac{m+m_\pi}{m}(a_1+2a_3)m_\pi ,\\ \lambda_2&=\frac{1}{6}\frac{m+m_\pi}{m}(a_1-a_3)m_\pi , \end{aligned} \tag{10a}$$

where the factor $(m+m_\pi)/m$ is the typical kinematical factor $\sqrt{s}/m$, at threshold, which relates the usual f amplitude ($|f|^2=d\sigma/d\Omega$) with the invariant T matrix that would come directly from the effective Hamiltonian (2). The numerical values are then given by

$$\lambda_1(t=0)=0.0075 \text{ and } \lambda_2(t=0)=0.053 . \tag{10b}$$

Some models have been made for the t dependence of these parameters.[11] For the second term in (10) which is supposed to account for ρ exchange in the t channel, this dependence should naturally be the t dependence of the ρ propagator, as is assumed in Ref. 11. In our case, although the pion might be off shell, t will always be equal to zero since the pion keeps its energy and momentum. Thus, we simply take the on-shell values at $t=0$ for λ_1 and λ_2.

In the last term of the second term of (5) the b function should not include this s-wave π self-energy in the pion propagator since this piece is meant to subtract the self-energy pieces when $\rho=0$. The final formulas are much simpler if the same b function is taken in both terms. Estimates of the errors induced by this change give us enough confidence to make it, since the k^0 and $\vec{k}$ dependence of Σ is barely affected by the pion s wave.

In order to perform the q^0 integration in (5) we find it useful to make a Wick rotation, but for such purposes it is important to know the analytical structure of U_r, which is shown in the Appendix. If μ_n and μ_p are the neutron and proton chemical potentials, we find that U_{π^-} has an analytic cut in the real axis from $-\infty$ to $\mu_n-\mu_p$, with the function continuous below the real axis. Analogously U_{π^+} has the cut from $-(\mu_n-\mu_p)$ to ∞ with the function continuous above the real axis. Finally for a π^0 the cut goes along the real axis with the function continuous above it for $\mathrm{Re}q^0>0$ and below it for $\mathrm{Re}q^0<0$.

This suggests the Wick rotations shown in Fig. 3 and as a consequence the integrals for real q^0 will be substituted by similar ones along a complex line parallel to the imaginary axis and the contribution from a pole, with the following result:

$$
\begin{aligned}
\Sigma_{tt'}(k)=&-3\delta_{ss'}\delta_{tt'}\sum_r C(\tfrac{1}{2},1,\tfrac{1}{2};t,-r,t-r)^2\frac{1}{(2\pi)^2}\int_0^\infty q^2dq \\
&\times\left\{\int_{-\infty}^{\infty}\frac{dx}{2\pi}\frac{m}{k\cdot q}\ln\frac{k^0-ix-\mu_r-\dfrac{(k-q)^2}{2m}}{k^0-ix-\mu_r-\dfrac{(k+q)^2}{2m}}M_r(q^0,q)\Big|_{q^0=\mu_r+ix}\right.\\
&-\int_{-1}^{1}d(\cos\alpha)\theta[k^0-\epsilon(\vec{k}-\vec{q})-\mu_r]M_r(q^0,q)\Big|_{q^0=k^0-\epsilon(\vec{k}-\vec{q})}\\
&\left.+\int_{-1}^{1}d(\cos\alpha)n(\vec{k}-\vec{q})\frac{1}{1-aU_r}\left[3a+\frac{bq^2}{1-aU_r-bq^2U_r}\right]_{q^0=k^0-\epsilon(\vec{k}-\vec{q})}\right\}
\end{aligned}
\tag{11}
$$

where

$$
M_r(q^0,q)=\frac{U_r}{1-aU_r-bq^2U_r}\left[3a^2+2abq^2+b^2q^4-\frac{2a^2bq^2U_r}{1-aU_r}\right]
\tag{12}
$$

and

$$\mu_r=-r(\mu_n-\mu_p)\ .$$

The remaining two integrations are performed numerically.

