Eigenvalues of the Yakubovskii equation kernel for a four-nucleon system

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Low-lying states of ^{4}He have been studied by calculating the trajectories of the first two eigenvalues of the kernels of the Yakubovskii four-body equation as the total energy E increases from $-\infty$ to $\infty+i\epsilon$. The two-particle interactions used are of the separable Yamaguchi type and include spin-dependent forces. The integral equations are derived for each state with values of spin S, isospin T, and total angular momentum L. To obtain a set of single variable integral equations, the Schmidt expansion is applied. The deformation of the integration contour is performed for the complex eigenvalues, and the eigenvalue problems for these equations are solved to determine the binding energy or the resonance energy including $l=1$ subamplitudes for 3 + 1 subsystems. The binding energies for the ground and the first excited state are -45.009 MeV and -11.529 MeV, respectively. A resonance state is found to be about -4.889 MeV in the state with $ST=10, L=1$ corresponding to the degenerate state of the second, third, and ninth excited states of the ^{4}He nucleus.

[NUCLEAR STRUCTURE ^{4}He; calculated levels. Four-body, separable potential model.]

I. INTRODUCTION

It is of current interest to extend the method of calculating the scattering problem established in the three-body system to the four-body scattering problem based on the Faddeev-Yakubovskii (FY) formulation[1] with a separable potential. The success in the three-body scattering problem[2-4] is founded on the fact that the singularities appearing in the kernel of the integral equation have been treated by the contour rotation method.[5] As regards four-body scattering, some representation for the amplitudes describing 3 + 1 and 2 + 2 subsystems at energies in the continuous spectrum region is needed in order to reduce the FY equations in two variables to a set of single variable integral equations. It was shown in a previous paper[6] that the introduction of the Schmidt expansion makes it possible to answer these questions. This expansion is applied to the calculation of the eigenvalues of the kernel of the FY equation to seek a resonance state. A more realistic analysis with spin dependent forces and with due regard to the orbital angular momentum decomposition is required. Such a study plays an important role in checking the rate of convergence of the Padé approximation which will be used for the calculation of the four-body scattering. We could also examine whether we can get a general experimental level scheme of the ^{4}He nucleus.

The level scheme of ^{4}He is shown in Fig. 1 (see Ref. 7). As we have central s-wave forces, the spin algebra does not involve the orbital angular momentum, and we can perform the spin-isospin analysis independently of the oribtal angular momentum structure. Therefore, in our formulation, total spin J is not conserved, so we must assign the degenerate states of the spin-isospin supermultiplets to each state of the ^{4}He nucleus with $J^{\pi}T$. From Table I (see Ref. 7), the ground and the first excited state correspond to our $ST=00, L=0$ state; the second, third, and ninth excited states correspond to our $ST=10, L=1$ state; the ninth excited state corresponds to our $ST=00, L=1$ state; and the fifth, sixth, seventh, and eighth excited states approximately correspond to our $ST=11, L=1$ state. In this paper we would like to seek these resonance states using our resonance theory.[8]

In Sec. II the separable expressions for 3 + 1 and 2 + 2 amplitudes are obtained. The four-body formalism used in this paper is the one introduced by Narodetskii.[9] The necessary generalization of his method for the problem at hand is given in Sec. III with the expansion obtained in the preceding section. Section IV contains the obtained results and a discussion. A summary is given in Sec. V.

II. EXPANSION OF TWO- AND THREE-BODY SUBAMPLITUDES

In this section we shall give the expression for the amplitude for 3 + 1 and 2 + 2 subsystems, taking account of

TABLE I. Degenerate states of the one- and fifteen-dimensional spin-isospin supermultiplets. Adapted from Table 3.0.1 of Ref. 7.

				J^{π}	
[N]	S	T	$L=0$	$L=1$	$L=2$
[1]	0	0	0^+	(1^-)	2^+
	1	0	1^+	$0^-1^-2^-$	$1^+2^+3^+$
[15]	0	1	0^+	1^-	2^+
	1	1	1^+	$0^-1^-2^-$	$1^+2^+3^+$

