Dirac single-particle wave functions in inelastic electron scattering

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We formulate inelastic electron scattering using Dirac single-particle wave functions to describe the nuclear states. The lower components of these wave functions are much larger than those obtained assuming a free space relation with the upper component taken to be a Schrödinger single-particle wave function. We note that the impulse approximation to the nuclear current operator is ambiguous in the present case. However, one of the two possible forms yields an explicitly conserved current in the single particle limit. We identify an inelastic amplitude which is linear in the lower components and is therefore very sensitive to the "relativity" of the bound nucleons. This amplitude is found to be dominant for transverse isoscalar transitions.

I. INTRODUCTION

Electron scattering has long been recognized as one of the most important probes of nuclear structure. Because the fundamental interaction (one photon exchange) is presumably well understood, one can focus on the nuclear structure aspects of the problem without complications arising from uncertainties in the reaction mechanism. In addition to providing valuable nuclear structure information itself, electron scattering also serves as a means of calibrating more versatile probes (such as hadrons) whose fundamental interactions are less well understood.

The standard formulation of electron scattering is, at the outset, relativistic. The electron-nucleus S matrix[1] is proportional to

$$j_e \cdot J_{fi} \ , \tag{1}$$

where j_e^μ is the electron four current and

$$J_{fi}^\mu = \int d\vec{r} \exp(-i\vec{q}\cdot\vec{r}) \langle f \mid \gamma^0 \hat{J}^\mu \mid i \rangle \tag{2}$$

is the nuclear transition four current. The nuclear wave functions in Eq. (2) are implicitly Dirac spinors consisting of upper and lower components. Almost without exception in the existing literature, the upper component is taken to be a standard Schrödinger wave function and the lower component is obtained by assuming the free space relation between upper and lower components. The relativistic modifications of the nuclear wave functions due to the nuclear potentials are generally treated (if at all) by calculating corrections[2-4] in powers of v/c. Since the potential energies encountered in the usual Schrödinger treatment of nuclear bound states are small on the scale of the nuclear mass, the corrections are also small and can be treated in powers of v/c.

Recently a new approach to the nucleon-nucleus interaction based on the use of the Dirac equation has been developed. This approach has provided a very appealing description of infinite nuclear matter[5,6] as well as finite nuclei using the Dirac-Hartree[7,8] or Dirac-Hartree-Fock[9] methods. Proton-nucleus elastic scattering has also been shown[10,11] to be very well described via the Dirac impulse approximation.[12] All of these treatments are characterized by the fact that nucleons interact with nuclei (or, more generally, nuclear matter) via scalar and timelike vector potentials with strengths on the order of the nucleon mass. This implies that the nucleon-nucleus problem is fundamentally relativistic and that it cannot effectively be treated via v/c corrections.

The upper components of Dirac single particle wave functions are generally quite similar to the corresponding Schrödinger wave functions. However, the lower components of the Dirac wave function are quite different from those obtained from the Schrödinger wave function (assuming the free space relation between upper and lower components). Since the lower components of nuclear bound state wave functions appear explicitly in electron-nucleus inelastic scattering, it is of interest to formulate the problem using Dirac wave functions and to identify quantities which directly reflect the properties of the lower components and, hence, might be very different in the Dirac and Schrödinger approaches.

Miller[13] and Serot[14] have investigated the role of Dirac wave functions in exclusive electron-nucleus elastic scattering. Miller restricted his inquiry to elastic scattering from spherically symmetric spin-zero nuclei and found only very small relativistic effects. Serot[14] examined the consequences of using a Dirac $1h_{9/2}$ proton single particle wave function in calculating the magnetic contribution to $e^- + {}^{209}Bi$ elastic scattering. He found large differences between Dirac and Schrödinger calculations, but unfortunately these differences occur only in a region of momentum transfer where no data exist.

In the present work, we present the first calculations of inelastic electron scattering form factors employing Dirac single particle wave functions. As will be seen below, the

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extension to inelastic scattering allows us to identify various types of transitions sensitive to several different aspects of the calculations. Furthermore, reasonable experimental data exist for such transitions in just the range of momentum transfer where such differences are appreciable.

II. THE NUCLEAR CURRENT OPERATOR

We wish to evaluate the nuclear transition current of Eq. (2) using initial and final nuclear wave functions expanded in a Dirac shell model basis. Assuming $\tilde{J}^\mu$ to be a single particle operator, we write

$$\langle f \mid \gamma^0 \tilde{J}^\mu \mid i \rangle = \langle J_f M_f \mid \gamma^0 \tilde{J}^\mu \mid J_i M_i \rangle = -\sum_J (J_i J M_i M \mid J_f M_f) \sum_{j_f j_i} \left[\frac{2j_f+1}{2J+1} \right]^{1/2} A_J^{J_f J_i}(j_f j_i) \langle \psi_{j_f} || \{ \psi_{j_i} (\gamma^0 \tilde{J}^\mu)_J \}_{j_f} \rangle \ , \tag{3}$$

where $(\gamma^0 \tilde{J}^\mu)_J$ is the Jth multipole of $e^{-i\vec{q}\cdot\vec{r}} \gamma^0 \tilde{J}^\mu$. The usual one-body spectroscopic amplitude is defined by

$$A_J^{J_f J_i}(j_f j_i) = \langle \psi_{J_f} || \{ \psi_{J_i} [a_{j_f}^\dagger b_{j_i}^\dagger]_J \}_{J_f} \rangle \tag{4}$$

and where the ψ_j are Dirac single particle bound state wave functions, with $a^\dagger$ and $b^\dagger$ being the associated particle and hole creation operators, respectively. We use the gamma matrix notation of Ref. 15 and the following definition of the reduced matrix element:

$$\langle \Psi_{J_f M_f} \mid T_{Kq} \mid \Psi_{J_i M_i} \rangle = \langle J_i K M_i q \mid J_f M_f \rangle \langle \Psi_{J_f} || \{ \Psi_{J_i} T_K \}_{J_f} \rangle \ . \tag{5}$$