We should note at this point that the whole formal development is based on a ground state of pure neutron matter. As we already know, when compressing the neutron matter at higher density there might be the chance to create a pion condensate, and the ground state of the system would now accommodate a certain amount of condensed pions.[15–18] Hence, the formal development should now be changed to account for these pions as well. This poses a natural limit to the validity of our formal approach at the threshold for pion condensation in neutron matter. The onset of this phase transition has been thoroughly investigated by us,[19] by looking carefully at

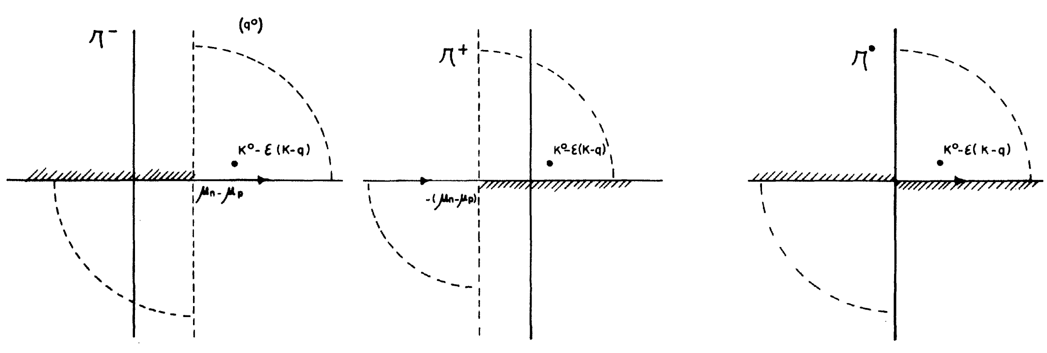

FIG. 3. Diagrammatic representation of the q^0 complex plane with the analytical cuts and the nucleon pole of the integrand in Eq. (5). The dashed curve shows the path followed in the complex integration.

the different branches that the pions develop in a neutron medium. The conclusion which is relevant for this paper is that for values of g' in the commonly accepted range $g'=0.5$—0.7 there are no pion condensates at any density. The natural limit for this approach would come now at another phase transition, when the neutrons would lose their identity and would give way to the quark degrees of freedom.

Another point is worth noting in this approach. When calculating the binding energy from the nucleon propagator that we have evaluated, we would be considering a subseries of the Brueckner-Bethe expansion, formed by the Ring diagrams. It is interesting to note that this series was shown to be asymptotic in Ref. 20 for symmetric nuclear matter, with growing oscillations starting at relatively low order for normal nuclear matter density. The method used here also has the virtue of summing exactly this series, avoiding the nonconvergence of the series expansion. However, a partial convergence was found in Ref. 20 for low orders of the series expansion, to which they linked the apparent convergence of the Brueckner expansion. This convergence was found to become worse for increasing densities. The exact sum of the Ring series that we do here also makes our approach more reliable at large densities than a conventional Brueckner expansion.

As stated in Sec. II, we have selected only a part of the p-h interaction which is the one with $T=1$ in the t channel, where one pion exchange plays a major role. As was the case in Ref. 5, the pion plays an important role in providing the energy and momentum dependence of the nucleon self-energy, essentially because of its small mass. This makes the pion propagator, once the self-energy is included, very sensitive to changes in the momentum and energy variables, since the self-energy can be appreciably larger than m_π^2.

This can be seen better in Fig. 1 if we replace $k-q\rightarrow k$. If we take a renormalized one pion exchange for the dashed line, the nucleon self-energy would contain the product of propagators $G_0(q)D(k-q)$, and thus the dependence on k^0 would be contained in the pion propagator, $D(k-q)$, given by

$$[(k^0-q^0)^2-(\vec{k}-\vec{q})^2-\mu^2-\Pi(k^0-q^0,\vec{k}-\vec{q})]^{-1}$$

where Π is the pion self-energy. The k^0 dependence of Σ is stressed because the pion mass is small. Also, the attractive character of Π goes in towards canceling the pion kinetic energy, which once again stresses the k^0 dependence of the pion propagator and hence of the nucleon self-energy.

One may wonder about the contribution from other pieces of the p-h interaction, for example, from two pion exchange. We can think of a dispersion theoretical approach where the p-h interaction would have the pion pole and the exchange of a continuous mass distribution, which would begin at $m^2=4m_\pi^2$.

The one pion exchange piece, given essentially by the pion propagator $[(k^0-q^0)^2-(\vec{k}-\vec{q})^2-\mu^2]^{-1}$ would then be replaced by

$$\int_{4m_\pi^2}^{\infty} dt\rho_{2\pi}(t)/[(k^0-q^0)^2-(\vec{k}-\vec{q})^2-t] ,$$

where $\rho_{2\pi}(t)$ is the spectral function representing the mass spectrum of the exchanged 2π system, and which can be obtained from the $\pi\pi\rightarrow$NN helicity amplitudes.[30] The $T=0$ channel in the 2π system has a mass distribution peaking around 550 MeV while the $T=1$ channel peaks around 750 MeV and is fairly well reproduced by the ρ meson distribution. In all cases the range of t in the dispersion integral is fairly larger than m_π^2, and this automatically reduces the k^0 and $\vec{k}$ dependence of the dispersion integral. Indeed, the $T=1$ part, that we replace in our calculation by the ρ-meson pole, will be shown explicitly to contribute little to the k^0 dependence of Σ when we evaluate the strength factor. The $\vec{k}$ dependence will be somewhat more important because of the extra momentum dependence of the ρNN vertex. The same can be said about other irreducible pieces, like the two pion exchange in the $T=0$ channel, which is normally accounted for in terms of a "σ" exchange. Calculations done by Holinde and Machleidt, mentioned in Ref. 5 in the same context as here, again show that the k^0 dependence from these pieces is negligible and the $\vec{k}$ dependence is very small compared to the results that we obtain here.