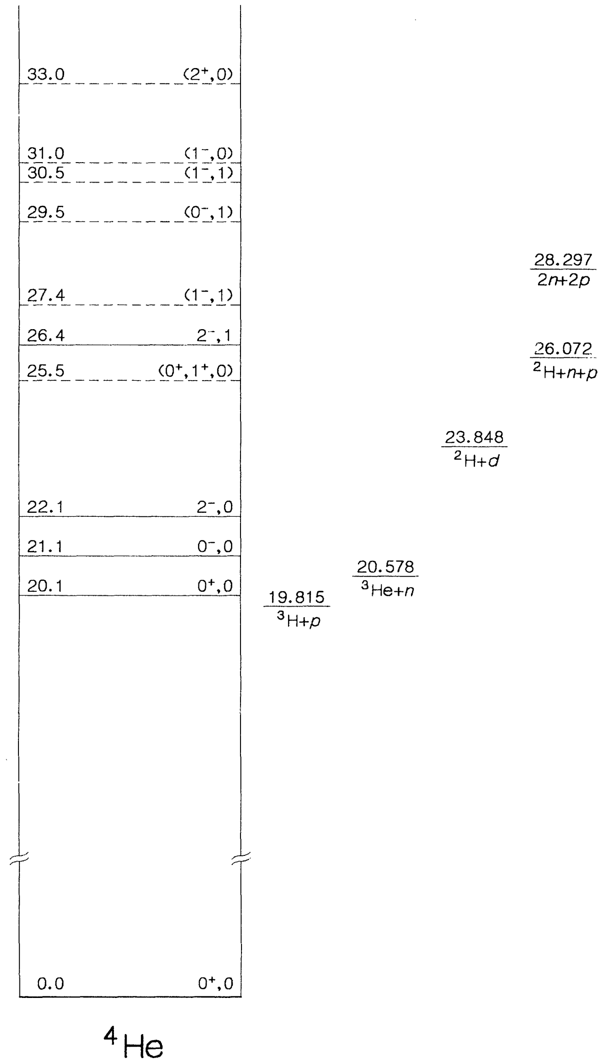

FIG. 1. Energy levels of ^{4}He. Adapted from Fig. 2 of Ref. 7.

spin, isospin, and angular momentum.

The triplet and singlet two-bdoy potentials were assumed to be s-wave separable potentials of Yamaguchi form. The corresponding T matrix is of the form

$$T_i(k,k';z)=-\frac{\lambda_i}{2m}\frac{g_i(k)g_i(k')}{d_i(z)}\ ,$$
$$g_i(k)=(k^2+\beta_i^2)^{-1}\ , \qquad (1)$$
$$d_i(z)=1-\frac{\pi^2\lambda}{\beta_i(\beta_i+\sqrt{-2mz}\,)^2}\ ,$$

where z is the energy parameter, m is the nucleon mass, and the label $i=0,1$ denotes the isospin state. The values of the parameter in Eq. (1) are the same as those of Ref. 9:

$$\beta_0=\sqrt{2}\times 1.450\ \text{fm}^{-1}\ ,$$
$$\beta_1=\sqrt{2}\times 1.165\ \text{fm}^{-1}\ ,$$
$$\lambda_0=2\sqrt{2}\times 0.4156\ \text{fm}^{-3}\ ,$$
$$\lambda_1=2\sqrt{2}\times 0.149\ \text{fm}^{-3}\ ,$$
$$\alpha=\sqrt{2}\times 0.232\ \text{fm}^{-1}\ .$$

A. The 3 + 1 channel

To apply the Schmidt expansions, the eigenvalue problem must be defined for the eigenvalue $\eta^{\sigma\tau}_{l,n}(z)$ and the eigenfunction $\varphi^{\sigma\tau,l}_{i,n}(p;z)$:

$$\eta^{\sigma\tau}_{l,n}(z)\varphi^{\sigma\tau,l}_{i,n}(p;z)=\int_0^\infty \sum_j K^{\sigma\tau,l}_{ij;R}(p,p';z)\varphi^{\sigma\tau,l}_{j,n}(p';z)p'^2dp'\ , \qquad (2)$$

where

$$K^{\sigma\tau,l}_{ij;R}(p,p';z)=\int_0^\infty \sum_k K^{\sigma\tau,l}_{ik}(p,p'';z)K^{\sigma\tau,l}_{jk}(p'',p';z)p''^2dp''\ ,$$

$$K^{\sigma\tau,l}_{ik}(p,p';z)=-8\pi\left[d_i^{-1}\left[z-\frac{p^2}{2m}\right]\right]\times V^{\sigma\tau,l}_{ik}(p,p';z)\left[d_k^{-1}\left[z-\frac{p'^2}{2m}\right]\right]\ ,$$

$$V^{\sigma\tau,l}_{ik}(p,p';z)=\frac{\sqrt{\lambda_i\lambda_k}}{2}\left[\frac{2}{\sqrt{3}}\right]^3\times\int_{-1}^{1}\frac{g_i(p_1)g_k(p_2)P_l(x)}{2mz-\frac{4}{3}(p^2+p'^2+pp'x)}\Lambda^{\sigma\tau}_{ij}dx\ ,$$

$$p_1^2=\tfrac{1}{3}p^2+\tfrac{4}{3}p'^2+\tfrac{4}{3}pp'x,\quad p_2^2=\tfrac{4}{3}p^2+\tfrac{1}{3}p'^2+\tfrac{4}{3}pp'x\ ,$$

and $\Lambda^{\sigma\tau}_{ij}$ are spin-isospin matrices

$$\Lambda^{1/2\,1/2}=\begin{bmatrix}\frac14 & \frac34\\ \frac34 & \frac14\end{bmatrix},$$

