

Comparison of effective particle-hole and particle-particle forces in the random-phase approximation

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The possibility of “universal” effective forces for the particle-hole and particle-particle problem in the renormalized random-phase approximation approach is analyzed using Green’s function theory. A numerical comparison in the lead region for a simple semiphenomenological force and a more microscopic force deduced from the bare nucleon-nucleon interaction is given and discussed.

Several extensive investigations have been reported in the literature to find “universal effective interactions,” which reproduce both ground and excited states of nuclei. Details and further references can be found in recent Skyrme force fits¹ by Nayak² and by Warroquier, Heyde, and Wenes.³ It seems an interesting but difficult question to find out whether in principle there exists such a “universal potential” describing a variety of nuclear properties. For this purpose we start from the microscopic definition of effective forces by means of the well known Green’s function theory.⁴⁻⁷ In principle these “pure” effective forces determine the nuclear spectra, but one is forced to use *different* forces for the excited (ph) states and for the two-particle (two-hole) valence nucleon spectra.⁴⁻⁷ This is illustrated shortly in the first section of this paper. However, as known from many investigations one incorporates in particle-hole (ph) [particle-particle (pp)] random-phase approximation (RPA) calculations—at least implicitly—renormalization procedures due to basis truncation and quasiparticle behavior.⁴⁻⁷ The consequences of this treatment are outlined in the next step. From these considerations it seems possible to find a candidate for a “universal potential.” Since the validity of the assumptions involved are difficult to prove, we have tested the possibility of a simultaneous description of the pp and ph spectra for two potentials in the third part.

Since the Green’s function theory is given in many text books and articles (for instance in the book by Migdal⁴), we shall only give the relevant equations. The ph spectra are determined by the generalized response function L defined by⁷

$$L_{abmn} = -i(G_{ambn}^{(2)} - G_{am}G_{bn}) . \quad (1)$$

The Latin indices (a, b, \dots) stand for the quantum numbers (α, β, \dots) and time variables (t_a, t_b, \dots). Transforming to energy space and integrating over the relative energy variables gives the desired quantities^{7,8} ($E_N^0 = 0$):

$$L_{\alpha\beta\mu\nu}(\omega) = (2\pi)^{-1} \oint d\epsilon \oint d\epsilon' L_{\alpha\beta\mu\nu}(\epsilon, \epsilon'; \omega) \quad (2a)$$

$$= \sum_{|N\rangle \neq |0\rangle} \left[\frac{\rho_{\alpha\mu}^N(\rho_{\nu\beta}^N)^*}{\omega - E_N + i\eta} - \frac{\rho_{\beta\nu}^N(\rho_{\mu\alpha}^N)^*}{\omega + E_N - i\eta} \right] , \quad (2b)$$

with

$$\rho_{\alpha\beta}^N = \langle 0 | a_{\beta}^{\dagger} a_{\alpha} | N \rangle .$$

In an analogous manner the pp spectra are given by the

two-particle propagator $G^{(2)}$. L and $G^{(2)}$ obey the following Bethe-Salpeter equations:^{4,6,9}

$$L_{kmjn} = iG_{kc}G_{bj}(\delta_{bm}\delta_{nc} - K_{cubv}^{\text{ph}}L_{vumn}) , \quad (3)$$

$$G_{abcd}^{(2)} = G_{am}G_{bn}(\delta_{mc}\delta_{nd} - \delta_{md}\delta_{nc} + \frac{i}{2}K_{mnlk}^{\text{pp}}G_{klcd}^{(2)}) , \quad (4)$$

where G_{mn} denotes the single particle propagator. We use the convention to sum or integrate, respectively, over all doubly occurring variables. $K^{\text{ph}}(K^{\text{pp}})$ are the ph- (pp) forces (irreducible vertices), defined as the sum of all irreducible graphs in the ph (pp) channels. An alternative but equivalent definition of K uses functional derivatives.^{6,9,10} [In the appendix of Ref. 9 an extensive list of general relations between $K^{\text{pp}}, K^{\text{ph}}$ and different six-point functions is given which contain also the core polarization problem (see Refs. 11 and 12, for instance)]. By performing the energy integrations in Eqs. (3) and (4)—as in procedure (2a)—and taking the limit $\omega \rightarrow E^N(E^{N \pm 2})$, respectively, one obtains directly the unrenormalized equations for the ph and pp spectra.^{6,7} According to the given derivation one has to use the effective force $K^{\text{ph}}(E^N)$ for the N particle spectra, but for the $N \pm 2$ spectra $K^{\text{pp}}(\pm E^{N \pm 2})$ enters as an effective force. Within this (pure) scheme no mathematical and physical reason can be seen for a common description of K^{ph} and K^{pp} , since these are defined completely differently. For instance, K^{ph} is not necessarily antisymmetric in contrast to K^{pp} .^{4,5} The situation is different with respect to a common potential for the ground state properties and the ph spectrum only. The ground state properties are determined by the nucleon self-energy M (irreducible mass operator). In ladder-type approximations one obtains^{4,10}

