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Electromagnetically induced nuclear beta decay calculated by a Green's function method

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The transition probability for enhancement of forbidden nuclear beta decay by an applied plane-wave electromagnetic field is calculated in a nonrelativistic spinless approximation by a Green's function method. The calculation involves a stationary-phase approximation. The stationary phase points in the presence of an intense field are located in very different positions than they are in the field-free case. In order-of-magnitude terms, the results are completely consistent with an earlier, much more complete wave-function calculation which includes spin and relativistic effects. Both the present Green's function calculation and the earlier wave function calculation give electromagnetic contributions in first-forbidden nuclear beta decay matrix elements which are of order $(R_0/\lambda\!\!\!^{-}_C)^2$ with respect to allowed decays, where R_0 is the nuclear radius and $\lambda\!\!\!^{-}_C$ is the electron Compton wavelength.

I. INTRODUCTION

In Ref. 1 (hereafter referred to as I), the acceleration of nuclear beta decay by application of a plane-wave electromagnetic field was calculated by a straightforward wave function approach. In particular, the procedure involves execution of integrations over time at an early stage, which leads to the appearance of an energy-conserving delta function. Once this delta function is exhibited, it is then possible to perform the unambiguous separation of contributions representing allowed beta decay, conventional forbidden beta decay (from "retardation" or lepton momentum contributions), and electromagnetically induced beta decay. An alternative procedure, suggested by Becker, Schlicher, and Scully[2] (hereafter referred to as BSS), is based on the use of Green's functions. Their procedure involves an early execution of integrations over momentum states, with time integrations left for later. Since no energy delta function is extracted, it is not possible to make simple distinctions between allowed, conventional forbidden, and electromagnetically enhanced portions of the transition probability. They all remain intimately mixed in the integrand of the time integrations. The purpose of this paper is to show how to carry the Green's function calculation to completion, with concomitant identification of allowed and forbidden constituents. The relative orders of magnitude of conventional forbidden and electromagnetically induced terms with respect to the allowed term are shown to be in agreement with the results of I.

II. ANALYSIS

A. Conditions

Since the intent of this work is qualitative and not quantitative, the simplest approximations introduced by BSS are adopted here. Spin terms are dropped, the electromagnetic field is taken to be circularly polarized, the decay electron is treated nonrelativistically, and the longitudinal part of the electron momentum is neglected.

B. Transition probability

The transition probability per unit time for field interaction with a beta decay system, reflecting the conditions and limitations stated above, is given by BSS as

$$W=\frac{1}{(2\pi)^6}\lim_{T\to\infty}\frac{1}{T}\int d^3r\int d^3r'\int dt\int dt' e^{iE_0(t'-t)}G_v(r',t';r,t)$$
$$\times\left[\frac{2m\pi}{i(t'-t)}\right]^{3/2}\exp\left[i\frac{m}{2}\frac{(\vec{r}\,'-\vec{r})^2}{(t'-t)}\right]\exp\left[ie\int_t^{t'}d\tau\,\vec{R}(\tau)\cdot\vec{E}(\tau)\right]$$
$$\times\exp\left[-\frac{ie^2}{2m}\left\{\int_t^{t'}d\tau\,\vec{A}^{\,2}(\tau)-\frac{1}{t'-t}\left[\int_t^{t'}d\tau\,\vec{A}(\tau)\right]^2\right\}\right]$$
$$\times[\phi_f^*(\vec{r})(gV)\phi_i(\vec{r})][\phi_i^*(\vec{r}\,')(gV)^*\phi_f(\vec{r}\,')]\ . \tag{1}$$