$$\Lambda^{3/2\,1/2}=\begin{bmatrix}-\frac12 & 0\\ 0 & 0\end{bmatrix},$$

$$\Lambda^{1/2\,3/2}=\begin{bmatrix}0 & 0\\ 0 & -\frac12\end{bmatrix}.$$

The orthonormality conditions may be written in the form

$$\sum_{i=0,1}\int_0^\infty \varphi^{1/2\,1/2,l}_{i,m}(p;z)\varphi^{1/2\,1/2,l}_{i,n}(p;z)p^2dp=\delta_{mn} \qquad (3)$$

for $\sigma=\tau=\frac12$ and in the form

$$\int_0^\infty \varphi^{\sigma\tau,l}_{i,m}(p;z)\varphi^{\sigma\tau,l}_{i,n}(p;z)p^2dp=\delta_{mn}$$

for $\sigma=\frac12$, $\tau=\frac32$ or $\sigma=\frac32$, $\tau=\frac12$.

If we set

$$[\eta^{\sigma\tau}_{l,n}(z)]^{1/2}\psi^{\sigma\tau,l}_{i,n}(p;z)=\int_0^\infty \sum_j \overline{K^{\sigma\tau,l}_{ji}}(p',p;z)\varphi^{\sigma\tau,l}_{j,n}(p';z)p'^2dp'\ ,$$

then this new function $\varphi^{\sigma\tau,l}_{i,n}(p;z)$ will be an eigenfunction of $K^{\sigma\tau,l}_{ij;L}(p,p';z)$ corresponding to the same eigenvalue η

and will satisfy the similar othogonality condition. The kernel $K_{ij;L}^{\sigma\tau,l}(p;p';z)$ is defined as follows:

$$K_{ij;L}^{\sigma\tau,l}(p,p';z)=\int_0^\infty \sum_k \overline{K_{ki}^{\sigma\tau,l}}(p'',p;z)K_{kj}^{\sigma\tau,l}(p'',p';z)p''^2dp'' .$$

Following our method in Ref. 6 the separable expression for the 3 + 1 amplitude is obtained:

$$w_{i,j}^{\sigma\tau,l}(p,p';z)=\sum_n \frac{-1}{4\pi}\varphi_{j,n}^{\sigma\tau,l}(p;z)F_{jn}^{\sigma\tau,l}(p';z) . \quad (4)$$

The unknown function $F_{jn}^{\sigma\tau,l}(p';z)$ is determined from the following relation:

$$F_{jn}^{\sigma\tau,l}(p;z)=[\eta_{l,n}^{\sigma\tau}(z)]^{1/2}\left[\psi_{j,n}^{\sigma\tau,l}(p;z)+\sum_k F_{jk}^{\sigma\tau,l}(p;z)f_{nk}^{ji}\right] , \quad (5)$$

where

$$f_{nk}^{ji}=\int_0^\infty \psi_{j,n}^{\sigma\tau,l}(p;z)\varphi_{i,k}^{\sigma\tau,l}(p;z)p^2dp .$$

B. The 2 + 2 channel

The expression for the 2 + 2 subamplitude for all cases but $S=T=1$ is already given in Ref. 6, and so we will discuss this case only.

From Narodetskii's formulae we have the integral equation for the 2 + 2 amplitude:

$$u_{ii'}^{(\pm)}(p,p';z)=\frac{-1}{4\pi}K_{ii'}(p,p';z)+\int_0^\infty K_{ii''}(p,p'';z)u_{i''i'}^{(\pm)}(p'',p';z)p''^2dp'' ,$$

where the labels i and i' take the same values, $i=1$, $i'=0$ or $i=0$, $i'=1$, and

$$K_{ii'}(p,p';z)=-\left[d_i^{-1}\left[z-\frac{p^2}{2m}\right]\right]\times V_{ii'}(p,p';z)\left[d_{i'}^{-1}\left[z-\frac{p'^2}{2m}\right]\right] ,$$

$$V_{ii'}(p,p';z)=-\sqrt{\lambda_i\lambda_{i'}}\frac{g_i(p)g_{i'}(p')}{2mz-p^2-p'^2}(1-\delta_{ii'}) .$$

The eigenvalue problem is defined as follows:

$$\zeta_m(z)\omega_{i,m}(p;z)=\int_0^\infty K_{ii';R}(p,p';z)\omega_{i,m}(p';z)p'^2dp' ,$$

and the orthogonality condition

$$\int_0^\infty \omega_{i,m}(p;z)\omega_{i,n}(p;z)p^2dp=\delta_{mn} .$$

Introducing another eigenfunction by the definition

$$[\zeta_m(z)]^{1/2}\kappa_{i,m}(p;z)=\int_0^\infty \overline{K_{ii'}}(p',p;z)\omega_{i',m}(p';z)p'^2dp' ,$$

the separable expression for the 2 + 2 amplitude is obtained:

$$u_{i,i}^{+}(p,p';z)=\sum_m \frac{-1}{4\pi}\omega_{i,m}(p;z)F_{im}^{+}(p';z) ,$$
$$u_{i,i'}^{-}(p,p';z)=\sum_m \frac{-1}{4\pi}\omega_{i,m}(p;z)F_{i'm}^{-}(p';z) . \quad (6)$$