$$M_{ab} = -i\Gamma_{abcd}^L G_{dc} , \quad (5)$$

and^{6,8,10}

$$K_{abcd}^{\text{ph},L} = \Gamma_{abcd}^L + i\Gamma_{kmdn}^L G_{ns} G_{tm} \Gamma_{bsct}^L . \quad (6)$$

The effective scattering matrix Γ^L can be considered as an effective potential.^{13,14} Therefore, if the second term in Eq. (6) is small⁸ one may use a common potential for the N -particle properties. Possible choices for Γ^L are either Brueckner’s G matrix^{13,15} or the so-called Λ approximations of the Green’s function theory.¹⁰

So far the presented scheme is very general. Practical nuclear calculations are usually done in a truncated basis under the assumption that the nucleus has a dominantly shell-

model structure, i.e.,

$$G_{\mu\nu}(\epsilon) = z_\nu \delta_{\nu\mu} \left(\frac{1 - n_\nu}{\epsilon - \epsilon_\nu + i\eta} + \frac{n_\nu}{\epsilon - \epsilon_\nu - i\eta} \right) + G_{\nu\mu}^R(\epsilon) \\ = \tilde{G}_{\nu\mu}(\epsilon) + G_{\nu\mu}^R(\epsilon) . \quad (7)$$

Here, $z_\nu (\leq 1)$ denotes Migdal's renormalization constant.⁴ Both effects can be taken into account by changing the interaction utilizing standard procedures.^{4,5,7} The advantage of this procedure is the replacement of the two-point function by the shell-model propagator in the Bethe-Salpeter equations with renormalized forces. By applying the same procedure as in the first part one obtains the renormalized RPA equations which agree formally with the standard RPA equations.^{4,6,7} But instead of the pure interactions one has to use the following renormalized forces \hat{F} , given by^{4,6,7}

$$\hat{F}_{\alpha\beta\sigma\lambda}^{\text{pp}}(\pm E^N \pm 2) = z_\alpha z_\beta F_{\alpha\beta\sigma\lambda}^{\text{pp}}(\pm E^N \pm 2) ; \quad (8) \\ \hat{F}_{\alpha\beta\sigma\lambda}^{\text{ph}}(E^N) = z_\alpha z_\beta F_{\alpha\beta\sigma\lambda}^{\text{ph}}(E^N) .$$

The new kernels F of the Bethe-Salpeter equations in the restricted basis obey the following integral equations:^{4,5,16}

$$F_{abcd}^{\text{ph}} = \{\delta_{aj}\delta_{kc} - iF_{amcn}^{\text{ph}}[G_{nj}G_{km} - (P\tilde{G}_{nj}\tilde{G}_{km})]\}K_{jbkd}^{\text{ph}} , \quad (9)$$

$$F_{abcd}^{\text{pp}} = \{\delta_{ak}\delta_{bj} + \frac{i}{2}F_{abmn}^{\text{pp}}(\tilde{G}_{mk}G_{nj} - [P\tilde{G}_{mk}\tilde{G}_{nj}])\}K_{kjcd}^{\text{pp}} . \quad (10)$$

The symbol P denotes projection onto the truncated space. Furthermore one can also derive Bethe-Salpeter equations for the effective scattering matrix Γ with the new ph or pp forces, respectively:¹⁶

$$\Gamma_{abcd} = F_{amcn}^{\text{ph}}[\delta_{mb}\delta_{nd} - i(P\tilde{G}_{nj}\tilde{G}_{km})\Gamma_{jbkd}] , \quad (11)$$

or

$$\Gamma_{abcd} = F_{abmn}^{\text{pp}}[\delta_{mc}\delta_{dn} + \frac{i}{2}(P\tilde{G}_{mk}\tilde{G}_{nj})\Gamma_{kjcd}] . \quad (12)$$

