Scaling violation in the separated response functions of ^{12}C

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The properties of deep inelastic electron scattering from ^{12}C are analyzed in the context of relativistic y scaling. A large enhancement of the transverse response relative to the longitudinal response is observed in the quasielastic region which is in contradiction to impulse approximation predictions assuming a free one-body current. Scaling behavior is recovered when a relativistic effective mass is included in the nucleon current operator. The first two moments of the longitudinal scaling function are also evaluated.

I. INTRODUCTION

The reaction mechanism of electron scattering at the quasielastic peak is usually described in the impulse approximation (IA) as a one-step process with the virtual photon coupling to a free nucleon current. The incoherent scattering limit is valid because, at the momentum transfers relevant to quasielastic scattering, the wavelength of the probe is less than or equal to the nucleon's dimensions. Energetic constraints on the reaction process are small when the excitation energy is large compared to the single nucleon knockout threshold. These assumptions imply that the longitudinal and transverse response functions should exhibit y scaling.[1–3] The scaling behavior (or absence thereof) is a direct measure of the validity of the impulse approximation. Modifications of the nucleon properties, internal nucleonic degrees of freedom, the presence of exchange forces, and two-body correlations in the initial state are possible scale breaking mechanisms.

We have performed a scaling analysis of the separated ^{12}C(e,e′) data from Saclay[4] and have found substantial deviations from scaling behavior. We have treated the problem in a fully relativistic manner since errors introduced by a nonrelativistic reduction of the current operator can be as large as many of the nuclear modifications to the reaction.

The longitudinal and transverse response functions each behave as a function of a single variable in the region of the quasielastic peak consistent with the scaling hypothesis. However, the peak positions and widths disagree with those predicted assuming free kinematics and, more seriously, the relative strengths of the longitudinal and transverse response functions deviate by more than 50% from calculations employing a nucleon current unmodified by the nuclear medium. This latter type of scaling violation is totally unexpected and indicates that complications in the reaction mechanism extend throughout the quasielastic region and not just in the high energy-loss "dip" region as previously thought.[5]

We have found that scaling behavior can be recovered by introducing a relativistic effective mass based on a Dirac phenomenological model. In particular, the shift, broadening, and relative strengths of the response functions over the entire range of momentum transfers can be successfully accounted for in terms of this single parameter. We emphasize the fact that we found it necessary to treat this effective mass as a truly dynamical effect, modifying the nucleon current as well as kinematical factors. We feel an unambiguous interpretation of the recovery of the y scaling behavior of the ^{12}C(e,e′) response functions by this parametrization cannot be justified at this time due to the number of possible modifications to the basic reaction process.

II. QUASIELASTIC ELECTRON SCATTERING

Figure 1 shows a diagrammatic description of quasielastic scattering. The four-momentum transfer to the nuclear system by the exchanged virtual photon is denoted by $q=(\omega,\vec{q})$, and the photon is absorbed on a single nucleon of initial momentum $k=(k_0,\vec{k})$ in a one-step process. At high momentum transfers the photon incoherently samples the nucleon momentum distribution $n(\vec{k})$. There is no necessary implication that a nucleon of final momentum $\vec{k}+\vec{q}$ is actually emitted from the nu-

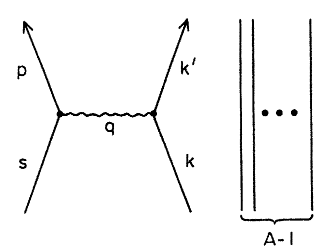

FIG. 1. Quasifree electron scattering kinematics.

cleus since the recoiling nucleon will suffer a number of final state interactions (FSI) as it propagates through the nuclear medium. Rather the diagram is usually understood to imply closure over all final states.[6] Implicit in a quasifree analysis is the assumption that the reaction channel sampled by the virtual photon is sensitive only to the local properties of the medium and that the reaction is relatively insensitive to the details of the final state propagation of the nucleon. We note, however, that this is not a universally accepted point of view.[7] This means that for the quasifree hypothesis to be valid one must also require that the excitation energy be significantly higher than the single nucleon separation energies in addition to requiring high momentum transfers. This ensures that the density of final states is large enough so that energetic constraints on the reaction process can be ignored.

