Six-quark compound state in deuteron

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Based on a recently proposed method of incorporating quark degrees of freedom in nuclei, we estimate the probability of the six-quark compound state of deuteron using the experimental data and quantum chromodynamics prediction for the deuteron form factor, and also using the partial width of the dibaryon resonance of Kamae *et al.* Both estimates are consistent with each other.

In this report, we present two independent estimates of the probability of the six-quark compound state in deuteron, the first estimate using the experimental data and quantum chromodynamics (QCD) prediction for the deuteron form factor[1,2] and the second using the partial width of the dibaryon resonance of Kamae *et al.*[3] and Grein *et al.*[4] Both estimates are based upon a recently proposed theory of the quark degrees of freedom in nuclei.[5]

In our formulation for the np system, the $^3S_1$ deuteron wave function (we neglect the $^3D_1$ component for simplicity) is written as

$$\psi_{\rm d}(r)=N(\epsilon_{\rm d})\phi_{\rm n}\phi_{\rm p}\chi^{\rm FL}_{\epsilon_{\rm d}}(r\geq R_1)+A(\epsilon_d)\phi^1_s(r\leq R_2)\ , \quad (1)$$

where $N(\epsilon_{\rm d})$ and $A(\epsilon_{\rm d})$ denote the probability amplitudes of the nuclear part and of the six-quark compound bag in the deuteron, respectively, and $R_1<R_2$. ϕ^1_s describes the lowest six-quark excitation which we identify with the dibaryon resonance of Kamae *et al.*[3] at 2.35 GeV or 0.48 GeV above the deuteron mass. $\phi_{\rm n}$ and $\phi_{\rm p}$ are the three-quark $(3q)$ colorless nucleon bags of radius R with normalization

$$\langle\phi_{\rm n}\,|\,\phi_{\rm n}\rangle=1=\langle\phi_{\rm p}\,|\,\phi_{\rm p}\rangle\ .$$

$\chi^{\rm FL}_{\epsilon_{\rm d}}(r\geq r_0)$ is the deuteron wave function generated from the boundary-condition model (BCM) potential of Feshbach and Lomon[6,7] (FL) and is normalized, $\langle\chi^{\rm FL}_{\epsilon_{\rm d}}\,|\,\chi^{\rm FL}_{\epsilon_{\rm d}}\rangle=1$. $\phi^1_s(r\leq R_2)$ with its amplitude $A(\epsilon_{\rm d})$ is also normalized, $\langle\phi^1_s\,|\,\phi^1_s\rangle=1$, and represents a quark compound bag (QCB), consisting of gluons and six quarks $(6q)$, which we call "*soul*" and denote by letter s. r_0 is the effective radius which represents the transition region $R_1<r_0<R_2$ between the quark and nucleon domains. A choice of r_0 will be made based on our method as discussed later.

The continuity of QCD and conventional nuclear physics demonstrates the need for a unified description of nuclear charge from factors. The "democratic chain" QCD model by Brodsky and Chertok[1] predicts the behavior of a system of n "valence" quarks and antiquarks for asymptotic Q^2

$$F_n(Q^2)=\left[1+\frac{Q^2}{m_n^2}\right]^{-n+1}\ , \quad (2)$$

where the mass m_n^2 is proportional to the number of constituent quark and antiquarks. This limit can be accommodated with appropriate weighting factors consistent with the probability of the *soul* component in our deuteron wave function ansatz (1). Therefore this study may offer an independent estimate of the same probability from the experimental data and QCD prediction for the deuteron form factor.

According to our ansatz [Eq. (1)] for the deuteron wave function, the deuteron form factor $F_{\rm d}(Q^2)$ can be written as the following:

$$F_{\rm d}(Q^2)=N^2F^{\rm np}(Q^2)+2ANF^{\rm int}(Q^2)+A^2F^s(Q^2)\ , \quad (3)$$

where $F^{\rm np}(Q^2)$ is the ordinary deuteron form factor calculated with the nonrelativistic Lomon-Feshbach wave function, $F^s(Q^2)$ is the form factor due to the QCB state, and $F^{\rm int}(Q^2)$ is the interference term. There is experimental evidence that at large momenta transfer the deuteron form factor is in agreement with QCD scaling rules.[1,8] In the region of large Q^2 the QCB component $F^s(Q^2)$ is therefore expected to play a role in explaining the experimental data. This expected behavior may allow us to extract the QCB probability directly from the experimental data.