With respect to the relevance of the q^0 dependence of the medium modified NN interaction we can just quote the results of Ref. 5 on the density dependence of the effective mass. This magnitude was shown to increase with the density, and the q^0 dependence was the reason for it. This is in contrast to the results of Futami *et al.*,[31] where essentially the same model was used to evaluate the effective mass, but the q^0 dependence of the interaction was neglected. In the latter work the effective mass decreased with the density instead of increasing. Also, at normal nuclear density our results for m^* at the Fermi surface gave $m^*\approx 1.1m$, in fair agreement with experiment, while their approach inevitably gave $m^*<m$ ($m^*\approx 0.6m$) because of the absence of the energy dependence.

One more word should be said about the s-wave pion self-energy, Eq. (10). This expression was derived from the effective Lagrangian (2) and the parameters λ_1, λ_2 were kept fixed without worrying about their energy dependence. Note however that the isovector term in (10) is proportional to q^0, giving thus a fair q^0 dependence to the most important part of the s-wave pion self-energy in neutron matter. We should also note that the k^0 dependence of the nucleon self-energy through $\Pi^{(s)}$ comes from the last two terms of Eq. (11) where $q^0=k^0-\epsilon(\vec{k}-\vec{q})$. The values of q^0 involved there are actually small $q^0\approx 0$—60 MeV (at $\rho\simeq\rho_0$), and thus we are dealing with low frequency pions for which Eqs. (2) and (10) are fair approximations. With respect to the isovector piece of $\Pi^{(s)}$, which comes from the isovector part of the s-wave πN interaction, we would like to mention that it is fairly well described in terms of the vector part of the ρ exchange in the t channel,[30,32] which leads to the q^0 dependence written in (10). In any case, we should also mention that the s-wave part of the pion self-energy is at least one order of magnitude smaller than the p-wave self-energy in the pion kinematical region of largest contribution to Σ, which comes around small values of q^0, and $|\vec{q}|\approx 3$—5μ. Thus, as we mentioned before, the s-wave pion self-energy does not play an important role in the quasiparticle properties of the nucleons.

III. QUASIPARTICLE PROPERTIES

A direct evaluation of (11) gives us the nucleon self-energy as a complex function of the independent variables $k^0, \vec{k}$

$$\Sigma(k^0,\vec{k})=\Sigma_R(k^0,\vec{k})+iW(k^0,\vec{k}) . \quad (13)$$

For a Fermi sea of particles the imaginary part is known to vanish at $k^0=\epsilon_F$, the Fermi energy, and to behave as[21]

$$W(k^0,\vec{k})\approx S_k(k^0-\epsilon_F)^2\mathrm{sgn}(\epsilon_F-k^0) , \quad k^0\to\epsilon_F . \quad (14)$$

Here we find the first difference in the behavior of protons and neutrons. For neutrons the structure of (14) results in our case from the analytic structure of (11) and comes out numerically from a cancellation between the imaginary parts of the different terms. Instead, for protons, where there is no Fermi sea, the imaginary part rises monotonically with the energy. The particular structure of (14) makes the quasiparticle approximation very useful. There the neutron propagator can be written approximately as

$$G_n(k^0,\vec{k})=\frac{Z_k}{k^0-\dfrac{\vec{k}^2}{2m_n^*}-iZ_k W\left[\dfrac{\vec{k}^2}{2m_n^*},\vec{k}\right]} , \quad (15)$$

where Z_k is the residue of the propagator at the pole, or pole strength, and m_n^* the neutron effective mass, given, respectively, by

$$Z_k=\left[1-\frac{\partial\Sigma(k^0,\vec{k})}{\partial k^0}\right]^{-1}_{k^0=\vec{k}^2/2m^*} , \quad (16)$$

$$m^*=m\left.\frac{1-(\partial\Sigma(k^0,\vec{k})/\partial k^0)}{1+2m(\partial\Sigma(k^0,\vec{k})/\partial k^2)}\right|_{k^0=\vec{k}^2/2m^*} . \quad (17)$$

By taking $\vec{k}^2/2m_n^*$ for the neutron energy, we have already set the energy origin at the energy of a neutron with zero momentum.