In Eq. (1), E_0 is the difference between initial and final nuclear energies in the transition; $G_{(\nu)}$ is the neutrino Green's function, given by

$$G_{(\nu)}(r',t';r,t)=\int d^3E_\nu e^{-iE_\nu(t'-t)}e^{i\vec{k}_\nu\cdot(\vec{r}'-\vec{r})}\ , \tag{2}$$

where $E_\nu=|\vec{k}_\nu|$; $\vec{R}$ is a spatial displacement parameter defined by

$$\vec{R}(\tau)=\vec{r}'-(\vec{r}'-\vec{r})(t'-\tau)/(t'-t)\ ; \tag{3}$$

$\vec{E}(t)$ is the electric field vector in the long-wavelength approximation; $\vec{A}(t)$ is the Coulomb-gauge vector potential of the plane-wave field, also in the long-wavelength approximation; the $\phi(\vec{r})$ functions give the ordinary nuclear wave functions in final and initial states; and gV represents the weak interaction. The spatial integrals in Eq. (1) will reflect the usual selection rules of beta decay. In particular, if the rules for Fermi transitions (there are no Gamow-Teller transitions since spin is neglected) are not met, then the overlap of the ϕ_f and ϕ_i wave functions will give a zero result. Spatial dependence arising from parts of Eq. (1) other than ϕ_f and ϕ_i are necessary to achieve a nonvanishing result when the selection rules are violated.

C. Integrals over time variables

The t and t' integrals in Eq. (1) must be done in order to assess the forbiddenness-removing properties of the $\vec{r}$ and $\vec{r}'$ dependent factors which occur in Eq. (1). From Eqs. (1) and (2), the integrations over t and t' are

$$I\equiv\int_{-T/2}^{T/2}dt\int_{-T/2}^{T/2}dt'\left[\frac{2m\pi}{i(t'-t)}\right]^{3/2}\exp[i(E_0-E_\nu)(t'-t)]\exp\left[\frac{im}{2}\frac{(\vec{r}'-\vec{r})^2}{(t'-t)}\right]$$
$$\times\exp\left[ie\int_t^{t'}d\tau\,\vec{R}(\tau)\cdot\vec{E}(\tau)\right]\exp\left[-\frac{ie^2}{2m}\left\{\int_t^{t'}d\tau\,\vec{A}^2(\tau)-\frac{1}{t'-t}\left[\int_t^{t'}d\tau\,\vec{A}(\tau)\right]^2\right\}\right]. \tag{4}$$

The analysis of Eq. (4) is expedited if an explicit form for the plane-wave electromagnetic field is specified. The field is to be treated in the long-wavelength approximation in Coulomb gauge, so the assignments

$$\vec{A}=a\vec{\epsilon}\cos\omega t\ ,\qquad \vec{E}=\omega a\vec{\epsilon}\sin\omega t \tag{5}$$

are appropriate. When Eqs. (3) and (5) are substituted into Eq. (4), the result is

$$I=\int_{-T/2}^{T/2}dt\int_{-T/2}^{T/2}dt'\left[\frac{2m\pi}{i(t'-t)}\right]^{3/2}e^{if(t',t)}\ , \tag{6}$$

where

$$f(t',t)=(E_0-E_\nu)(t'-t)-\frac{e^2a^2}{4m}\left[(t'-t)+\frac{1}{2\omega}(\sin2\omega t'-\sin2\omega t)-\frac{2}{\omega^2(t'-t)}(\sin\omega t'-\sin\omega t)^2\right]$$
$$+ea\vec{\epsilon}\cdot\left[-\vec{r}'\cos\omega t'+\vec{r}\cos\omega t+\left[\frac{\vec{r}'-\vec{r}}{t'-t}\right]\frac{(\sin\omega t'-\sin\omega t)}{\omega}\right]+\frac{m}{2}\frac{(\vec{r}'-\vec{r})^2}{(t'-t)}\ . \tag{7}$$