The single particle wave functions are assumed to be time-independent solutions of

$$[\not{p} - m - S(r) - \gamma^0 V(r)] \psi_{nljm}(\vec{r}) = 0 \ , \tag{6}$$

where S and V are (real) scalar and timelike vector potentials. These wave functions then have the form

$$\psi_{nljm}(\vec{r}) = \begin{bmatrix} u_{nlj}(r) Y_{ljm}(\hat{r}) \\ i w_{nlj}(r) Y_{l'jm}(\hat{r}) \end{bmatrix} , \tag{7}$$

where

$$Y_{ljm}(\hat{r}) = [Y_l(\hat{r}) \chi_{1/2}]_{jm} \ , \tag{8}$$

and where $l' = l \pm 1$ for $j = l \pm \frac{1}{2}$. Using Eq. (6), we find that the radial piece of the large component, u, is related to that of the lower component, w, by

$$w_{nlj}(r) = \frac{1}{\mathscr{E}(r) + \mathscr{M}(r)} \left[\frac{d}{dr} - \frac{\kappa_{lj}}{r} \right] u_{nlj}(r) \ , \tag{9}$$

where

$$\kappa_{lj} = \begin{cases} l & \text{for } j = l + \frac{1}{2} \\ -l-1 & \text{for } j = l - \frac{1}{2} \ . \end{cases} \tag{10}$$

In the preceding we have used

$$\begin{aligned} \mathscr{E}(r) &\equiv E - V(r) \ , \\ \mathscr{M}(r) &\equiv m + S(r) \ , \end{aligned} \tag{11}$$

where E is the asymptotic total energy of the bound nucleon and m is the free nucleon rest mass. Note that, in standard nonrelativistic formulations of electron scattering, the Schrödinger wave function is identified with the upper component of a Dirac wave function and the lower component is obtained from it via the relation

$$w_{nlj}^{(\mathrm{Sch})}(r) = \frac{1}{E+m} \left[\frac{d}{dr} - \frac{\kappa_{lj}}{r} \right] u_{nlj}^{(\mathrm{Sch})}(r) \ , \tag{12}$$

which follows from the *free* Dirac equation. To the extent that the large components of our Dirac single particle wave functions can be identified with the usual Schrödinger wave functions, the main difference between our relativistic treatment and standard formulations lies in the nature of the small components. Comparing Eqs. (9) and (12), we find

$$w_{nlj}^{(\mathrm{Dir})}(r) = \frac{E+m}{E+m-V(r)+S(r)} w_{nlj}^{(\mathrm{Sch})}(r) \ . \tag{13}$$

In most relativistic models, $V \simeq +350$ MeV and $S \simeq -450$ MeV in the nuclear interior.[5,6] Therefore, the lower components of the Dirac wave functions are enhanced by a factor of about 1.7 in the nuclear interior. This is a direct consequence of the large potentials which characterize the model and is directly related to the magnitude of the nucleon-nucleus spin-orbit potential. In fact, this enhancement of the lower components also implies an enhancement of the nucleon-nucleus spin-orbit interaction in the nuclear interior and has been shown to be partly responsible for the success of the Dirac impulse approximation for proton-nucleus elastic scattering mentioned above.[10,11]

Before going further, we must decide on the form of the nucleon current operator. Since we do not have a complete relativistic quantum field theory of nuclear structure for which a consistent, conserved electromagnetic current can be identified, we use the impulse approximation and take $\tilde{J}^\mu$ to be the free nucleon current operator. In the present case, however, even the impulse approximation is an ambiguous prescription.[16] The ambiguity arises because the free nucleon matrix elements of the free current operator can be written, using the Gordon decomposition, in two equivalent forms which are both consistent with the notion of minimal coupling. Specifically, we have[15,16]

$$\tilde{J}^\mu(1) = F_1(q^2) \gamma^\mu + \frac{i\kappa}{2m} F_2(q^2) \sigma^{\mu\nu} q_\nu \ , \tag{14}$$

$$\tilde{J}^\mu(2) = F_3'(q^2) \gamma^\mu + \kappa F_1'(q^2) Q^\mu \ , \tag{15}$$

where F_1 and F_2 are the usual free nucleon form factors, κ is the nuclear anomalous magnetic moment, and where

$F_3'=F_1+\kappa F_2$ and $F_1'=-\kappa F_2/(2m)$. Also, we have used $q^\mu=(p_f-p_i)^\mu$ and $Q^\mu=(p_f+p_i)^\mu$, where p_i (p_f) is the initial (final) nucleon four-momentum. The equality

$$\bar{u}(p_f,s_f)\tilde{J}^\mu(1)u(p_i,s_i)=\bar{u}(p_f,s_f)\tilde{J}^\mu(2)u(p_i,s_i) \tag{16}$$

obtained by the Gordon decomposition depends on the fact that the free spinors satisfy the free Dirac equation, viz.,

$$(\not{p}-m)u(p,s)=0\ . \tag{17}$$

Since our single particle wave functions instead satisfy Eq. (6), we have the inequality,

$$\bar{\psi}_f\tilde{J}^\mu(1)\psi_i\neq\bar{\psi}_f\tilde{J}^\mu(2)\psi_i\ , \tag{18}$$

in general. We can use a generalized Gordon decomposition (see Appendix A) to recast the transition current given by $\tilde{J}^\mu(2)$ so as to facilitate comparison of the two currents. Assuming the initial and final (real) bound state potentials to be equal ($S_f^*,=S_i$, $V_f^*=V_i$; see Appendix A), we find

$$\bar{\psi}_f\tilde{J}^\mu(2)\psi_i=\bar{\psi}_fF(q^2)\{[1-\kappa S(r)/m]\gamma^\mu -\kappa V(r)/m\delta^{\mu 0}+i\kappa\sigma^{\mu\nu}q_\nu/(2m)\}\psi_i\ , \tag{19}$$

where we have also assumed $F_1(q^2)=F_2(q^2)=F(q^2)$. (This approximation can be easily removed.) This is to be compared with

$$\bar{\psi}_f\tilde{J}^\mu(1)\psi_i=\bar{\psi}_fF(q^2)\{\gamma^\mu+i\kappa\sigma^{\mu\nu}q_\nu/(2m)\}\psi_i\ . \tag{20}$$