The nuclear response to longitudinal and transverse virtual photon absorption can be written in terms of dimensionless structure functions, f_L and f_T, as

$$R_L(q^2,\omega)=\frac{A}{|\vec{q}\,|}\tilde{G}_E^2(q^2)\left[\frac{|\vec{q}\,|^2}{-q^2}\right]f_L(q^2,\omega) \qquad \text{(1a)}$$

and

$$R_T(q^2,\omega)=\frac{A}{|\vec{q}\,|}\tilde{G}_M^2(q^2)\left[\frac{-q^2}{2M^2}\right]f_T(q^2,\omega)\ , \qquad \text{(1b)}$$

where A is the number of nucleons, M is the nucleon mass, $\tilde{G}_E$ and $\tilde{G}_M$ are the mean nucleon electromagnetic Sachs form factors in the medium, and R_L and R_T are the usual electron scattering response functions (see the Appendix). If one neglects a small component of the reaction mechanism proportional to $(\vec{k}\times\hat{q})^2/M^2$, the nuclear structure functions can be expressed as functions of a single dimensionless variable in the high $\vec{q}$ limit

$$f_L(y)=f_T(y)=\pi M\int_{(My)^2}^{\infty}\left[\frac{M}{k_0}\right]n(\vec{k}^{\,2})d\vec{k}^{\,2}\ , \qquad \text{(2)}$$

where

$$y=\left[\frac{\vec{k}_{\min}\cdot\hat{q}}{M}\right]=-\frac{|\vec{q}\,|}{2M}+\frac{\omega}{|\vec{q}\,|}\nu\ , \qquad \text{(3a)}$$

$$\nu=\left[\frac{|\vec{q}\,|^2}{-q^2}\left[1-\frac{q^2}{4M^2}\right]\right]^{1/2}. \qquad \text{(3b)}$$

The factor (M/k_0) in the integrand arises from the relativistic contraction of the three-volume element. The nonrelativistic form of this scaling variable is given by the limit $\nu\to 1$. Nonrelativistically, energy-momentum conservation requires that $\vec{k}\cdot\hat{q}$ be a constant for fixed $\vec{q}$ and ω so that the y scaling variable is the constant initial proton velocity component along $\vec{q}$. Relativistically, the relevant constant quantity is $k_\mu q^\mu=-q^2/2$. The response function in the scaling limit should form a peak which is symmetric and whose maximum is at $y=0$, corresponding to the quasielastic peak at $\omega=-q^2/2M$.

One immediate result from this scaling analysis is a data-to-data relation between the separated response functions in terms of the elementary nucleon response

$$\frac{R_T}{R_L}\approx\frac{q^2}{2M^2}\frac{q^2}{|\vec{q}\,|^2}\frac{\tilde{G}_M^2}{\tilde{G}_E^2}\ . \qquad \text{(4)}$$

One can see that the reduced nuclear structure or scaling functions cancel in the ratio. This result is independent of the choice of the scaling variable and is more general than the impulse approximation analysis might indicate.

The results of the relativistic scaling analysis of the Saclay data are shown in Figs. 2 and 3 assuming free kinematics and the free nucleon one-body current. For the sake of clarity, only data for $\omega>30$ MeV are shown. In the vicinity of the quasielastic peak the longitudinal and transverse data individually exhibit $\vec{q}$ independence. The rapid increase of the transverse response at large ω (large positive y) is due to the opening up of the $\Delta(3,3)$ isobaric resonance channel. There is a net shift of the quasielastic peak to positive y, a result which is well known if not well understood. The most striking feature is the large relative enhancement of the transverse response relative to the longitudinal response at the quasielastic peak, a 50% effect over the entire momentum transfer range (200—600 MeV/c) of the available ^{12}C(e,e′) data.