A few notational changes will be made in Eqs. (6) and (7). The variables of integration will be changed from t',t to the dimensionless variables x,y, where

$$x=\omega(t'-t),\quad y=\omega t\ . \tag{8}$$

Then the double integral is

$$I=\frac{1}{\omega^2}\left[\frac{2m\pi\omega}{i}\right]^{3/2}\int dx\int dy\frac{1}{x^{3/2}}e^{i(m/\omega)g(x,y)}\ , \tag{9}$$

where

$$g(x,y)=\frac{\Delta E}{m}x-\frac{z_f}{2}\left\{x+\tfrac{1}{2}\sin2(x+y)-\tfrac{1}{2}\sin2y-\frac{2}{x}[\sin(x+y)-\sin y]^2\right\}$$
$$-(2z_f)^{1/2}\omega\vec{\epsilon}\cdot\vec{r}'\cos(x+y)+(2z_f)^{1/2}\omega\vec{\epsilon}\cdot\vec{r}\cos y+(2z_f)^{1/2}\omega\vec{\epsilon}\cdot\Delta\vec{r}\frac{1}{x}[\sin(x+y)-\sin y]+\tfrac{1}{2}(\omega\Delta\vec{r})^2\frac{1}{x}\ , \tag{10}$$

with

$$\Delta E = E_0 - E_\nu, \quad \Delta\vec{r} = \vec{r}\,' - \vec{r}\ , \tag{11}$$

and with the intensity parameter given by

$$z_f = \frac{e^2 a^2}{2m^2} \ . \tag{12}$$

D. Stationary phase points

The purpose of exhibiting the factor m/ω as a multiplier of $g(x,y)$ in the exponential function in Eq. (9) is to make explicit the presence of a large parameter in the exponential. The hypothesis is made that the electromagnetic field which intervenes in the beta decay is a low-frequency field. The ratio m/ω is then an extremely large number, and so a stationary phase approximation can be employed to evaluate one of the integrals in Eq. (9). The integral over x will be evaluated in this fashion.

An important prelude to the stationary phase calculation is the estimation of the orders of magnitude of the terms in $g(x,y)$ given in Eq. (10). The magnitude of the first term is set by the nature of beta decay, which yields

$$O(\Delta E/m) = 1 \ . \tag{13}$$

Some statment must be made about the field frequencies to be considered. Suppose that the highest frequencies of interest are in the visible or near-uv region. This can be stated as

$$O\left[\frac{\omega}{m}\right] < 10^{-5} \ . \tag{14}$$

Since the r and r' coordinates are bounded by R_0, the nuclear radius, and since the nuclear radius measured in terms of the electron Compton wavelength is

$$O(mR_0) = 10^{-2} \tag{15}$$

for essentially all nuclei, then Eqs. (14) and (15) give the bound

$$O(|\omega\vec{r}\,|) < 10^{-7} \ . \tag{16}$$

It will be further presumed that the intensity parameter is relatively large, so as to be in the most likely region for electromagnetically enhanced beta decay. This is stated as

$$O(z_f) > 1 \ . \tag{17}$$

If z_f is taken to be large, then a stationary phase point occurs (as will be shown below) for a value of x such that

$$x = O(z_f^{-1/2}) \ . \tag{18}$$

The end result of Eqs. (13), and (16)–(18) is that the dominant terms in $g(x,y)$ in Eq. (10) are

$$g(x,y) \approx \frac{\Delta E}{m}x - \frac{z_f}{2}\left\{ x + \tfrac{1}{2}\sin 2(x+y) - \tfrac{1}{2}\sin 2y - \frac{2}{x}[\sin(x+y) - \sin y]^2 \right\} . \tag{19}$$

Equation (19) is entirely adequate to establish the location of stationary phase points. It yields

$$\frac{\partial g}{\partial x} = \frac{\Delta E}{m} - z_f\left\{ \cos(x+y) - \frac{1}{x}[\sin(x+y) - \sin y] \right\}^2 . \tag{20}$$