Taking the two-nucleon nature of the deuteron into account, Brodsky and Chertok[1] proposed the following parametrization of $F_{\rm d}(Q^2)$:

$$F^{\rm BC}(Q^2)\sim f_{\rm d}(Q^2)F^2_{\rm N}(Q^2/4)\ , \quad (4)$$

where

$$f_{\rm d}(Q^2)=(1+Q^2/0.28\ {\rm GeV}^2)^{-1}$$

is the so-called deuteron reduced form factor, and

$$F_{\rm N}(Q^2)=(1+Q^2/0.71\ {\rm GeV}^2)^{-2}$$

is the nucleon form factor. The comparison of Eq. (4) with the experimental data at large Q^2 (~ 4 GeV^2) yields

$$F^{\rm exp}_{\rm d}(Q^2)/F^{\rm BC}_{\rm d}(Q^2)=A^2=12\%\ ,$$

assuming $A^2(Q^2)=A^2(\epsilon_{\rm d})$. Such extrapolation, however, is expected to be unreliable. In principle, in Eq. (4) we have to consider the "running" QCD coupling constant $\alpha(Q^2)$ (Refs. 2 and 9) as an additional factor, whose behavior is known only at large Q^2, where $\alpha(Q^2)$ decreases logarithmically with increasing Q^2. Apart from this factor the parametrization (4) is not valid at small Q^2. Therefore we can regard $A^2=12\%$ as an upper bound for the QCB probability.

A general QCD prediction for asymptotic form factor of an object with n quarks, in which each quark is treated in a democratic way, is given by Eq. (2), where $m_n^2=n\beta^2=n(0.235\ \mathrm{GeV}^2)$. We note that in the case of pion ($n=2$) and proton ($n=3$) which are pure quark compound bags by themselves, formula (2) fit the experimental data in the *entire* Q^2 region. It is tempting to generalize Eq. (2) to describe any genuine multiquark compound state for any range of Q^2. If we use Eq. (2) for deuteron ($n=6$) with $m_6^2=6\beta^2$, we obtain for the six-quark probability

$$F_{\mathrm{d}}^{\mathrm{exp}}(Q^2)/F_6(Q^2)=A^2=20\% .$$

Since 20% seems to be unreasonably large, we conclude that the relation $m_n^2=n\beta^2$ is not valid for the multiquark compound bag with $n\geq 6$.

In order to make Eq. (2) compatible with our model, we require that the rms radius calculated from Eq. (2) is the same as our six-quark bag radius given by the Feshbach-Lomon boundary condition[7] $r_0=0.7$ fm. Since

$$r_{\mathrm{rms}}^2=1.2\ (\mathrm{GeV\,fm})^2/m_6^2$$

for Eq. (2) we obtain $m_6^2=2.45\ \mathrm{GeV}^2$ instead of 1.41 GeV^2. Using parametrization (2) with the new value m_6^2, we obtain for the QCB probability $A^2=3-4\,\%$, which is in good agreement with our subsequent determination of A^2 using a highly excited QCB resonance. Such consistency of our model justifies the use of the boundary condition model with $r_0=0.7$ fm. We note that $r_0=0.7$ fm for the Feshbach-Lomon potential was chosen to give best fits to the phase shifts.[7]

Now we turn to the second independent estimate of the same probability using the partial width of the dibaryon resonance of Kamae *et al.*[3] and Grein *et al.*[4] For simplicity, we consider the most simple case of the n-p system with only one open (s-wave elastic) channel and write our ansatz (1) as

$$\psi_{\mathrm{np}}=\phi_{\mathrm{n}}\phi_{\mathrm{p}}\chi(r>R_1)+\phi_s(r<R_2)\ , \tag{5}$$

with $R_1\leq r_0\leq R_2$.