The unknown function $F_{im}^{\pm}(p';z)$ is determined by the relation

$$F_{im}^{-}(p;z)=[\zeta_m(z)]^{1/2}\left[\kappa_{i,m}(p;z)+\sum_{m'}F_{im'}^{+}(p;z)f_{mm'}^{ii}\right] ,$$
$$F_{im}^{+}(p;z)=[\zeta_m(z)]^{1/2}\sum_{m'}F_{im'}^{-}(p;z)f_{mm'}^{i'i'} , \quad (7)$$

where

$$f_{mm'}^{ii}=\int_0^\infty \kappa_{i,m}(p;z)\omega_{i,m'}(p;z)p^2dp .$$

III. FOUR-BODY FORMALISM

We shall discuss the question of what modifications must be made in Narodetskii's equations (19) and (20) in Ref. 9 constructed under the spin-isospin analysis, if the orbital angular momentum decomposition is performed.

Let l_p be the angular momentum of a given triplet of particles and l_q be the angular momentum of the remaining particle relative to the center of mass of the other three. The orbital momenta l_p and l_q are then coupled together to form a state in which the total angular momentum L is diagonal. The partial wave decomposition of the function has the form

$$\alpha_{i\sigma\tau}^{ST}(\vec{p},\vec{q};z)=\sum_{Ll_pl_q}\alpha_{Ll_pl_q}^{ST,i\sigma\tau}(p,q;z)Y_{LMl_pl_q}(\vec{p},\vec{q}) , \quad (8)$$

where

$$Y_{LMl_pl_q}(\vec{p},\vec{q})=\sum_{m_pm_q}(l_pl_qL\mid m_pm_qM)Y_{l_pm_p}(\vec{p})Y_{l_qm_q}(\vec{q}) . \quad (9)$$

Because of our restriction to s-wave two-body forces, the partial decomposition of the function $b_{ij}^{ST}(p,\vec{q};z)$ is just

$$b_{ij}^{ST}(p,\vec{q};z)=\sum_L b_L^{ST,ij}(p,q;z)Y_{LM}(\vec{q}) . \quad (10)$$

Hereafter to the end of this paragraph we follow the Kharchenko formulation,[10] which is based on the method proposed by Ahmadzadeh and Tjon[11] in the three-body problem. Substituting expansions (8) and (10) in Narodetskii's integral equations (19) and (20) in Ref. 9 and projecting (19) and (20) onto the state (9) and $Y_{LM}(\vec{q})$, we obtain the following system of two-dimensional integral equations for the expansion coefficients of $a_{Ll_pl_q}^{ST,i\sigma\tau}$, $b_L^{ST,ij}$:

$$a_{Ll_pl_q}^{ST,i\sigma\tau}(p,q;z)=\frac{-1}{e(z)}\left\{\left[\frac{3}{2\sqrt{2}}\right]^3\sum_{i'\sigma'\tau'}C_{i'\sigma'\tau'}^{i\sigma\tau}(ST)\right.$$

$$\times\int w_{l_p}^{\sigma\tau,ii'}(p,Q_1;s)d_{i'}^{-1}\left[z-\frac{Q_2^2}{2m}-\frac{q'^2}{2m}\right]\sum_{l_p'l_q'}A_{l_p'l_q'}^{Ll_pl_q}(x)\alpha_{Ll_p'l_q'}^{ST,i'\sigma'\tau'}(Q_2,q';z)q'^2dq'dx$$

$$+(\sqrt{3/2})^3\eta^p\sum_{i'j'}C_{i'j'}^{i\sigma\tau}(ST)\int w_{l_p}^{\sigma\tau,ii'}(p,R_1;s)d_{i'}^{-1}\left[z-\frac{R_2^2}{2m}-\frac{q'^2}{2m}\right]$$

$$\left.\times B_{l_pl_q}^{L}(x)b_L^{ST,i'j'}(R_2,q';z)q'^2dq'dx\right\}, \tag{11}$$

$$b_L^{ST,ij}(p,q;z)=-\frac{2}{e(z)}\eta^P(-)^{S+T}(\sqrt{3/2})^3$$

$$\times\sum_{i'\sigma'\tau'}C_{i'\sigma'\tau'}^{ij}(ST)\int v_{ij,i'j'}^{\pm}\left[p,R_1;z-\frac{q^2}{2m}\right]d_{i'}^{-1}\left[z-\frac{R_2^2}{2m}-\frac{q'^2}{2m}\right]\times\sum_{l_p'l_q'}D_{l_p'l_q'}^{L}(x)\alpha_{Ll_p'l_q'}^{ST,i'\sigma'\tau'}(R_2,q';z)q'^2dq'dx \tag{12}$$

with

$$\vec{Q}_1=\frac{\vec{q}+3\vec{q}\,'}{2\sqrt{2}},\quad \vec{Q}_2=\frac{3\vec{q}+\vec{q}\,'}{2\sqrt{2}},\quad \vec{R}_1=\frac{\vec{q}+\sqrt{3}\vec{q}\,'}{\sqrt{2}},\quad \vec{R}_2=\frac{\sqrt{3}\vec{q}+\vec{q}\,'}{\sqrt{2}},$$

$$z_1=z-\frac{1}{2m}\frac{9}{8}(q^2+q'^2+\tfrac{3}{2}qq'x),\quad z_2=z-\frac{1}{2m}\frac{3}{2}\left[q^2+q'^2+\frac{2}{\sqrt{3}}qq'x\right],\quad x=\vec{q}\cdot\vec{q}\,'.$$