Final state interactions by themselves cannot explain such an enhancement, because the final state interactions, insofar as they factor from the initial reaction channel, cannot depend on the polarization state of the virtual photon. This will be the case for most optical potential calculations of the FSI. The explanation for the relative enhancement must therefore lie in the nature of the reaction mechanism.

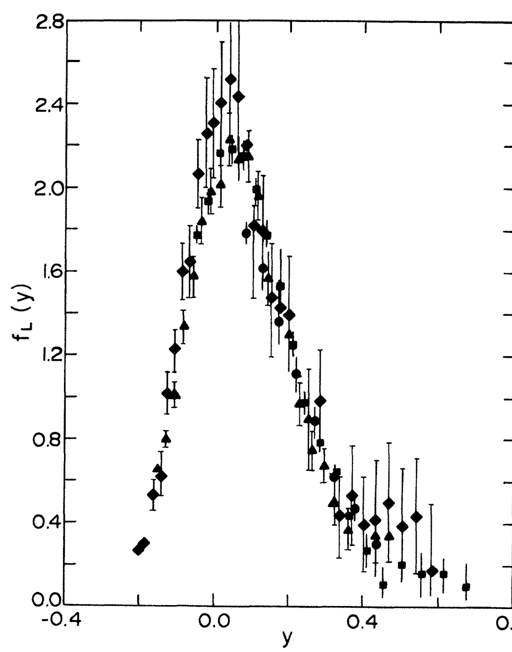

FIG. 2. The scaling function f_L for free kinematics and free current operator for ^{12}C(e,e′) is shown at $q=250$ MeV/c (circles), $q=350$ MeV/c (squares), $q=450$ MeV/c (triangles), and 550 MeV/c (diamonds).

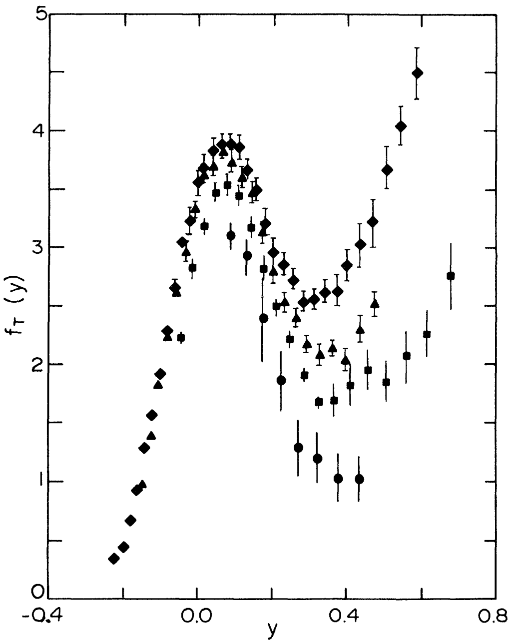

FIG. 3. The scaling function f_T for free kinematics and free current operator for $^{12}C(e,e')$ is shown at $q=250$ MeV/c (circles), $q=350$ MeV/c (squares), $q=450$ MeV/c (triangles), and 550 MeV/c (diamonds).