The quasiparticle approximation for protons would be less useful, except at small energies, where the imaginary parts are still small. At large energies, of around the neutron Fermi energy, Z_k and m^*, calculated according to (16) and (17), develop important imaginary parts.

There is another difference with respect to symmetric nuclear matter. Once the energy origin is set, the protons of zero momentum will have a certain energy not necessarily zero. This magnitude is calculated self-consistently by means of the equation

$$\epsilon_p=\Sigma_p(\epsilon_p,0)-\Sigma_n(0,0) , \quad (18)$$

and is found to converge to a value around -20 MeV, rather independent of the neutron density. Because of this fairly large starting energy, the proton energy can be quite well approximated by

$$\epsilon_p(\vec{k})\approx\epsilon_p+\frac{\vec{k}^2}{2m} \quad (19)$$

for momenta smaller than the neutron Fermi momentum. This is the approximation made for the proton energy in the evaluation of our integrals. The neutron propagator is instead substituted by the quasiparticle propagator with the aim of attaining a self-consistent solution. Here we refer the reader to Ref. 5 for a detailed discussion of the quality of the approximation.

IV. CHEMICAL POTENTIAL

As stated before, one important ingredient in the analytical structure of $\Sigma(k)$ is the difference between the chemical potentials for neutrons and protons. As shown in the Appendix, this magnitude, in the framework of the quasiparticle approximation, is the maximum possible difference between the energy of a neutron of the Fermi sea and a proton; hence, it is the difference between the neutron Fermi energy and the energy of a proton at zero momentum. It is clear that in the present approach we lack contributions to Σ that would mainly come from a G matrix constructed from σ and ω mesons.

This contribution can be appreciable but it was argued in Ref. 5 that it was rather independent of energy and momentum. On the other hand, this part of the interaction is also isospin independent which might suggest its equal contribution to the self-energy of neutrons or protons. This would be so in the lowest order contribution in the interaction, but for these short range pieces the ladder iteration to generate the G matrix is essential in order to get a meaningful answer, and there the symmetry with respect to protons and neutrons would break down. In other words, although the interaction is an isospin invariant quantity, the ground state is not, and we should expect different contributions to Σ_n and Σ_p from these isospin independent pieces. This implies that our approach, neglecting these last pieces, is not appropriate to calculate $\mu_n-\mu_p$.

On the other hand, $\mu_n-\mu_p$ is a necessary input in this calculation as it was shown when making the Wick rotations. For reasons of consistency it is clear that one has to use in the calculation the same $\mu_n-\mu_p$ that the model provides in order to get the appropriate analytical structure of the Green's function. We will comment later on the repercussions of using this value of $\mu_n-\mu_p$ instead of a more realistic one, but before that let us see the way to calculate it.

In the framework of the model, $\mu_n-\mu_p$ can be calculated as

$$\mu_n-\mu_p=\frac{k_F^2}{2m}+\Sigma_n\left[\frac{k_F^2}{2m_n^*},k_F\right]-\Sigma_p(\epsilon_p,0) , \quad (20)$$

and also, by making use of the quasiparticle approximation, as

$$\mu_n-\mu_p=\frac{k_F^2}{2m_n^*}-\epsilon_p . \quad (21)$$

The numerical agreement ($\sim 2\%$ difference) in both ways of calculating $\mu_n-\mu_p$ is an indirect check of the quality of the quasiparticle approximation.

It is clear that ϵ_p, once the origin of energies is set at the

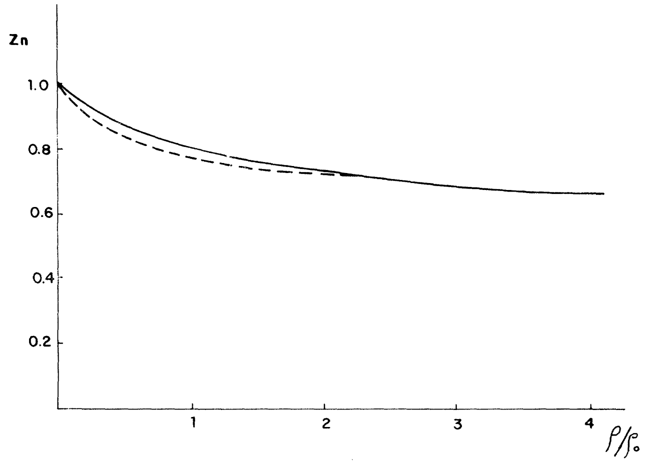

FIG. 4. Neutron pole strength factor Z_n, calculated at the Fermi surface, as a function of the neutron density. Full line, $C_\rho=1.7$; dashed line $C_\rho=2.3$.