The interior state ϕ_s is written as a superposition of quark compound state functions ϕ_s^α with energy-dependent amplitudes $A_\alpha(E)$:

$$\phi_s=\sum_\alpha A_\alpha(E)\phi_s^\alpha\ . \tag{6}$$

The coefficients $A_\alpha(E)$ describe the enhancement of the component ϕ_s^α at the pertinent resonance energy and are to be determined from a dynamical equation. The expectation value

$$E_s^\alpha=\langle\phi_s^\alpha\,|\,H_{\mathrm{QCD}}(r\leq R_2)\,|\,\phi_s^\alpha\rangle$$

describes approximately (up to a resonance shift due to the coupling to the open channels) the corresponding mass of the resonance.

Because of the quark confinement, we expect the spectrum of the *soul* state to be discrete in the absence of the exterior part. It then follows that any compound structure can be expressed as a superposition of eigensolutions $\hat{\phi}_i$ with energy-independent amplitudes $\phi_s^\alpha=\sum b_k^\alpha\hat{\phi}_k$, where $\hat{\phi}_k$ diagonalize the Hamiltonian H_{QCD} in the region $[0,R_2]$:

$$H_{\mathrm{QCD}}(r\leq R_2)\hat{\phi}_i=\epsilon_i\hat{\phi}_i\ . \tag{7}$$

It is reasonable to expect that ϕ_s^α may be described by one or few $\hat{\phi}_k$'s which give the dominant contribution. Note, however, that ϕ_s^α's do not necessarily form an orthonormal set, whereas $\hat{\phi}_k$'s do. For simplicity, we consider only the first (lowest) excitation, $\phi_s\cong A_1(E)\phi_s^1$. We assume that the cluster dynamics in the exterior region can be described satisfactorily within a static (or nonstatic) potential mode such as the boundary condition model (BCM) of Feshbach and Lomon. The total wave function ψ satisfies

$$H_{\mathrm{QCD}}\psi=E\psi\ . \tag{8}$$

Using the resonating group method,[10] we derive a set of dynamical equations from our ansatz (5) by considering the test function space

$$\delta\psi=\phi_{\mathrm{n}}\phi_{\mathrm{p}}\delta\chi+\phi_s^1\delta A_1(E)\ .$$

By projecting $\delta\psi$ on Eq. (8). i.e.,

$$\langle\delta\psi\,|\,H_{\mathrm{QCD}}-E\,|\,\psi\rangle=0\ ,$$

we obtain the following set of equations $[A(E)=A_1(E)$ and $H(6q)=H_{\mathrm{QCD}}]$:

$$\langle\phi_{\mathrm{n}}\phi_{\mathrm{p}}\delta\chi\,|\,H(6q)-E\,|\,\phi_{\mathrm{n}}\phi_{\mathrm{p}}\chi\rangle + A(E)\langle\phi_{\mathrm{n}}\phi_{\mathrm{p}}\delta\chi\,|\,H(6q)-E\,|\,\phi_s^1\rangle=0\ , \tag{9}$$

$$\langle\phi_s^1\,|\,H(6q)-E\,|\,\phi_{\mathrm{n}}\phi_{\mathrm{p}}\chi\rangle + A(E)\langle\phi_s^1\,|\,H(6q)-E\,|\,\phi_s^1\rangle=0\ . \tag{10}$$

From the last equation we can eliminate $A(E)$:

$$A(E)=-\frac{\langle\phi_s^1\,|\,H(6q)-E\,|\,\phi_{\mathrm{n}}\phi_{\mathrm{p}}\chi\rangle}{\langle\phi_s^1\,|\,H(6q)-E\,|\,\phi_s^1\rangle}\ . \tag{11}$$

According to our assumption,

$$\langle\phi_{\mathrm{n}}\phi_{\mathrm{p}}\delta\chi\,|\,H(6q)-E\,|\,\phi_{\mathrm{n}}\phi_{\mathrm{p}}\chi\rangle$$

in Eq. (9) goes upon integration of cluster internal degrees of freedom in the region $r\geq r_0$ into a potential Hamiltonian which can be best represented by the Feshbach-Lomon boundary condition model (see our discussion below), i.e., $(T+V_{\mathrm{FL}}-e)\chi^{\mathrm{FL}}=0$ for the relative motion wave function χ^{FL}. At this point we should note that the boundary value of BCM, $r_0=0.7$ fm, was determined as the best value around the range of two-pion mass which gives the best fit to the NN-phase shifts. A single value of $r_0=0.7$ fm for all channels need not be a final choice if it turns out that a different value of r_0, $R_1<r_0<R_2$, for each channel can give a better overall description of the phase shift, QCB resonances, and bound-state properties. We note also that the boundary r_0 is related directly to that of the multiquark bag considered first in the work by Jaffe and Low.[11]