Stationary phase points are found from setting $\partial g/\partial x = 0$, or, from Eq. (20),

$$\frac{\sin(x+y) - \sin y}{x} = \cos(x+y) \pm \left[\frac{\Delta E/m}{z_f}\right]^{1/2} . \tag{21}$$

Equation (21) is difficult to solve, but a simple closed form answer emerges if the assumption is made that

$$|x| \ll 1 \ . \tag{22}$$

From Eq. (18), the implication is that $z_f \gg 1$. With Eq. (22), the solution of Eq. (21) is

$$x_0 = \pm \frac{2}{\sin y}\left[\frac{\Delta E/m}{z_f}\right]^{1/2}; \quad y \not\approx 0(\text{modulo } \pi) \ , \tag{23}$$

which justifies Eq. (18).

The proviso in Eq. (23) that y should not be in the neighborhood of the zeros of $\sin y$ is to provide consistency with the constraint in Eq. (22). With z_f sufficiently large, the limitation on y in Eq. (23) excludes only a small fraction of the y space. It will be shown shortly that the outcome of the stationary phase calculation is to provide a dependence on the spatial r,r' coordinates which removes forbiddenness in the nuclear transition matrix elements. The stipulation in Eq. (23) that $\sin y \not\approx 0$ simply means that there is a portion of the integration over y which does not contribute to forbiddenness removal.

E. Forbiddenness removal due to the field

Now that stationary phase points have been located, they can be used to evaluate the integral over x. However, all that is really of interest here is to examine the way in which introduction of the electromagnetic field overcomes forbiddenness in the beta decay. This can then be compared with analogous terms which arise in the field-free case from orbital angular momentum contributions from the leptons emitted in the beta decay.

Upon stationary phase evaluation of the integral over x, a factor

$$\exp[i(m/\omega)g(x_0,y)] \tag{24}$$

will emerge, where x_0 is a stationary phase point, as found in Eq. (23). As a prelude to substitution of x_0 into $g(x,y)$, the small-x form of $g(x,y)$ will be written. This form is

$$g(x,y) = \frac{\Delta E}{m}x + \tfrac{7}{12}z_f x^3 \sin^2 y + \left[\frac{z_f}{2}\right]^{1/2} \omega\vec{\epsilon}\cdot(\vec{r}\,' + \vec{r}\,)x\sin y \ , \tag{25}$$

where the last term in Eq. (10) is omitted because of Eq. (16). The x_0 solution in Eq. (23), when substituted into Eq. (25), yields

$$\frac{m}{\omega}g(x_0,y)=\pm\frac{20}{3}\frac{\Delta E}{\omega}\left[\frac{\Delta E/m}{z_f}\right]^{1/2}\frac{1}{\sin y}$$
$$\pm(2m\Delta E)^{1/2}\vec{\epsilon}\cdot(\vec{r}\,'+\vec{r})\ . \qquad (26)$$

The essential content of Eq. (26) is in the dependence on $\vec{r}$ and $\vec{r}\,'$ in the last term. From Eq. (13), and the fact that r and r' are of the order of the nuclear radius R_0, then

$$(2m\Delta E)^{1/2}\vec{\epsilon}\cdot(\vec{r}\,'+\vec{r})=O(mR_0)$$
$$=O(10^{-2})\ , \qquad (27)$$

where the last statement was expressed in Eq. (15). In consequence of Eq. (27), when Eq. (26) is employed in the exponential given in Eq. (24), the spatial dependence in the exponential can be expanded as

$$\exp[i(m/\omega)g(x_0,y)]\rightarrow 1\pm i(2m\Delta E)^{1/2}\epsilon\cdot(\vec{r}\,'+\vec{r})\ . \qquad (28)$$

If a beta decay is first forbidden without intervention of an electromagnetic field, then the first term on the right-hand side of Eq. (28) will make no contribution, but the second term will. This last term introduces a change in parity into the transition matrix element, as well as a Y_1 spherical harmonic. It thus gives a nontrivial transition probability in a first-forbidden beta decay. Expansion terms of higher order than those exhibited in Eq. (28) will give contributions in more highly forbidden beta decays.