The two currents become equal in the limits $S=0$, $V=0$, or $\kappa=0$, as we expect. In Appendix A, we show that, again assuming initial and final state potentials to be equal, the transition current obtained with $\tilde{J}^\mu(1)$ [Eq. (19)] is conserved while that given by $\tilde{J}^\mu(2)$ [Eq. (20)] is not. While this property is not particularly relevant except in the extreme single particle limit (i.e., no residual interactions in the nuclear Hamiltonian), it nevertheless leads us to prefer $\tilde{J}^\mu(1)$. We note that in the usual nonrelativistic formulation using Schrödinger wave functions and Eq. (12) to generate the lower components, the transition current obtained using $\tilde{J}\mu(1)$ is *not* conserved when the wave function is generated using velocity-dependent potentials such as the spin-orbit interaction.[17]

III. EVALUATION OF TRANSITION FORM FACTORS

In order to evaluate electron scattering form factors, we rewrite $\tilde{J}^\mu$ in such a way that the spin dependence and the upper/lower component dependence are separated. We define matrices which operate only in component space:

$$\Gamma_1=\begin{bmatrix}1&0\\0&1\end{bmatrix},\quad \Gamma_2=\begin{bmatrix}1&0\\0&-1\end{bmatrix},$$
$$\Gamma_3=\begin{bmatrix}0&1\\1&0\end{bmatrix},\quad \Gamma_4=\begin{bmatrix}0&1\\-1&0\end{bmatrix}. \tag{21}$$

We then have from Eq. (20), where $\vec{\sigma}$ is the usual Pauli spin matrix,

$$\tilde{J}^\mu(1)=F(q^2)\{\Gamma_2+\kappa/(2m)\vec{\sigma}\cdot\vec{q}\Gamma_3,\ \vec{\sigma}\Gamma_4+\kappa/(2m)q_0\vec{\sigma}\Gamma_3+i\kappa/(2m)\vec{\sigma}\times\vec{q}\Gamma_1\}\ . \tag{22}$$

A similar expression is obtained for $\tilde{J}^\mu(2)$ using Eq. (19). Transition form factors evaluated using $\tilde{J}^\mu(1)$ are given explicitly in Appendix B.

Since the major difference between the standard nonrelativistic formulation and our present relativistic one is in the magnitude of the lower components of the bound state wave functions, electron scattering amplitudes which are linear in the lower components should be most affected by use of Dirac single particle wave functions. Such amplitudes correspond to terms of $\tilde{J}^\mu$ [Eq. (22)] involving Γ_3 or Γ_4. Two of these terms are multiplied by factors of $1/2m$ and therefore are generally negligible. The third, proportional to $\vec{\gamma}=\vec{\sigma}\Gamma_4$, can be dominant in certain transitions. Specifically, this term contributes to transverse excitations and competes with the $\vec{\sigma}\times\vec{q}$ term. This latter term connects upper components to upper components and thus is not strongly affected by using Dirac wave functions. It is also linear in the anomalous magnetic moments. Since $\kappa_p=1.79$ and $\kappa_n=-1.91$, we have

$$\kappa_{T=1}/\kappa_{T=0}=3.70/-0.12=-31\ .$$

This implies that the importance of the $\vec{\sigma}\times\vec{q}$ term is strongly isospin dependent and that the effects of using Dirac wave functions are likely to be most pronounced for isoscalar transverse transitions where the $\vec{\sigma}\times\vec{q}$ term is relatively small.

We have calculated transverse form factors for the transitions to the 4.44 MeV 2^+ $T=0$, 12.71 MeV 1^+ $T=0$, and 15.11 MeV 1^+ $T=1$ levels in ^{12}C using the current operator $\tilde{J}^\mu(1)$ of Eq. (22). The single particle wave functions were obtained by numerically solving Eq. (6) with real scalar and vector Woods-Saxon potentials which are the same for the initial and final states. The potential strengths and Woods-Saxon geometries were chosen (1) to be qualitatively consistent with Dirac low-energy, proton-nucleus elastic scattering potentials,[18] (2) to reproduce the experimental $0s_{1/2}$, $0p_{3/2}$, and $0p_{1/2}$ nucleon binding energies,[19] and (3) to be consistent with e^-+^{12}C elastic scattering. The fit to the elastic scattering data[20] is shown in Fig. 1. The resulting potential strengths are $V(0)=+361$ MeV and $S(0)=-430$ MeV, while the radius parameter is $R=1.275\times(11)^{1/3}$ and the diffuseness parameter is $a=0.635$ fm.

It is well known[21,22] that the nuclear structure of these levels is *not* simple. Since no nuclear structure calculations using a Dirac single particle basis exist, we adopt one-body spectroscopic amplitudes [Eq. (4)] obtained from standard nonrelativistic nuclear structure calculations. Specifically, we have used the amplitudes of Lee and

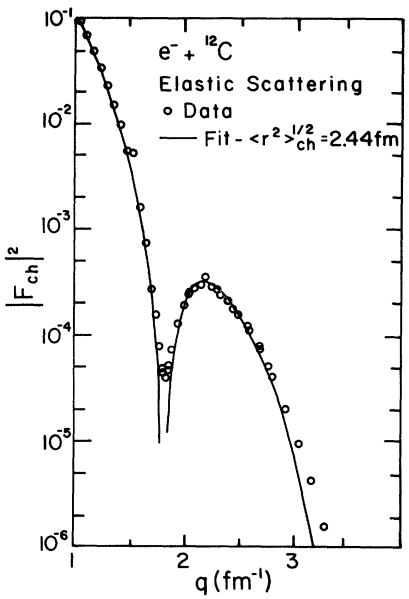

FIG. 1. The fit used to fix the ^{12}C bound state potentials is compared with the $e^- + {}^{12}C$ elastic scattering data of Ref. 20. The charge density is computed using the proton $0s_{1/2}$, $0p_{3/2}$, and $0p_{1/2}$ Dirac single particle wave functions determined by the potentials described in the text.