The evaluation of the numerator in Eq. (11) gives

$$\langle \phi_s^1 \mid H(6q)-E \mid \phi_n\phi_p\chi_E \rangle = \int_{R_1}^{R_2} dr\, w^1(r)\chi_E(r) = (R_2-R_1)w^1(r^*)\chi_E(r^*) ,$$

with

$$R_1 \le r^* \le R_2 ,$$

and projected functions,

$$w^1(r) = \langle \phi_s^1 \mid H(6q)-E \mid \phi_n\phi_p r(\le R_2)\rangle$$

and

$$\chi_E(r) = \langle \phi_n\phi_p r(\ge R_1) \mid \phi_n\phi_p\chi_E \rangle .$$

Assuming that r^* depends only weakly on energy, we can regard $r_0 = r^*$ as a reasonable approximation and define the form factor ψ^1 as

$$\psi^1(r) = \langle \phi_s^1 \mid H(6q)-E \mid \phi_n\phi_p r \ge R_1 \rangle = c^1\delta(r-r_0) ,$$

where $c^1 = (R_2-R_1)w^1(r^*)$ depends weakly on energy and represents the effective coupling strength of $H(6q)$ between the interior quark and the exterior nuclear phase of matter.

Thus, the elimination of the six-quark bag gives rise to an additional potential of a separable form with energy-dependent strength $\lambda(E) = (e^1-e)^{-1}$. We obtain therefore from Eq. (9),

$$\left[T + V_{\rm FL} - \frac{|\psi^1\rangle\langle\psi^1|}{e^1-e} - e \right]\chi = 0 . \tag{12}$$

Here T is the kinetic energy operator of the n-p system, $V_{\rm FL}$ is the Feshbach-Lomon potential plus the boundary condition at r_0, e and e^1 are defined by $e = E - e_{\rm n} - e_{\rm p}$ and by $e^1 = E^1 - e_{\rm n} - e_{\rm p}$, respectively, where $e_{\rm n(p)}$ is the nucleon mass corresponding to a QCD solution of $H(3q)\phi_{\rm n(p)} = e_{\rm n(p)}\phi_{\rm n(p)}$.

We turn now to the derivation of the one-channel resonance formula. The solutions of the Feshbach-Lomon BCM equation, $(T+V_{\rm FL}-e)\chi^{\rm FL} = 0$, are known and can now be used in order to solve Eq. (12) explicitly:

$$\chi^+ = \chi^{\rm FL} + G^{\rm FL}(e)\frac{|\psi^1\rangle\langle\psi^1|\chi^+\rangle}{e^1-e} . \tag{13}$$

Due to the separability of the resonance potential, we obtain for χ^+:

$$\chi^+ = \chi^{\rm FL+} + G^{\rm FL}(e)\frac{|\psi^1\rangle\langle\psi^1|\chi^{\rm FL+}\rangle}{e^1-e-\langle\psi^1|G^{\rm FL}(e)|\psi^1\rangle} . \tag{14}$$

Here $G^{\rm FL}(e)$ is the Green's function of the Feshbach-Lomon Hamiltonian. Using the operator identity

$$\lim_{\epsilon\to 0^+} \frac{1}{e'-e-i\epsilon} = P\frac{1}{e'-e} + i\pi\delta(e'-e) ,$$

the non-Hermitian Green's operator $G^{\rm FL}(e)$ can be separated into a Hermitian and an anti-Hermitian part:

$$G^{\rm FL}(e) = G^r(e) + i\pi G^i(e) , \tag{15}$$

with

$$G^r(e) = P \sum\!\!\!\!\!\!\int_{e'} \frac{|\chi_{e'}^{\rm FL}\rangle\langle\chi_{e'}^{\rm FL}|}{e'-e} \tag{16}$$

and

$$G^i(e) = |\chi_e^{\rm FL+}\rangle \frac{1}{e} \langle\chi_e^{\rm FL+}| . \tag{17}$$