F. Field-free case

The field-free case can be treated very simply by setting z_f equal to zero. Equation (10) then has only the two terms

$$g(x,y)=\frac{\Delta E}{m}x+\tfrac{1}{2}(\omega\Delta\vec{r})^2\frac{1}{x}\ , \qquad (29)$$

which yields

$$\frac{\partial g}{\partial x}=\frac{\Delta E}{m}-\frac{1}{2}\frac{(\omega\Delta\vec{r})^2}{x^2}\ . \qquad (30)$$

The stationary phase point is found from equating Eq. (30) to zero, which yields

$$x_0=\pm\frac{\omega\,|\Delta\vec{r}\,|}{\sqrt{2\Delta E/m}}\ . \qquad (31)$$

[This stationary phase point location was presumed by BSS to apply both to the field-free case and to the situation where a field is present. That presumption is incorrect. Equation (23) gives the stationary phase point location in the presence of a field.] Equation (31), inserted into Eq. (29), gives a value to Eq. (24) of

$$\exp[i(m/\omega)g(x_0,y)]=\exp(\pm i\sqrt{2m\Delta E}\ |\vec{r}\,'-\vec{r}\,|)\ . \qquad (32)$$

In view of the small magnitude of the argument of the exponential in Eq. (32), the expansion

$$\exp[i(m/\omega)g(x_0,y)]\approx 1\pm i\sqrt{2m\Delta E}\ |\vec{r}\,'-\vec{r}\,| \qquad (33)$$

is appropriate.

The first term in Eq. (33) is the usual allowed term in beta decay. The second term in Eq. (33) is the retardation term of ordinary (i.e., not field-induced) forbidden beta decay. It does not have quite the customary form because the sequence of operations in this Green's function approach differs from the conventional approach. Ordinarily, the correction term to allowed beta decay has the form

$$i(\vec{p}_e+\vec{k}_\nu)\cdot(\vec{r}\,'-\vec{r})\ , \qquad (34)$$

where $\vec{p}_e$ and $\vec{k}_\nu$ are the momenta of the beta particle and the antineutrino, respectively. However, here an integration over all the components of $\vec{p}_e$ has already been done. One way to establish the identity of the second term in Eq. (33) is to note that if $\vec{p}_e$ and $\vec{k}_\nu$ are neglected at an early stage in the (field-free) calculation, then the second term in Eq. (33) never appears.

A very important result is that the conventional forbidden beta decay term in Eq. (33) is of exactly the same order of magnitude as the term examined in Eq. (27). That is, the conventional retardation term in forbidden beta decay and the electromagnetic-field-induced term in forbidden beta decay are of the same order of magnitude.

III. CONCLUDING REMARKS

It has just been shown in Eqs. (28) and (33) that the electromagnetic enhancement term and the conventional retardation term play exactly the same role in the nuclear matrix element of forbidden beta decay. There are some qualifications to this parallelism which must be made.

In the foregoing analysis, the explicit spin term in the Volkov solution for the beta particle was neglected. The spin term will make an additional contribution of the same order of magnitude as the scalar Volkov term retained here. Furthermore, the longitudinal contribution from the beta decay electron has been entirely neglected. As shown in I, this contributes an additional amplitude of equal magnitude to the transverse term treated here. Altogether, the electromagnetic contribution to the nuclear matrix element of forbidden beta decay will be larger than that provided by the retardation term. However, in electromagnetically enhanced beta decay, the field occurs in other factors which lie outside the nuclear matrix element. [See Eq. (6) of Ref. 3.] Overall results for electromagnetically enhanced decay can then either exceed or fall short of ordinary forbidden beta decay transition probabilities. The actual comparison depends on the particular nuclear beta decay, and on the parameters of the electromagnetic field.

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