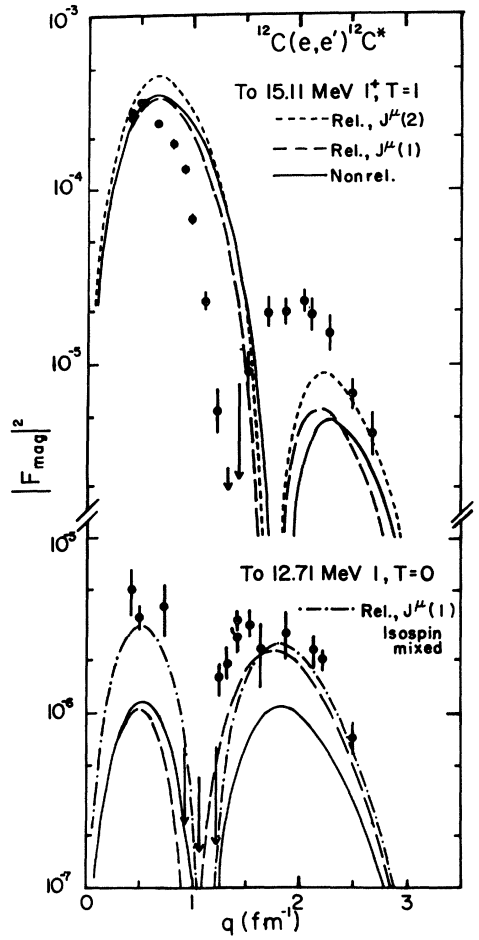

FIG. 2. Calculations are compared with experimental form factors (Ref. 25) for the first 1^+ levels in ^{12}C. The relativistic calculations for the $T=1$ level use the two forms of the current operator given in Eqs. (19) and (20). The nonrelativistic calculations are the same as the $\tilde{J}^\mu(1)$ relativistic calculations, except that Eq. (12) is used to compute the lower components of the bound nuclear wave functions. The spectroscopic amplitudes of Ref. 22 are used. A relativistic isospin-mixed calculation using $\tilde{J}^\mu(1)$ is compared with the 12.71 MeV data using an isospin-mixing element of 140 keV (Ref. 25).

Kurath.[22] For the 2^+ level, we have also used the projected Hartree-Fock basis (PHFBA) amplitudes of Amos and Morrison.[23,24] Since the amplitudes and the single particle basis we use are inconsistent, comparison of our results with data may not be meaningful. We nevertheless include relevant data in Figs. 2 and 3 where our calculations are presented in order to set a scale by which to judge the magnitude of the effects we find. We also note that using multiconfiguration transition amplitudes destroys the explicit current conservation discussed above and in Appendix A. It can be recovered only when a consistent relativistic formulation of the relevant nuclear structure exists.

Calculations for the 15.11 MeV 1^+ $T=1$ level appear in Fig. 2. The nonrelativistic result was obtained by setting the potentials appearing in the upper/lower component relation [Eq. (9)] to zero as discussed above. This result is quite consistent with conventional nonrelativistic calculations, e.g., the dashed curve in Fig. 1 of Ref. 21. The relativistic calculation (dashed line) differs only very slightly from the nonrelativistic one. This is because the $\vec{\sigma}\times\vec{q}$ term [Eq. (22)] is dominant for isovector transverse transitions and this term is little affected by the nature of the lower components, as discussed above.

Calculations for the 12.71 MeV 1^+ $T=0$ level appear at the bottom of Fig. 2 and again our nonrelativistic calculation (solid line) is very similar to conventional results, e.g., the nonisospin-mixed dashed curve in Fig. 3 of Ref. 25. In this case, the relativistic result (dashed line) is substantially different, being about a factor of 2 larger in the region of the second maximum ($q\simeq 1.7$ fm^{-1}). In this region of momentum transfer, the amplitude is dominated by the $\vec{\gamma}$ term in Eq. (22). This term is off diagonal in upper and lower components, hence the amplitude is linear in the lower components. Thus the factor of 2 enhancement in $|F_{\rm mag}|^2$ at $q\simeq 1.7$ fm^{-1} is a *direct reflection* of the enhancement of the lower components of the *bound* nucleons due to the strong scalar and vector potentials which bind them. We also show an isospin-mixed relativistic calculation (dashed-dot line) using the $T=1$ mixing matrix element of Flanz *et al.*[25]

Calculations of the 4.44 MeV 2^+ $T=0$ transverse (electric) form factor are displayed in Fig. 3. We have calculated the electric form factor directly and have not attempted to make it consistent with the longitudinal form factor in keeping with current conservation.[17,26] Our nonrelativistic results using the Lee and Kurath p-shell amplitudes (solid line) are again very similar to conventional results (see, e.g., the solid line, Fig. 6 of Ref. 23 or Fig. 1,

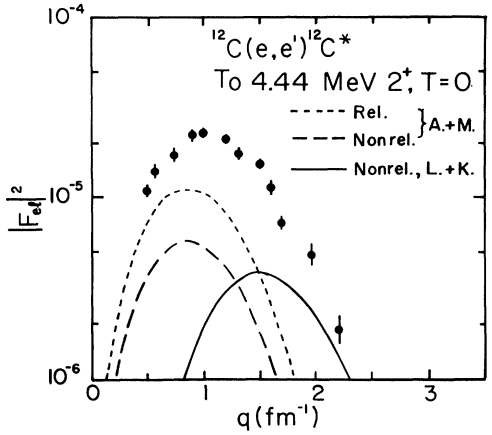

FIG. 3. Same as Fig. 2 using $\tilde{J}^\mu(1)$, but for the transverse form factor of the 4.44 MeV 2^+ $T=0$ transition. The solid curve was calculated using the amplitudes of Ref. 22; those of Ref. 23 were used for the other curves.