The spin-isospin recoupling coefficients $C_{i'\sigma'\tau'}^{i\sigma\tau}(ST)$ can readily be calculated and are given explicitly by Narodetskii and Grach.[12] The function for the angular momentum part takes the form:

$$A_{l_p'l_q'}^{Ll_pl_q}(x)=16\pi^{5/2}\sqrt{2l_q'+1}(-)^{L+l_p-l_q}\sum_{m_p\nu_pm_q}\begin{bmatrix}l_p & l_q & L\\ m_p & m_q & \nu_p\end{bmatrix}\begin{bmatrix}l_q' & l_p' & L\\ 0 & -\nu_p & \nu_p\end{bmatrix}Y_{l_pm_p}(\theta_{q'Q_1},0)Y_{l_p',-\nu_p}(\theta_{q'Q_2},0)Y_{l_qm_q}(\theta_{qq'},0),$$

$$B_{l_pl_q}^{L}(x)=8\pi^2\frac{1}{\sqrt{2L+1}}\sum_{m_p}(l_pm_pl_q-m_p\,|\,L\,0)Y_{l_pm_p}(\theta_{q'R_1},0)Y_{l_qm_p}(\theta_{qq'},0),$$

$$D_{l_pl_q}^{L}(x)=8\pi^2\frac{\sqrt{2l_q+1}}{2L+1}\sum_{m_p}(l_pm_pl_q0\,|\,Lm_p)Y_{l_pm_p}(\theta_{q'R_2},0)Y_{Lm_p}(\theta_{qq'},0).$$

Hereafter to the end of this section we confine our attention to the case $ST=11$ and arbitrary L values, since this state increases the number of Narodetskii's components of the unknown function $b(q;z)$ by using the Schmidt expansion.

The Schmidt expansions for the $3+1$ and $2+2$ amplitudes obtained in the previous section are of the following forms:

$$w_{l_p}^{1/2\,1/2,ii'}(p,p';z)=\sum_n\frac{-1}{4\pi}w_{i,n}^{1/2\,1/2;l_p}(p;z)\chi_{i',n}^{1/2\,1/2;l_p}(p';z),$$

$$w_{l_p}^{1/2\,3/2,11}(p,p';z)=\sum_n\frac{-1}{4\pi}w_{1,n}^{1/2\,3/2;l_p}(p;z)\chi_{1,n}^{1/2\,3/2;l_p}(p';z),$$

$$w_{l_p}^{3/2\,1/2,00}(p,p';z)=\sum_n\frac{-1}{4\pi}w_{0,n}^{3/2\,1/2;l_p}(p;z)\chi_{0,n}^{3/2\,1/2;l_p}(p';z), \tag{13}$$

$$v_{i,i'}^{+}(p,p';z)=\sum_n\frac{-1}{4\pi}v_{i,n}(p;z)\theta_{i',n}^{+}(p';z),$$

where

$$w_{i,n}^{\sigma\tau;l_p}(p;z)=\varphi_{i,n}^{\sigma\tau;l_p}(p;z)d_i^{1/2}\left[z-\frac{p^2}{2m}\right],$$

$$x_{j,n}^{\sigma\tau;l_p}(p;z)=F_{j,n}^{\sigma\tau;l_p}(p;z)d_j^{1/2}\left[z-\frac{p^2}{2m}\right],$$

$$v_{i,n}(p;z)=\omega_{i,n}(p;z)d_i^{1/2}\left[z-\frac{p^2}{2m}\right],$$

$$\theta_{i,n}(p;z)=F_{i,n}^{\pm}(p;z)d_i^{1/2}\left[z-\frac{p^2}{2m}\right].$$

A solution of integral equations (11) and (12) may be introduced in the following forms:

$$\begin{aligned}
\alpha_{Ll_pl_q}^{11,i1/2\,1/2}(p,q;z)&=\sum_n w_{i,n}^{1/2\,1/2;l_p}\left[p;z-\frac{q^2}{2m}\right]\alpha_{L,l_q,n}^{11,1/2\,1/2}(q;z)\,,\\
\alpha_{Ll_pl_q}^{11,11/2\,3/2}(p,q;z)&=\sum_n w_{1,n}^{1/2\,3/2;l_p}\left[p;z-\frac{q^2}{2m}\right]\alpha_{L,l_q,n}^{11,1/2\,3/2}(q;z)\,,\\
\alpha_{Ll_pl_q}^{11,03/2\,1/2}(p,q;z)&=\sum_n w_{0,n}^{3/2\,1/2;l_p}\left[p;z-\frac{q^2}{2m}\right]\alpha_{L,l_q,n}^{11,3/2\,1/2}(q;z)\,,\\
b_L^{11,10}(p,q;z)&=\sum_n v_{1,n}\left[p;z-\frac{q^2}{2m}\right]b_{L,n}^{11,1}(q;z)\,,\\
b_L^{11,01}(p,q;z)&=\sum_n v_{0,n}\left[p;z-\frac{q^2}{2m}\right]b_{L,n}^{11,0}(q;z)\,,
\end{aligned}\tag{14}$$

where the $a_{L,l_q,n}^{ST,\sigma\tau}(q;z)$ and $b_{L,n}^{ST,i}(q;z)$ are to be determined. Inserting Eqs. (13) and (14) into Eqs. (11) and (12), we may obtain the set of equations for the unknown functions a_n and b_n:

$$\begin{aligned}
\alpha_{L,l_q,n}^{11,1/2\,1/2}(q;z)=\frac{1}{e(z)}\sum_{i=0,1}\Bigg\{&-\tfrac{1}{3}\int_0^\infty\sum_{n'l_p'l_q'}R_{l_pl_q;l_p'l_q';L}^{1/2\,1/2;1/2\,1/2;i;nn'}(Q_1,Q_2;s,s')\alpha_{L,l_q',n'}(q';z)q'^2dq'\\
&+\frac{-2\sqrt{2}}{3}\int_0^\infty\sum_{n'l_p'l_q'}R_{l_pl_q;l_p'l_q';L}^{1/2\,1/2;\sigma\tau;i;nn'}(Q_1,Q_2;s,s')\alpha_{L,l_q',n'}^{\sigma\tau}(q';z)q'^2dq'\\
&+\frac{-1}{\sqrt{2}}\int_0^\infty\sum_{n'}S_{l_pl_q;L}^{1/2\,1/2;i;nn'}(R_1,R_2;s,s')b_{L,n'}^{i}(q';z)q'^2dq'\Bigg\}\,,
\end{aligned}\tag{15}$$

$$\begin{aligned}
\alpha_{L,l_q,n}^{11;\sigma\tau}(q;z)=\frac{1}{e(z)}\sum_{i=0,1}\Bigg\{&-\tfrac{1}{3}\int_0^\infty\sum_{n'l_p'l_q'}R_{l_pl_q;l_p'l_q';L}^{1/2\,1/2;1/2\,1/2;i;nn'}(Q_1,Q_2;s,s')\alpha_{L,l_q',n'}(q';z)q'^2dq'\\
&+\frac{-2\sqrt{2}}{3}\int_0^\infty\sum_{n'l_p'l_q'}R_{l_pl_q;l_p'l_q';L}^{1/2\,1/2;\sigma\tau;i;nn'}(Q_1,Q_2;s,s')\alpha_{L,l_q',n'}^{\sigma\tau}(q';z)q'^2dq'\\
&+\frac{-1}{\sqrt{2}}\int_0^\infty\sum_{n'}S_{l_pl_q;L}^{1/2\,1/2;i;nn'}(R_1,R_2;s,s')b_{L,n'}^{i}(q';z)q'^2dq'\Bigg\}\,,
\end{aligned}\tag{16}$$

$$b_{L,n}^{11,i}(q;z)=\frac{1}{e(z)}\sum_{i'=0,1}\left\{\frac{-1}{\sqrt{3}}\int_0^\infty \sum_{n'l_p'l_q'} T^{(s);1/2\,1/2;i';nn'}_{l_p'l_q';L}(R_1,R_2;s,s')\alpha^{11,1/2\,1/2}_{L,l_q',n'}(q';z)q'^2dq' + \frac{-2\sqrt{2}}{3}\int_0^\infty \sum_{n'l_p'l_q'} T^{(s);\sigma\tau;i';nn'}_{l_p'l_q';L}(R_1,R_2;s,s')\alpha^{11,\sigma\tau}_{L,l_q',n'}(q';z)q'^2dq'\right\}, \qquad (17)$$

where

$$R^{1/2\,1/2;\sigma\tau;i;nn'}_{l_pl_q,l_p'l_q';L}(Q_1,Q_2;s,s')=\frac{1}{2}\left[\frac{3}{2\sqrt{2}}\right]^3\int_{-1}^{1}\frac{\chi_{i,n}^{1/2\,1/2;l_p}(Q_1,s)w_{i,n'}^{\sigma\tau;l_p'}(Q_2,s')}{d_i\left[z-\frac{Q_1^2}{2m}-\frac{q^2}{2m}\right]}A^{Ll_pl_q}dx\ ,$$