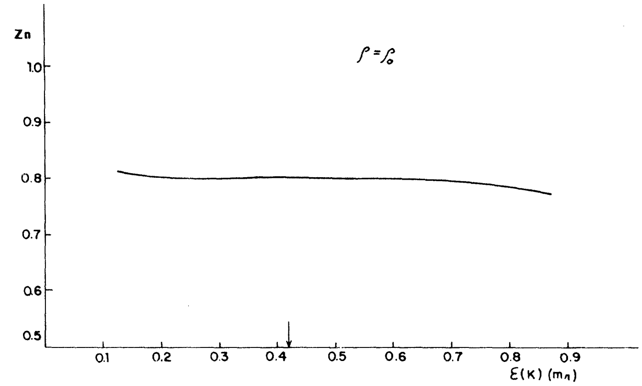

FIG. 6. Neutron pole strength factor, at $\rho=\rho_0$, as a function of the neutron energy, $\epsilon(\vec{k})=\vec{k}^2/2m_n^*$. The arrow signals the Fermi energy.

energy of a neutron with zero momentum, will lack a certain contribution from the missing pieces of the N-N interaction, but this is just a constant term for each neutron density. This would be the missing part in $\mu_n-\mu_p$ as seen in (21). A calculation using a proper value of ϵ_p would introduce a shift in the denominator of (A1) in the Appendix, or in the proton propagators in (6), but this would be a constant shift, independent of the energy and momentum and, hence, would practically not alter the $\vec{k}$ and k^0 dependence of the proton and neutron self-energy at the end. This simply means that we should use the calculated values of ϵ_p and $\mu_n-\mu_p$ as input for successive calculations in the computational procedure with the aim of attaining a self-consistent solution for Σ_n and Σ_p, but only the derivatives of these quantities with respect to k^0 and k^2 will be reliable and not the absolute values, nor the calculated values of ϵ_p and $\mu_n-\mu_p$. This is sufficient for the purpose of this paper which is to calculate the quasiparticle properties of neutrons and protons in a neutron star.

Our numerical procedure calculates Σ_p, Σ_n and then Z, m^*, ϵ_p, and $\mu_n-\mu_p$. The calculated values are used as input for a second calculation and the procedure is followed iteratively until convergence is found, which happens normally after five to ten iterations.

V. RESULTS

In Fig. 4 we have plotted the pole strength Z_k, calculated at the Fermi surface, as a function of the neutron density. This factor falls rather fast from 1 to about 0.8 at $\rho=\rho_0/2$ (ρ_0 is the normal nuclear matter density), and afterwards decreases smoothly to about 0.7 at $\rho=3\rho_0$. We have calculated it for two values of the ρ-meson coupling constant, $C_\rho=1.7$ ($g'=0.55$) and $c_\rho=2.3$ ($g'=0.7$) and, except at small densities, the results are practically the same. The strength factor and its dependence on C_ρ are practically the same in neutron matter as in symmetric nuclear matter.[5]

Figure 5 shows the value of the neutron effective mass, also calculated in the Fermi surface, for the same two values of the ρ-meson coupling constant. We can see a monotonous increase from m to about $1.2m$ at $\rho\approx 4\rho_0$. The effect of the ρ meson is more important in m^* than in Z because of the k dependence induced in Σ by the momentum dependent coupling of the ρ meson to the nucleons, a fact already noticed in symmetric nuclear matter.[22,5] In any case, the changes induced in m^* by the variation of C_ρ within a tolerable range are not very large as can be appreciated in the figure.

It is interesting to look at the dependence of Z_k and m^* on the energy at a given density. Figures 6 and 7 show the remarkable constancy of both Z_k and m^* as functions of

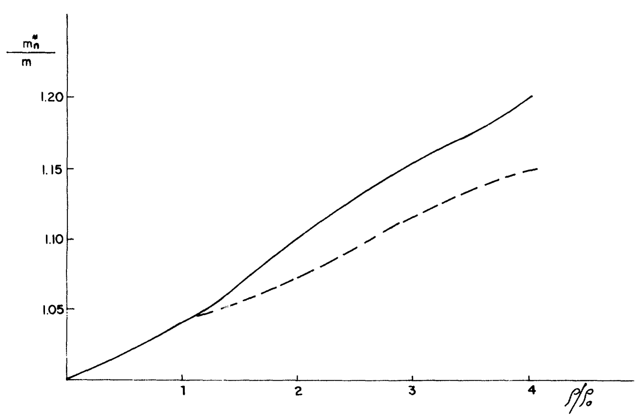

FIG. 5. Same as Fig. 4 for the neutron effective mass.