Ref. 27). Our nonrelativistic calculations using the projected Hartree-Fock basis amplitudes (dashed line) are roughly a factor of 2 lower than, but similar in shape to, the conventional calculations (i.e., the dashed curve, Fig. 6, Ref. 23) and therefore reproduce the shape of the experimental form factor reasonably well. The relativistic calculation using these amplitudes is uniformly about a factor of 2 larger than the nonrelativistic one, again due to the dominance of the $\vec{\gamma}$ term for isoscalar transverse transitions.

We have also examined the consequences of using the alternative form of the free nucleon current operator, $\tilde{J}^\mu(2)$. Since the two terms constituting the difference between $\tilde{J}^\mu(1)$ and $\tilde{J}^\mu(2)$ [Eqs. (19) and (20)] are proportional to κ, it is apparent that the difference arising from the use of $\tilde{J}^\mu(2)$ will be greater for isovector transitions. For longitudinal form factors, the leading terms of $\tilde{J}^\mu$ are

$$\tilde{J}^0(1)\simeq F(q^2)\gamma^0 \tag{23}$$

and

$$\tilde{J}^0(2)\simeq F(q^2)[\gamma^0(1-\kappa S/m)-\kappa V/m]\ . \tag{24}$$

Given the typical values of $S(r=0)$ and $V(r=0)$ quoted above, the quantity in Eq. (24), $|\kappa/m(S+V)|$, is about 0.4 in the nuclear interior for $T=1$ and less than 0.01 for $T=0$. Thus for *isoscalar* longitudinal form factors, the two terms of the current operator will give nearly identical results.

For *isovector* longitudinal form factors, substantially larger differences are anticipated. However, calculations of the longitudinal form factor[27] for the transition to the 16.1 MeV 2^+ $T=1$ level in ^{12}C using the amplitudes of Ref. 22 show that the $\tilde{J}^\mu(2)$ result differs uniformly by only about 20% from the $\tilde{J}^\mu(1)$ calculation, the former being larger. This suggests that such differences may generally be small compared to other uncertainties in the calculations.

The last two terms of $\tilde{J}^\mu(1)$ and $\tilde{J}^\mu(2)$ are identical. To the extent that these terms are dominant for isovector transverse transitions, the differences arising from the two forms of the current operator should be small. However, the $\vec{\gamma}$ terms are

$$\vec{\tilde{J}}(1)\simeq F(q^2)\vec{\gamma} \tag{25}$$

and

$$\vec{\tilde{J}}(2)\simeq F(q^2)\vec{\gamma}[1-3.7S(r)/m]$$
$$\rightarrow F(q^2)\vec{\gamma}\times 2.8 \text{ at } r=0\ , \tag{26}$$

which reflects a substantial enhancement of this term for $\tilde{J}^\mu(2)$. The short dashed curve for the 1^+ $T=1$ transition shown in Fig. 2 uses $\tilde{J}^\mu(2)$ [Eq. (19)] rather than $\tilde{J}\mu(1)$ [Eq. (20)] which was used to generate the long dashed curve. This comparison indicates that the differences in the form of the contribution from the $\vec{\gamma}$ term in the current operator can have non-negligible consequences for transverse isovector transitions even though this term is not dominant. Calculations of the 1^+ $T=0$ form factor using $\tilde{J}^\mu(1)$ and $\tilde{J}^\mu(2)$ are nearly identical, in keeping with the arguments presented above.

IV. SUMMARY AND CONCLUSIONS

We have evaluated directly the relativistic nuclear matrix elements which determine electron-nucleus inelastic scattering transition form factors using Dirac single particle bound state wave functions. These wave functions are calculated using real scalar and timelike vector potentials which are consistent with recently developed relativistic descriptions of infinite nuclear matter,[5,6] finite nuclei,[7–9] and proton-nucleus elastic scattering.[10–12] The major difference between the present approach and standard nonrelativistic treatments of inelastic electron scattering (e.g., Ref. 1) lies in the nature of the lower components of the bound nucleon. Specifically, they are nearly a factor of 2 larger in the nuclear interior for our Dirac wave functions.

Lacking a consistent relativistic treatment of the nuclear structure, we must approximate the nuclear current operator. We apply the impulse approximation but find that the two "equivalent" forms of the free nuclear current operator which are consistent with the idea of minimal coupling are not equivalent for nucleons bound by strong vector and scalar potentials. However, we find that, under the condition that the binding potentials are the same in the initial and final states, the more common form[1] of the current operator [Eqs. (14), (20), and (22)] yields an explicitly conserved current and is therefore preferable.

In evaluating the nuclear matrix elements using this current operator, we find that only transverse isoscalar form factors are strongly affected by the use of Dirac shell model wave functions. This is because the transverse isoscalar amplitude is dominated by the γ term of the current operator which implies a linear dependence on the lower components of the bound state wave functions. All other amplitudes are insensitive to the nature of the lower components.

The factor of 2 increase in transverse isoscalar form

factors which results when Dirac shell model wave functions are used is a direct reflection of the unique relativistic properties of bound nucleons in relativistic treatments of nuclear structure. The effect of using the alternate form of the impulse approximation nucleon current operator [Eqs. (15) and (19)] is found to be very small for isoscalar transitions and substantial only for isovector transverse excitations.