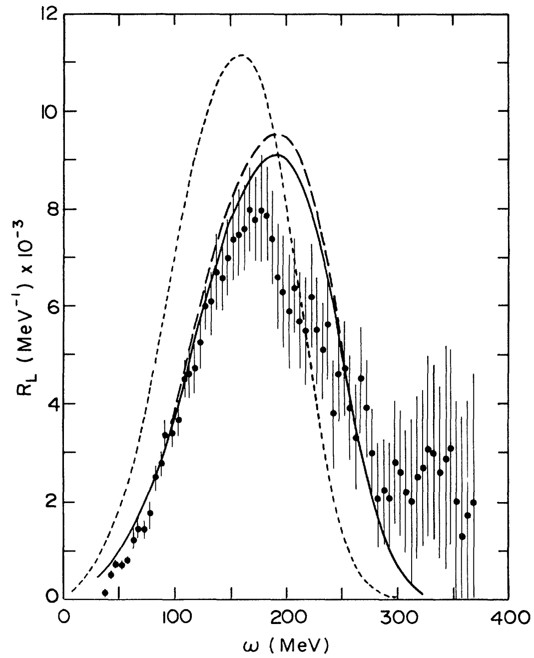

FIG. 4. Calculated quasielastic longitudinal response functions are shown using the Hartree-Fock momentum distribution (Ref. 8) with $M^*=M$ (dotted line), $M^*=0.82M$ with unmodified current (dashed), and $M^*=0.82M$ with modified current (solid) for $q=550$ MeV/c.

III. EFFECTIVE MASS APPROXIMATION

The IA with free kinematics fails to describe the data except in a qualitative sense. This is shown in Figs. 4 and 5, which compare the data at $\vec{q}=550$ MeV/c to a calculation using a Hartree-Fock momentum distribution[8] and free kinematics (dotted curve). Neglecting the absolute normalization for a moment, the observed peak position is shifted to higher excitation energies and the momentum distribution is broadened due to media effects.

A phenomenological way to account for modifications due to interaction effects is to introduce a relativistic effective mass. This shifts the quasielastic peak to an energy loss $\omega=-q^2/2M^*$ while simultaneously broadening the peak by approximately the right amount.

The relativistic effective mass approximation (REMA) can be given a physical interpretation in terms of an interacting Dirac particle in the mean field limit. In the Dirac phenomenological approach, spherical symmetry and other requirements are used to reduce the interaction to a scalar, s, and the fourth component of a vector potential, v_μ (Ref. 9). Replacing these potentials by their mean values gives the Dirac equation the momentum space form

$$\not{k}^*\Psi \equiv (\not{k}-\not{v})\Psi = M^*\Psi \equiv (M-s)\Psi \ . \tag{5}$$

Rosenfelder[10] has pointed out that in taking energy differences the vector component of the interaction cancels. Thus the net effect is a renormalization of the Dirac wave function, giving the particle an effective mass M^*. The nonrelativistic reduction of this equation generates a Schrödinger-type equation with an energy-dependent potential.[10,11]

A fundamental problem arises in extending the IA to interacting nucleons in the medium in that the nucleons are composite objects whose electromagnetic structure arises from strong interactions. In the absence of a com-

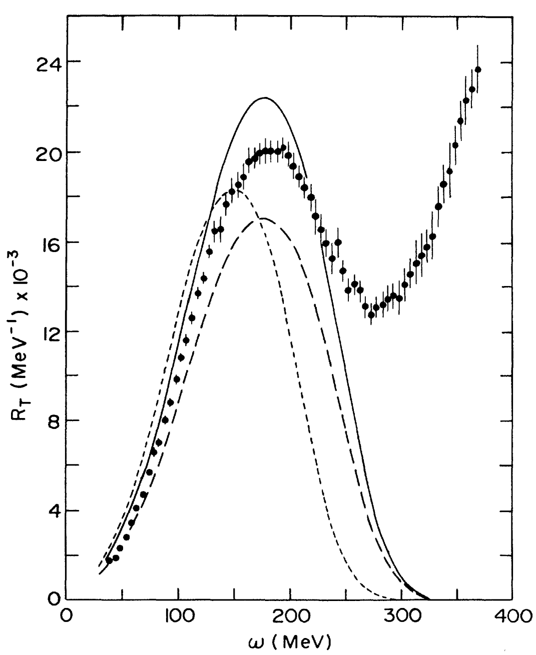

FIG. 5. Same as Fig. 4 except the transverse response is shown.