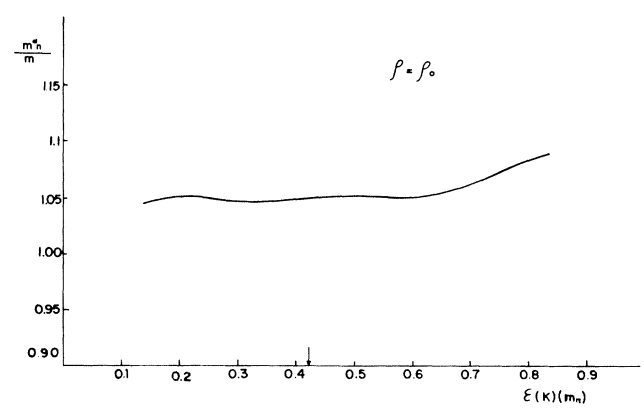

FIG. 7. Same as in Fig. 7 for the neutron effective mass.

the momentum. This is a feature that contrasts with the corresponding one in symmetric nuclear matter, where the effective mass shows a pronounced maximum around the Fermi energy and, on the contrary, Z_k exhibits a minimum at the same energy.[5,23,24] The enhancement of m^* and the dip of Z_k around the Fermi surface is a consequence of the existence of a filled Fermi sea together with the continuous choice for the nucleon single particle energy.[23] However a large part of the neutron self-energy comes from the exchange of charged mesons in Fig. 2, which implies protons in the intermediate states for which there is no Fermi sea. This fact, together with a slight shift in the energies due to the s-wave pion self-energy (which is ignored in symmetric nuclear matter) are responsible for the lack of such maximum or dip in the case of neutron matter.

The proton self-energy is less apt to be cast into a quasiparticle form because of the absence of a Fermi sea. At large momenta, comparable to the neutron Fermi momentum, the imaginary part of Σ is fairly large. Keeping this warning in mind we have used the same formulas (16) and (17) to obtain a complex renormalization factor, Z_p, and effective mass m_p^*. In Fig. 8 we can see Z_p^2 as a function of momentum for $\rho=\rho_0$. The real part is fairly constant, while the imaginary part, always negative, rises in modulus approximately as a linear function of k^2. For energies about double the neutron Fermi energy it can be as large as half the real part.

As for the proton effective mass there are similar features, as can be seen in Fig. 9. The real part is practically constant and very close to the nucleon free mass, while the imaginary part rises again approximately like a linear function of k^2 and it is, as an average, about one order of magnitude smaller than the real part, making more useful the concept of a real effective mass over a large range of momenta. We have thus calculated m_p^* as a function of the density and show it in Fig. 10. The derivatives are calculated at k_F as for neutrons. We can see a smooth decrease of m_p^* at densities below ρ_0 and then a steady increase up to $\rho=4\rho_0$ where m_p^* is around 1.4 m.

At small momenta compared to the neutron Fermi momentum, all the imaginary parts of Σ are small compared to the real parts, and the quasiparticle approximation for protons would also become a useful tool.

With respect to the neutrons, the imaginary part of Z is very small around k_F and about an order of magnitude smaller than the real part at large or small energies in the range of momentum of Fig. 6. Something similar holds for m_n^*/m where the imaginary part is about a factor 20 smaller than the real part for large and small momenta in that same range, and is negligible around k_F.

The actual values of m^* are very important to calculate the neutrino emissivities in neutron stars via the $nn\nu\bar{\nu}$, $np\nu\bar{\nu}$, or the modified URCA process.[25,26] The cooling rate is proportional to $(m^*)^4$. The values used in Ref. 25 for m^* are $m_n^*=m_p^*=0.8$m. With the values of m^* obtained here, always larger than one, the cooling rate would be immediately increased.

More relevant than this is the importance of m^* on the superfluid gap of neutrons and protons[27] which substan-

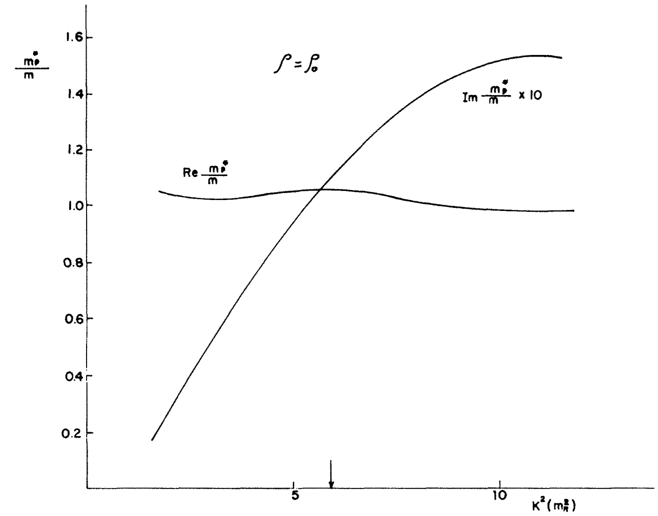

FIG. 9. Same as in Fig. 9 for the proton effective mass.