It is difficult to assess the significance of the present calculations by comparing with data since they have been performed without the benefit of a consistent relativistic description of the relevant nuclear structure. It is, however, encouraging that they appear to provide a more consistent description of $e^- + {}^{12}C$ inelastic scattering (especially for the isoscalar transverse form factors) than do standard nonrelativistic calculations[23,25,27] employing the same impulse-approximation nucleon current operator. In any case, the present work makes clear the need for the extension of currently existing relativistic descriptions of the ground states of spin-zero nuclei[7-9] to include ground states with spin as well as excited nuclear levels. Surely inelastic electron scattering will provide important tests of such a relativistic shell model. Furthermore, such shell model wave functions might also shed light on the reasons for the apparent failure[28,29] of the Dirac model to reproduce the magnetic moment of ^{15}N in the single particle limit. The explicit current conservation in the single particle limit which we have demonstrated suggests that the thorny problem of how to generate a general, self-consistent, conserved transition current may be easier to address within the framework of a relativistic shell model. Certainly it is within such a framework that ideas about relativistic medium modifications of the nucleon current[30] can best be tested.

The present treatment of inelastic scattering can easily be extended to include the (e,e'p) reaction[31] and the single-nucleon model[32] of the (p,γ) reaction. In fact, the use of Dirac bound state and continuum wave functions is likely to be important in instances where nonorthogonality effects[33] are large. This is because most of the strong energy dependence of the Schrödinger central and spin-orbit potentials is explicitly accounted for in Dirac phenomenology.[34] Put another way, the energy dependence of the Dirac scalar and vector potentials is much less than for the Schrödinger potentials. Therefore the nonorthogonality of Dirac bound and continuum wave functions can be expected to be much less than for their Schrödinger counterparts.

Finally, we note that, by using relativistic wave functions, we evaluate relativistic matrix elements "as they stand," without reductions to nonrelativistic forms which often throw out some of the physics and may obscure the structure of the amplitude. This should therefore be the ideal framework to use in further establishing the relationships among electromagnetic, weak, and hadronic probes of nuclear structure.[1] Furthermore, if the current relativistic formulations of nuclear phenomena with their strong scalar and vector nucleon-nucleus interactions are valid, the use of relativistic wave functions as we have done here is almost certainly the most economical calculational procedure.

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APPENDIX A

A. Generalized Gordon decomposition

We assume that the initial and final bound state wave functions satisfy the *same* Dirac equation containing real scalar and timelike vector potentials

$$(\tilde{p}-m-S-\gamma^0 V)\psi=0\ , \tag{A1}$$

where $\tilde{p}=i(\partial_0,-\vec{\nabla})$. The adjoint equation is

$$0=\bar{\psi}(-\tilde{p}-m-S-\gamma^0 V)\ . \tag{A2}$$

We can then write

$$\bar{\psi}_f(-\overleftarrow{\tilde{p}}-m-S-\gamma^0 V)a\psi_i + \bar{\psi}_f a(\vec{\tilde{p}}-m-S-\gamma^0 V)\psi_i=0\ , \tag{A3}$$

where the arrows indicate the direction in which the momentum operator acts and where a is an arbitrary, constant four-vector. We can then write

$$\bar{\psi}_f[-\overleftarrow{\tilde{p}}a+a\vec{\tilde{p}}-2ma-2Sa-V(\gamma^0 a+a\gamma^0)]\psi_i=0\ . \tag{A4}$$

Using the properties of the γ matrices, this can be rewritten as

$$\bar{\psi}_f[(-\overleftarrow{\tilde{p}}+\vec{\tilde{p}})\cdot a-i\sigma^{\mu\nu}(-\overleftarrow{\tilde{p}}-\vec{\tilde{p}})_\mu a_\nu-2a(m+S) - V(\gamma^0 a+a\gamma^0)]\psi_i=0\ . \tag{A5}$$

This can be further reorganized to give

$$\bar{\psi}_f(Q^\mu+i\sigma^{\mu\nu}q_\nu-2\mathcal{M}\gamma^\mu-2V\delta^{\mu 0})\psi_i=0\ , \tag{A6}$$

since a_μ is arbitrary and where we have defined $Q\equiv-\overleftarrow{\tilde{p}}+\vec{\tilde{p}}$, $q\equiv-\overleftarrow{\tilde{p}}-\vec{\tilde{p}}$, and $\mathcal{M}\equiv m+S$. Finally we have the generalized Gordon decomposition

$$\bar{\psi}_f\gamma^\mu\psi_i=\bar{\psi}_f[Q^\mu/(2\mathcal{M})-V/(2\mathcal{M})\delta^{\mu 0} + i\sigma^{\mu\nu}q_\nu/(2\mathcal{M})]\psi_i\ , \tag{A7}$$

where we understand that the operators Q^μ and q_ν *do not* act on $\mathcal{M}(r)$.

We now use Eq. (A7) to recast the second form of the transition current

$$J^\mu_{fi}(2)=\bar{\psi}_f[F'_3(q^2)\gamma^\mu+\kappa F'_1(q^2)Q^\mu]\psi_i =\bar{\psi}_f F(q^2)[(1+\kappa)\gamma^\mu-\kappa/(2m)Q^\mu]\psi_i\ ,$$

where we have assumed the free space relations between the F_1,F_2 and F'_3,F'_1, i.e., $F'_1=-\kappa F_2/(2m)$ and $F'_3=F_1+\kappa F_2$, and where we have also assumed $F_1=F_2=F$. Using Eq. (A7) to eliminate Q^μ, we have

$$J^\mu_{fi}(2)=\bar{\psi}_f F(q^2)[(1-\kappa S/m)\gamma^\mu-\kappa V/m\delta^{\mu 0} + i\kappa/(2m)\sigma^{\mu\nu}q_\nu]\psi_i\ . \tag{A8}$$

B. Current conservation

We first examine the four-divergence of the Dirac transition current,

$$\partial_\mu J^\mu_{fi}(D)\equiv\partial_\mu\bar{\psi}_f\gamma^\mu\psi_i \ . \tag{A9}$$