$$R^{\sigma\tau;1/2\,1/2;i;nn'}_{l_pl_q,l_p'l_q';L}(Q_1,Q_2;s,s')=\frac{1}{2}\left[\frac{3}{2\sqrt{2}}\right]^3\int_{-1}^{1}\frac{\chi_{i,n}^{\sigma\tau;l_p}(Q_1,s)w_{i,n'}^{1/2\,1/2;l_p'}(Q_2,s')}{d_i\left[z-\frac{Q_1^2}{2m}-\frac{q^2}{2m}\right]}A^{Ll_pl_q}dx\ ,$$

$$S^{\sigma\tau;i;nn'}_{l_pl_q;L}(R_1,R_2;s,s')=\frac{1}{2}(\sqrt{3/2})^3\eta^P\int_{-1}^{1}\frac{\chi_{i,n}^{\sigma\tau;l_p}(R_1,s)v_{i,n'}(R_2,s')}{d_i\left[z-\frac{R_1^2}{2m}-\frac{q^2}{2m}\right]}B^L_{l_pl_q}dx\ ,$$

$$T^{(s);\sigma\tau;i;nn'}_{l_pl_q;L}(R_1,R_2;s,s')=\frac{1}{2}(\sqrt{3/2})^3\eta^P(-)^{S+T}\int_{-1}^{1}\frac{\theta_{i,n}^{(s)}(R_1,s)w_{i,n'}^{\sigma\tau;l_p}(R_2,s')}{d_i\left[z-\frac{R_1^2}{2m}-\frac{q^2}{2m}\right]}D^L_{l_pl_q}dx\ .$$

In Eqs. (15)–(17) the labels σ, τ, i, and (s) take the same values: $\sigma=\frac{1}{2}$, $\tau=\frac{3}{2}$, $i=1$, $(s)=+,-$ $(i'=1,0)$ or $\sigma=\frac{3}{2}$, $\tau=\frac{1}{2}$, $i=0$, $(s)=-,+$ $(i'=1,0)$.

In the numerical analysis, we must take the finite sum instead of an infinite sum in Eqs. (15)–(17). Let the number of the unknown functions in the set of Eqs. (15), (16), and (17) be N_w and N_v, respectively.

IV. RESULTS AND DISCUSSION

In this section the numerical results are given and discussed. We have plotted in Figs. 2–5 the trajectories of the first two eigenvalues or the second one only of the kernel of the FY equations for $ST=00,10,11$, $L=0$ or 1 states which are thought to correspond to the experimental energy levels, as the total energy z increases from $-\infty$ to $\infty+i\epsilon$ of the ^{4}He nucleus.

In Fig. 2 the two largest eigenvalues are shown with a circle of radius 1 for the state of $ST=0,L=0$. The largest eigenvalue passes through unity in the real part, and this point can be assigned to the ground state. The second eigenvalue seems to pass through the circle of radius 1 in the complex part. But this is because of a numerical error. As we discuss later, we must regard this point of intersection assigned to the first excited state as a bound

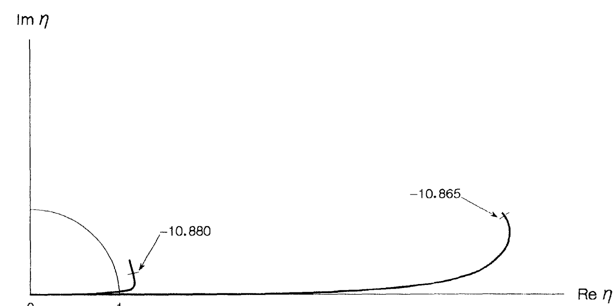

FIG. 2. Trajectories of the first two eigenvalues $e(z)$ for the $ST=00,L=0$ state. The numbers give the energies in MeV.

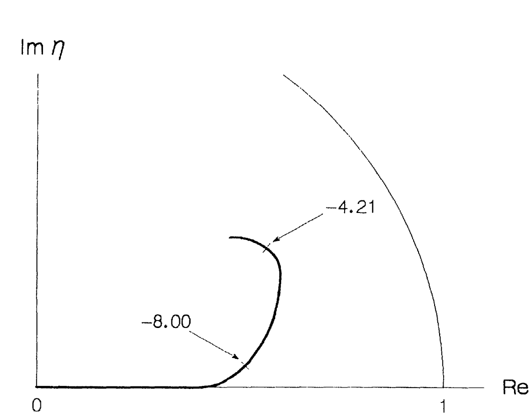

FIG. 3. Trajectory of the second eigenvalue $e(z)$ for the $ST=00,L=1$ state. The numbers give the energies in MeV.