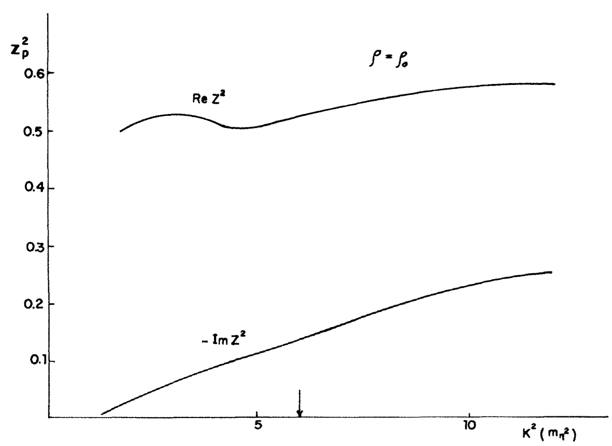

FIG. 8. Real and imaginary parts for the proton pole strength factor, at $\rho=\rho_0$, as a function of momentum. The arrow signals k_F^2 (k_F, neutron Fermi momentum).

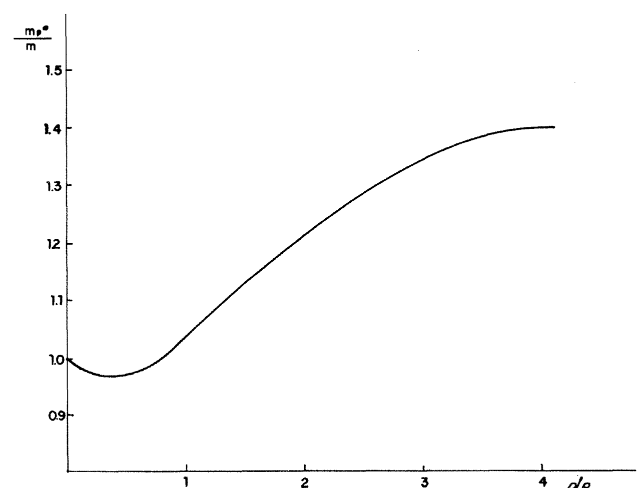

FIG. 10. Proton effective mass, calculated at the neutron Fermi momentum, as a function of the neutron density.

tially changes the cooling scenarios. The gaps are roughly proportional to $\exp[-1/N(0)V_0]$, where $N(0)=m^*k_F/\pi^2$ is the density of states at the Fermi surface and V_0 is an average pairing potential. In the early stages of the cooling of the star when $T>10^9$ K the superfluid gaps suppress the phase space available to the nucleons in the neutrino emission process. However, later on, for temperatures somewhat below the neutron superfluid transition temperature, the superfluidity induces an acceleration of the cooling process due to a drastic reduction in the specific heat. This new cooling mechanism depends strongly on m^*, because of the dependence of the transition temperature on this magnitude.

As an example, let us quote from Ref. 26 the calculated effective surface temperatures of the Crab pulsar with two values of m^*. For $m_n^*=m$, $T_e=1.6\times10^6$ K, while for $m_n^*=0.8m$, $T_e=3.8\times10^6$ K. With our calculated values of $m^*\approx1.2m$, the effective surface temperature would be substantially lower than 1.6×10^6 K to compare with the present observability limit for this star of $T_{\mathrm{Crab}}=4.7\times10^6$ K.

This more efficient cooling mechanism would make it easier to explain present temperatures of other stars,[28] without referring to the even more efficient mechanism based on the β decay of pion condensates.[29]

VI. CONCLUSIONS

We have used a many body scheme, based on a nonstatic boson exchange picture for the N-N interaction, which treats the mesons and the nucleons on the same footing, allowing all of them to interact with the medium. The problem is solved self-consistently and the quasiparticle properties are calculated for neutrons and protons. The renormalized one pion exchange is seen to play a major role with respect to these quasiparticle properties, though heavier mesons play an important indirect role as the responsible agents for the nuclear short range correlations.