With these definitions, we can write $(e_r \approx e^1)$

$$\langle\psi^1|G^{\rm FL}(e)|\psi^1\rangle = \Delta(e) + i\tfrac{1}{2}\Gamma(e) \tag{18}$$

with

$$\Delta(e) = P \sum\!\!\!\!\!\!\int_{e'} \frac{(c^1)^2|\chi_{e'}^{\rm FL}(r_0)|^2}{e'-e} \tag{19}$$

and

$$\Gamma(e) \cong 2\pi(c^1)^2|\chi_{e_r}^{\rm FL}(r_0)|^2/e_r . \tag{20}$$

In order to extract the S-matrix element for the solution χ^+, we use the asymptotic behavior of the Green's function to obtain for the S-matrix element S_l in a state of angular momentum l

$$S_l = \exp[2i\delta_l^{\rm FL}(e)] \frac{e^1-e-\Delta(e)+\frac{1}{2}i\Gamma(e)}{e^1-e-\Delta(e)-\frac{1}{2}i\Gamma(e)} . \tag{21}$$

The exponential function $\exp[2i\delta_l^{\rm FL}(e)]$ contains the background phase shift $\delta_l^{\rm FL}(e)$ of the Feshbach-Lomon BCM solutions, i.e., without the knowledge of the six-quark excitation. A generalization of our formulation to the multichannel and coupled-channel resonances will be given elsewhere.[5]

We will now give explicit formulae for the bound state deuteron wave function, from which we can determine the probability for the nuclear $N^2(\epsilon_{\rm d})$ and $A^2(\epsilon_{\rm d})$ parts. We obtain from Eqs. (1) and (11),

$$A = -N\frac{c^1\chi_{\epsilon_{\rm d}}^{\rm FL}(r_0)}{e^1+\epsilon_{\rm d}} \tag{22}$$

and

$$N^2 = \frac{1}{1+\dfrac{(c^1)^2|\chi_{\epsilon_{\rm d}}^{\rm FL}(r_0)|^2}{(e^1+\epsilon_{\rm d})^2}} . \tag{23}$$

Equations (22) and (23) show that from the knowledge of the experimental resonance width in the n-p channel and from the ratio $|\chi_{\epsilon_{\rm d}}^{\rm FL}(r_0)/\chi_e^{\rm FL}(r_0)|$ we can determine the probability amplitude of six-quark compound bag in deuteron. Using Eqs. (20) and (23) we can write

$$c^1\chi_{\epsilon_{\rm d}}^{\rm FL}(r_0) = [\Gamma(e_r)e_r/2\pi]^{1/2}x$$

and

$$N^2 = \left[1 + \frac{\Gamma_{\rm np}e_r x^2}{2\pi(e_r+\epsilon_{\rm d})^2}\right]^{-1} ,$$

where $\Gamma_{\rm np} = \Gamma(e_r)$ and

$$x = \chi^{\rm FL}_{\epsilon_{\rm d}}(r_0) / \chi^{\rm FL}_{e_r}(r_0) \ .$$

χ's are normalized so that, for $r \to \infty$, $r\chi^{\rm FL}_{e}(r) \to e^{-\beta r}$ and $r\chi^{\rm FL}_{\epsilon_r}(r) \to e^{+i\kappa r}$ with $\kappa \to i\beta$. When evaluated from the coupled-channel BCM interaction of Lomon[12] with $r_0 = 0.7$ fm, the numerical values are $\chi^{\rm FL}_{e_r}(r_0) = -1.2$ at the Kamae resonance energy (2352 MeV),[3] $\chi^{\rm FL}_{\epsilon_{\rm d}}(r_0) = 2.9$, and $x^2 = 5.86$. Using $\Gamma_{\rm np} \approx 12$—30 MeV,[4] we obtain the probability of ϕ_s,

$$| A(\epsilon_{\rm d}) |^2 = 1 - N^2 = 2.2\text{—}5.5\% \ ,$$

which is consistent with the previous estimate of 3—4 % described above.

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