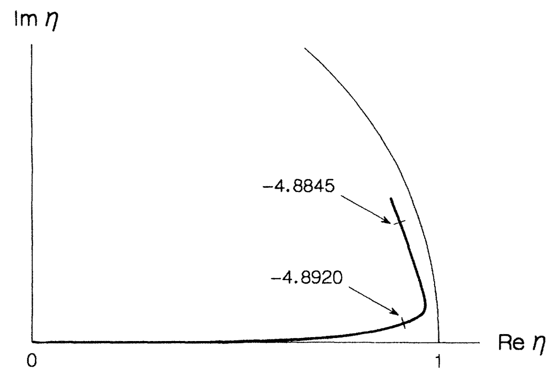

FIG. 4. Trajectory of the second eigenvalue $e(z)$ for the $ST=10,L=1$ state.

state. At the indicated points in the two eigenvalues, the values of the corresponding energies are given. In spite of full use of our computer's memory, the insufficiency of the mesh points in the Gaussian quadrature has brought about an inaccuracy of the numerical value of the eigenvalues. So we have had to give up the plotting too soon.

In Fig. 3 we have plotted the second eigenvalue among the largest two for the state of $ST=00,L=1$. As the first eigenvalue has a minus sign, we have not represented it on Fig. 3. The second eigenvalue does not pass through the circle and so we could not find the energy level corresponding to the ninth excited state in the $ST=00,L=1$ state.

Figure 4 shows the second eigenvalue of the $ST=10,L=1$ state. The eigenvalue could be considered to almost touch the circle of radius 1 in the neighborhood of the turning point of the curve, at which the energy is approximately -4.889 MeV. So we can assign this point to the resonance state of the second, third, and ninth excited states of the ^{4}He nucleus. The first eigenvalue has a minus sign.

Figure 5 shows the second eigenvalue of the $ST=11,L=1$ state. The eigenvalue is small and we cannot find the energy level in this state corresponding to the fifth, sixth, seventh, or eighth excited state.

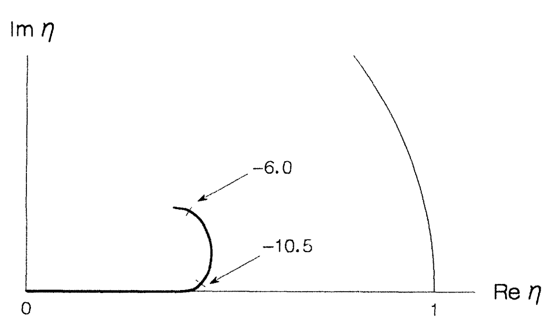

FIG. 5. Trajectory of the second eigenvalue $e(z)$ for the $ST=11,L=1$ state.

TABLE II. The 0^+ state of ^{4}He with the $l=0$ state only or with $l=0,1$ states for the (3 + 1) subamplitude.

N_w (N_v)	z_1 (MeV) s wave	z_1 (MeV) s,p wave	z_2 (MeV) s wave	z_2 (MeV) s,p wave
3	-44.826	-44.962	-11.274	-11.280
4	-45.262	-45.009	-11.627	-11.529

In all of these cases, when the eigenvalue becomes complex with a positive imaginary part, this rise to the complex plane is initially horizontal. We could find all of the energy levels in Fig. 1 but the bound states corresponding to the ground and first excited states and the resonance state corresponding to the second, third, and ninth excited state of the ^{4}He nucleus.

Bound state. We solved the eigenvalue problem for the $ST=00,L=0$ state at energies on the left-hand side of the point from which the continuous spectrum starts to run on the real axis. We obtained Narodetskii's result that the first excited state is in the bound state. By the way, this state is regarded to be in the resonance state by experimentalists. The effect of including the $l=1$ (3 + 1) subamplitude is apparent from Table II, if we compare two pairs of columns for the calculated energy levels. The results show that the overboundness could be improved only slightly by including the $l=1$ (3 + 1) subamplitude; the effect of increasing the values of N_ω and N_v from 3 to 4 is greater.

Resonance state. Since the second eigenvalue does not pass through the circle of radius 1, we have calculated the eigenvalue at energies on the second sheet of the complex plane.[13] Though the calculated eigenvalue showed a tendency to move on toward 1 as the imaginary part of the energy increased in minus value, unfortunately owing to the insufficiency of the number of mesh points in our numerical analysis the accuracy of the calculated eigenvalue became worse before its arrival at 1. Since the result[14] for the $J^\pi T=1^+0$ state in the ^{6}Li nucleus is that the value of the phase shift is 90°, even though the corresponding second eigenvalue does not pass through the circle of radius 1, we would like to conclude that there is a resonance state in the vicinity of the value -4.889 MeV of the energy in the $ST=10,L=1$ state. We cannot determine the level width.

V. SUMMARY

In our formulation we could find the binding energies corresponding to the ground and first excited states in the $ST=00,L=0$ state and the resonance energies corresponding to the second, third, and ninth excited states of the ^{4}He nucleus in the $ST=10,L=1$ state, but none of the other states whose existence is recognized by experimentalists. Calculated binding energies of -45.009 and -11.529 MeV are too overbound compared to the experimental values of -28.297 and -8.197 MeV, while the calculated resonance energy −4.889 MeV is underbound compared to the experimental value, more or less −6.0 MeV. Nevertheless, we think that the present paper will give further stimulation to the calculation of four-body scattering.

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