The effective masses for neutrons and protons are very close to unity, increasing smoothly with the neutron density. The pole strength factor as a function of the density follows the same trend as in symmetric nuclear matter but there is an important difference with respect to the energy dependence. While in symmetric nuclear matter this factor had a minimum around the Fermi energy, in neutron matter it is a remarkable constant function with the energy. Similarly, the effective mass, which in symmetric nuclear matter had a maximum around the Fermi energy, is also remarkably constant as a function of the energy.

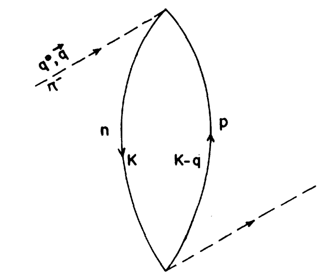

FIG. 11. Feynman diagram entering the calculation of the π^- Lindhard function.

Our results on the nucleon effective mass, have an immediate repercussion on astrophysical questions related to the cooling rate of neutron stars. These results show that this velocity can be substantially increased with respect to the available calculations. This would make the conventional cooling mechanisms more efficient, thus, probably making unnecessary the cooling mechanism based on a pion condensate in the medium.

We are particularly grateful to O. Maxwell for very helpful discussions on the mechanism of neutron star cooling.

APPENDIX: THE LINDHARD FUNCTION

For a π^-, the Lindhard function for p-h excitation (neutron-hole proton particle) takes into account the lowest order contribution to the pion self-energy, as shown in Fig. 11, and is given by

$$U_{\pi^-}(q^0,q)=4\int\frac{d^3k}{(2\pi)^3}\frac{n(\vec{k})}{-q^0-\dfrac{(\vec{k}-\vec{q})^2}{2m_p}+\dfrac{\vec{k}^2}{2m_n}-\epsilon_p+i\delta} \tag{A1}$$

with ϵ_p the proton energy at rest, after defining the energy origin for neutrons of zero momentum. Performing the integrations explicitly, we get

$$\mathrm{Re}U_{\pi^-}(q^0,q)=\frac{4}{(2\pi)^2}\frac{m_p}{q}\frac{1}{2b}\left[\left(bk_F^2+a-\frac{c^2}{2b}\right)\ln\left(\frac{a+bk_F^2+ck_F}{a+bk_F^2-ck_F}\right)+2ck_F-c\alpha\ln\left(\frac{bk_F^2-a+2\alpha k_F b}{bk_F^2-a-2\alpha k_F b}\right)\right], \tag{A2}$$

where

$$\begin{aligned} a&=-q^0-\frac{q^2}{2m_p}-\epsilon_p\,,\\ b&=\frac{1}{2m_n}-\frac{1}{2m_p}\,,\\ c&=\frac{q}{m_p}\,,\\ \alpha&=\left(-\frac{a}{b}+\frac{c^2}{4b^2}\right)^{1/2}. \end{aligned} \tag{A3}$$

If α is complex or the arguments of the ln are complex, we should take the complex ln of the arguments. When the arguments are real, then they should be substituted by their absolute values.

For the imaginary part we get

$$\begin{aligned} &\mathrm{Im}U_{\pi^-}(q^0,q)=0 \quad \text{if } c^2-4ab\le0\,,\\ &\mathrm{Im}U_{\pi^-}(q^0,q)=0\\ &\qquad\qquad \text{if } c^2-4ab>0\,,\ \text{but } |k_1|>k_F\,,\\ &\mathrm{Im}U_{\pi^-}(q^0,q)=-\frac{1}{2\pi c}(k_3^2-k_1^2)\ \text{otherwise}\,, \end{aligned} \tag{A4}$$

where

$$k_1 = \left| \frac{c}{2b} \right| - \alpha \;,$$

$$k_2 = \left| \frac{c}{2b} \right| + \alpha \;, \tag{A5}$$

$$k_3 = \min\, [k_F, k_2] \;.$$

For a π^+, we use the relationship

$$U_{\pi^+}(q^0,q) = U_{\pi^-}(-q^0,q) \;. \tag{A6}$$

With respect to the Lindhard function for π^0, or for Δ-h excitation, we can use the formulas of the appendix of Ref. 5 with trivial modifications.

The structure of (A1) clearly shows that if

$$q^0 > \max\, (E_n) - \min\, (E_p) \;, \tag{A7}$$

where E_n and E_p are the neutron and proton energies, then U_{π^-} does not have an imaginary part. The second member of (A7) defines the difference of neutron and proton chemical potentials in the quasiparticle approximation.

There are some particular cases where the Lindhard function presents strong cancellations if calculated with the above formulas. This is also a feature that appears in symmetric nuclear matter. The appropriate limits have been calculated analytically for these cases and used in the actual calculation.

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