As can be seen from Eqs. (9) and (10) the properties of the new effective ph (pp) forces F^{ph} (F^{pp}) depend strongly on the basis choice. One recovers from Eqs. (9) and (10) the former result of distinct ph and pp forces, i.e.,

$$F^{\text{ph}} \approx K^{\text{ph}}; \quad F^{\text{pp}} \approx K^{\text{pp}} ,$$

for the complete basis ($P=1, z \approx 1$). But by selecting a sufficiently small basis, one gets from Eqs. (11) and (12) a more suitable result, namely,

$$F^{\text{pp}} \approx F^{\text{ph}} \approx \Gamma .$$

Therefore one may conclude from this renormalization consideration a possibility for a universal interaction for an approximate overall description of N - and $(N \pm 2)$ -particle spectra. Further support is given by the usual ladder approximation for Γ , which takes the important hard core correlations into account.¹⁵ They seem to be an essential contribution from the excluded part of the Hilbert space. It

is a very complex task to justify the involved assumptions in detail. For this reason we make only a simplified test for a common pp and ph potential. The first attempt in this direction was done by Vary and Ginocchio,¹⁷ who used the (density-independent) Gillet forces for a simultaneous calculation of ph and pp spectra. For our purpose we have selected (i) a simple phenomenological interaction—namely, the surface delta interaction SDI—and (ii) a force RNT, deduced from a ladder-type approximation in nuclear matter and extended to finite nuclei by a local density approximation.^{7,18} Both forces have been employed in the lead region for ph spectra.^{7,18} For the ph treatment the simple SDI force gives fair results and the RNT force reproduces the experimental situation as well as the semiphenomenological forces of Bauer *et al.*¹⁹ We do not try a Landau-Migdal parametrization of the forces, since according to Klemt, Moszkowski, and Speth^{6,20} the L - M force ansatz should be different for the ph and pp problem. But at least for the RNT force, which makes no restrictions with respect to the momenta—as does the L - M ansatz—and includes energy dependence, one has a different situation. If one performs with the SDI and RNT forces the pp RPA, one obtains reasonable agreement with the spectra.¹⁶ Also the eigenvectors seem satisfactory, since, for instance, the spectroscopic factors for ^{210}Pb (p, d) ^{209}Pb are in agreement with the experiment. As in the ph case¹⁸ the strength parameters or Migdal's renormalization constants z , respectively, have been adjusted to the data. For the pp case a least-squares fit to the relevant energy eigenvalues of ^{206}Pb , ^{210}Pb , and ^{210}Bi was performed. For details see Ref. 16. The outcome is similar to the results of Klemt and Speth,²⁰ who also use the renormalized RPA. The parameters are given in Table I.

Comparison with the results of the ph RPA seems to suggest that the use of common forces for the pp and ph RPA is not adequate, but this conclusion can be too precipitate due to the role of the different isospin interactions. For the pp states with $T_z=0$ and the ph states both isospin parts of the interaction ($t=0, 1$) contribute, but the pp states with $T_z = \pm 1$ are determined by the ($t=1$) potential only. Therefore, it may be possible that by shifting the relative strength of both parts a common (phenomenological) force can be found for both problems (see also Ref. 6). This supposition is supported by the force SMII², which shows this trend in the strengths. For the force RNT this procedure would imply, that one has determined in the calculation of the pp states with $T_z=0$ and the ph states an average potential renormalization only. However, such an explanation seems to violate the spirit of the RNT approach, where one starts from a free interaction and should only need a common renormalization constant for the quasiparticles [see Eq. (8), $z_\alpha = z_\beta = z$].^{7,18} In our opinion it remains an open question, whether an improved calculation of the vertices would resolve and decide the question.

TABLE I. Comparison of the SDI strengths V_{st} (MeV fm⁴) and Migdal's renormalization constants z for the pp and ph RPA in the lead region.

pp	SDI		RNT	
	ph	pp	pp	ph
$V_{01} = -1290.$	$V_{01} = -1150.$	$z = 0.67$ ($T_z = 0$)		
$V_{10} = -1037.56$	$V_{10} = -2500.$	$z = 0.95$ ($T_z = \pm 1$)		$z = 0.64$

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