plete theory of nucleon structure it is impossible to predict how the nucleon's properties are modified by the surrounding medium. A variety of prescriptions[12–14] are available for treating the off-mass-shell behavior of the nucleon form factors. We have assumed that the Dirac and Pauli form factors are unchanged in the medium. However, it is unclear whether the effective mass should also be used in the current operator [Eq. (A1)]. Making this minimal replacement in the current has the effect of enhancing the transverse scattering which is proportional to $\tilde{G}_M^2/M^{*2}$ and to a lesser extent reducing the longitudinal scattering via the Darwin-Foldy correction. An unmodified current fails to reproduce the relative amplitudes for these response functions as can be seen in Fig. 6(b). Incorporating M^* in the current produces excellent agreement for the relative strengths of the longitudinal and transverse ^{12}C data [Fig. 6(c)]. This latter case corresponds to a replacement of M by M^* in all expressions. The intermediate case where the current is unmodified can be achieved by this same prescription and by simultaneously replacing the anomalous moment κ by $\kappa^* = \kappa(M^*/M)$. One can reconcile the separated ^{12}C data with the one-body IA only if the nucleon current is modified, a result independent of any assumed nucleon momentum distribution.

Rosenfelder,[10] using essentially the same method of analysis, finds a clear preference for an unmodified current operator in the unseparated ^{40}Ca data of Whitney *et al.*[15] Brieva and Dellafiore,[16] using a momentum dependent optical potential, obtain results similar to Rosenfelder. It is interesting to note that the ^{12}C data of Whitney *et al.* can be fit equally well with either prescription for the current and that these data are consistent with the Saclay results. It appears that there might be significant differences in the nuclear physics of these two nuclei. However, Whitney's data consist of only a single incident energy and angle on several nuclei. Without the additional information inherent in a longitudinal-transverse separation, one cannot distinguish between nuclear structure and reaction mechanism effects. The situation with respect to the separated ^{40}Ca data of Deady *et al.*[17] is unclear: The data at $\vec{q} = 330$ and 370 MeV/c exhibit a similar relative enhancement of the transverse response over the longitudinal response. The large systematic uncertainties in the 410 MeV/c data preclude any definitive statements. The early inferences that the nucleon current operator is unaffected by the medium need to be reexamined in light of the new separated data on ^{12}C.

Model calculations employing two different momentum distributions are displayed in Fig. 7. The Fermi gas and Hartree-Fock[8] momentum distributions both underestimate the high ω longitudinal response of the carbon data at 550 MeV/c. For these models, the scale breaking term in the IA proportional to $(\vec{k}\times\hat{q})^2/M^2$ is found to be about a 3% effect and is greatest in the vicinity of the quasielastic peak. The high energy loss tail observed in the longitudinal response may be due to high momentum components in the initial nuclear wave functions produced by ground state correlations or by photon induced dynamical correlations. Incoherent pion production and contributions from the $E2$ component of the $\Delta(3,3)$ resonance are also expected to contribute to the high energy loss tail.

Since pion production and meson exchange currents (MEC's) are known to be nonscaling processes and contribute primarily to the transverse response,[5] it is reasonable to use the longitudinal response as a calibrator for the one-body part of the reaction process. The $f_T - f_L$ difference spectra shown in Figs. 8(a)–(c) provide a measure of the nonquasielastic component of the reaction mechanism. With the assumption of a modified one-body current this difference maps smoothly into the $\Delta(3,3)$ resonance, and there is no remnant of the quasielastic peak.