We can write

$$\begin{aligned}\partial_\mu J^\mu_{fi}(D)&=\partial_\mu\psi^\dagger_f\gamma^0\gamma^\mu\psi_i\\&=(\partial\psi_f)^\dagger\gamma^0\psi_i+\psi^\dagger_f\gamma^0(\partial\psi_i) \ .\end{aligned} \tag{A10}$$

Using $\partial\!\!\!/=-i\tilde{p}\!\!\!/$ and Eq. (A1), we have

$$\partial_\mu J^\mu_{fi}(D)=-i\bar{\psi}_f[(S_i-S^*_f)+\gamma^0(V_i-V^*_f)]\psi_i \ . \tag{A11}$$

For real potentials which are the same for initial and final states,

$$\partial_\mu J^\mu_{fi}(D)=0 \ .$$

We now examine the four-divergence of the current arising from the second term of $\tilde{J}^\mu(2)$ [Eq. (15)],

$$\partial_\mu J^\mu_{fi}(Q)\equiv\partial_\mu\bar{\psi}_f Q^\mu\psi_i=\partial_\mu\bar{\psi}_f(-\overleftarrow{\tilde{\mathrm{p}}}^\mu+\vec{\tilde{\mathrm{p}}}^\mu)\psi_i \ . \tag{A12}$$

This can be rewritten as

$$\begin{aligned}\partial_\mu J^\mu_{fi}(Q)&=-i\tilde{p}_\mu\bar{\psi}_f(-\overleftarrow{\tilde{\mathrm{p}}}^\mu+\vec{\tilde{\mathrm{p}}}^\mu)\psi_i\\&=-i\bar{\psi}_f(-\overleftarrow{\tilde{\mathrm{p}}}^2+\vec{\tilde{\mathrm{p}}}^2)\psi_i \ .\end{aligned} \tag{A13}$$

We now use Eqs. (A1) and (A2) to show

$$\tilde{p}\!\!\!/^2\psi_i=\left[(m+S-\gamma^0V)(m+S+\gamma^0V)+2E_iV+i\vec{\gamma}\cdot\hat{r}S'-i\gamma^0\vec{\gamma}\cdot\hat{r}V'\right]\psi_i \tag{A14}$$

and

$$-\bar{\psi}_f\tilde{p}\!\!\!/^2=\bar{\psi}_f\left[-(m+S+\gamma^0V)(m+S-\gamma^0V)-2E_fV+i\vec{\gamma}\cdot\hat{r}S'+i\gamma^0\vec{\gamma}\cdot\hat{r}V'\right] , \tag{A15}$$

where the primes indicate radial derivatives. We then combine Eqs. (A13)–(A15) to obtain

$$\partial_\mu J^\mu_{fi}(Q)=2i\bar{\psi}_f\left[(E_f-E_i)V-i\vec{\gamma}\cdot\hat{r}S'\right]\psi_i \ , \tag{A16}$$

which implies $J^\mu_{fi}(2)$ is *not* conserved in general. Finally we consider the four-divergence of the current arising from the second term of $\tilde{J}^\mu(1)$ [Eq. (14)],

$$\partial_\mu J^\mu_{fi}(q)=-\partial_\mu\bar{\psi}_f\sigma^{\mu\nu}(\overleftarrow{\tilde{\mathrm{p}}}_\nu+\vec{\tilde{\mathrm{p}}}_\nu)\psi_i \ . \tag{A17}$$

Again using $\partial_\mu=-i\tilde{p}_\mu$, we have

$$\partial_\mu J^\mu_{fi}(q)=i\bar{\psi}_f\sigma^{\mu\nu}(\overleftarrow{\tilde{\mathrm{p}}}_\mu\overleftarrow{\tilde{\mathrm{p}}}_\nu+\overleftarrow{\tilde{\mathrm{p}}}_\nu\vec{\tilde{\mathrm{p}}}_\mu+\overleftarrow{\tilde{\mathrm{p}}}_\mu\vec{\tilde{\mathrm{p}}}_\nu+\vec{\tilde{\mathrm{p}}}_\mu\vec{\tilde{\mathrm{p}}}_\nu)\psi_i \ . \tag{A18}$$

Since the quantity in parentheses is symmetric under interchange of indices while $\sigma^{\mu\nu}$ is antisymmetric,

$$\partial_\mu J^\mu_{fi}(q)=0 \ , \tag{A19}$$

which taken together with Eq. (A11) implies $\partial_\mu J^\mu_{fi}(1)=0$ for $S^*_f=S_i$, $V^*_f=V_i$.

APPENDIX B

We now write explicitly the electron inelastic scattering form factors evaluated using $\tilde{J}^\mu(1)$ [Eqs. (14) and (22)]. In terms of these form factors the electron scattering cross section is

$$\frac{d\sigma}{d\Omega}=\frac{Z^2\sigma_M}{[1+2(E/M_T)\sin^2\theta/2]}F^2 \ , \tag{B1}$$

where F is the total form factor, E is the electron energy, $Z(M_T)$ is the target charge (mass), θ is the scattering angle, and σ_M is the point Mott cross section given by

$$\sigma_M=\frac{\alpha^2\cos^2\theta/2}{4E^2\sin^4\theta/2} \ , \tag{B2}$$

where, in turn, α is the fine structure constant. The total form factor is given by[1]

$$F^2=(-q^2/q'^2)^2\,|F_L|^2+[-q^2/(2q'^2)+\tan^2\theta/2]\,|F_T|^2 \ , \tag{B3}$$

where q (q') is the four- (three-) momentum transfer and F_L (F_T) is the Coulomb (transverse) form factor.

We now define[1] the following Coulomb and transverse multipole operators:

$$\hat{M}_{JM}(q')\equiv\int d\vec{\mathrm{r}}\,j_J(qr)Y_{JM}(\hat{r})J^0(\vec{\mathrm{r}}) \ , \tag{B4a}$$

$$\hat{T}^{\mathrm{el}}_{JM}(q')\equiv q^{-1}\int d\vec{\mathrm{r}}\{\vec{\nabla}\times j_J(qr)\vec{Y}_{J(J1)M}(\hat{r})\}\cdot\vec{\mathrm{J}}(\vec{\mathrm{r}}) \ , \tag{B4b}$$

$$\hat{T}^{\mathrm{mag}}_{JM}(q')\equiv\int d\vec{\mathrm{r}}\,j_J(qr)\vec{Y}_{J(J1)M}(\hat{r})\cdot\vec{\mathrm{J}}(\vec{\mathrm{r}}) \ , \tag{B4c}$$

where

$$\vec{Y}_{J(L1)M}(\hat{r})\equiv[Y_L\hat{e}_1]_{JM}$$

with $\hat{e}_{1M}$ the spherical unit vector.

The Coulomb and transverse transition form factors can then be written as

$$|F_L|^2=4\pi Z^{-2}\left[\frac{2J_f+1}{2J_i+1}\right]\sum_{J=0}^{\infty}|\langle\Psi_{J_f}||\gamma^0(\Psi_{J_i}\hat{M}_J)_{J_f}\rangle|^2 \ , \tag{B5a}$$

$$\begin{aligned}|F_T|^2&=|F_{\mathrm{el}}+F_{\mathrm{mag}}|^2\\&=4\pi Z^{-2}\left[\frac{2J_f+1}{2J_i+1}\right]\sum_{J=1}^{\infty}|\langle\Psi_{J_f}||\gamma^0[\Psi_{J_i}(\hat{T}^{\mathrm{el}}_j+\hat{T}^{\mathrm{mag}}_J)]_{J_f}\rangle|^2 \ ,\end{aligned} \tag{B5b}$$

where Ψ is the total nuclear wave function. Note that[1] $\hat{T}^{\mathrm{el}}_{JM}$ and $\hat{T}^{\mathrm{mag}}_{JM}$ have opposite parity so that one of these operators gives no contribution for a given J^π transfer. In deriving the cross section [Eqs. (B1) and (B3)] in terms of these

form factors, current conservation has been invoked[1] to eliminate the longitudinal contribution in favor of the Coulomb.

We can now write the explicit forms for $|F_L|^2$, $|F_{\rm el}|^2$, and $|F_{\rm mag}|^2$. It will be convenient to define the following quantity:

$$\mathscr{A}^{J_f J_i}_{n;J(LS)}(q') \equiv -\sum_{l_f l_i j_f j_i} \left[\frac{2j_f+1}{2J_i+1}\right]^{1/2} A^{J_f J_i}_{J(j_f j_i)} \langle \psi_{l_f j_f}(\vec{r}) || \Gamma_n j_L(q'r) \{\psi_{l_i j_i}(\vec{r})[Y_L(\hat{r})\sigma_S]_J\}_{j_f} \rangle \ . \tag{B6}$$

In Eq. (B6), $\psi_{lj}(r)$ is a Dirac single particle wave function with total angular momentum j and upper component orbital angular momentum l; $A^{J_f J_i}_{J(j_f j_i)}$ is the one-body spectroscopic amplitude defined in Eq. (4); Γ_n is one of the upper/lower component matrices defined in Eq. (21); and the "bra" in the reduced matrix element is the Hermitian conjugate wave function, i.e., $\langle \psi | = \psi^\dagger = \bar{\psi}\gamma^0$. We then have

$$|F_L(q')|^2 = 4\pi \left[\frac{F(q^2)}{Z}\right]^2 \left[\frac{2J_f+1}{2J_i+1}\right] \sum_{J=0}^{\infty} \left| \mathscr{A}_{1;J(J0)} + iq'\kappa/(2m) \left[\left[\frac{J+1}{2J+1}\right]^{1/2} \mathscr{A}_{4;J(J+1,1)} + \left[\frac{J}{2J+1}\right]^{1/2} \mathscr{A}_{4;J(J-1,1)} \right] \right|^2 , \tag{B7}$$

$$|F_{\rm el}(q')|^2 = 4\pi \left[\frac{F(q^2)}{Z}\right]^2 \left[\frac{2J_f+1}{2J_i+1}\right] \sum_{J=1}^{\infty} \left| \left[-\left[\frac{J}{2J+1}\right]^{1/2} \mathscr{A}_{3;J(J+1,1)} + \left[\frac{J+1}{2J+1}\right]^{1/2} \mathscr{A}_{3;J(J-1,1)} \right] \right.$$
$$\left. + q_0\kappa/(2m) \left[-\left[\frac{J}{2J+1}\right]^{1/2} \mathscr{A}_{4;J(J+1,1)} + \left[\frac{J+1}{2J+1}\right]^{1/2} \mathscr{A}_{4;J(J-1,1)} \right] - iq'\kappa/(2m)\mathscr{A}_{2;J(J1)} \right|^2 , \tag{B8}$$

$$|F_{\rm mag}(q')|^2 = 4\pi \left[\frac{F(q^2)}{Z}\right]^2 \left[\frac{2J_f+1}{2J_i+1}\right] \sum_{J=1}^{\infty} \left| \mathscr{A}_{3;J(J1)} + q_0\kappa/(2m)\mathscr{A}_{4;J(J1)} \right.$$
$$\left. + iq'\kappa/(2m) \left[-\left[\frac{J}{2J+1}\right]^{1/2} \mathscr{A}_{2;J(J+1,1)} + \left[\frac{J+1}{2J+1}\right]^{1/2} \mathscr{A}_{2;J(J-1,1)} \right] \right|^2 , \tag{B9}$$

where q_0 is the timelike component of the four-momentum transfer. In Eqs. (B2)—(B9), the dependence of the $\mathscr{A}$'s on J_f, J_i, and q' is implicit.

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