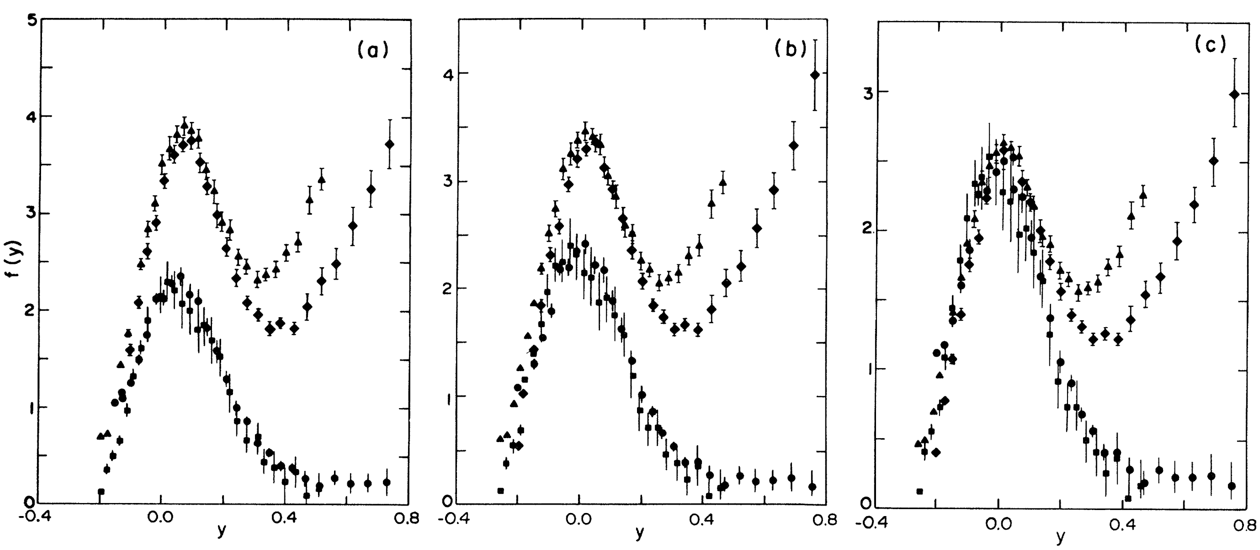

FIG. 6. Free kinematics and free current operator. (b) M replaced by $M^* = 0.82M$ in kinematics, free current retained. (c) Replacement of M by M^* in both kinematics and current. The longitudinal data are shown for $\vec{q} = 400$ MeV/c (circles) and 500 MeV/c (squares). The transverse 400 MeV/c data are shown by diamonds and the 500 MeV/c data by triangles. Note that the vertical scale is different for each figure.

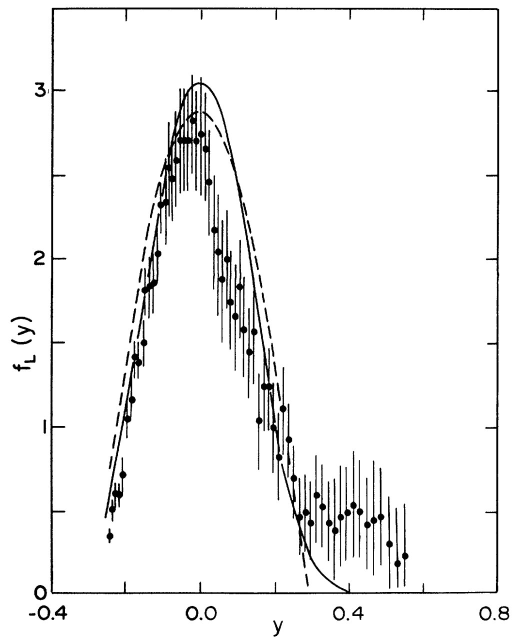

FIG. 7. Influence of the nucleon momentum distribution model. Shown are the calculated scaling functions for $\vec{q}=550$ MeV/c and $M^*=0.82M$ with modified current. The dashed line represents the relativistic Fermi gas model with $k_F=220$ MeV/c. The result using the Hartree-Fock distribution (Ref. 8) is shown by the solid curve.

IV. SUM RULES

One can define two sum rules for the longitudinal scaling function $f_L(y)$ analogous to the non-energy-weighted and energy-weighted sum rules. These are the integrated longitudinal strength,

$$S(\vec{q})=\int f_L(y)dy \ , \tag{6}$$

and the mean value of the scaling variable,

$$\overline{y_L(\vec{q})}=S(\vec{q})^{-1}\int y f_L(y)dy \ . \tag{7}$$

These sum rules are model dependent quantities since they depend on the specific IA model and ignore competing reaction processes. There is also dependence on the kinematics used to define y and on the choice of nucleon form factors. Although similar sum rules can be defined for the transverse scaling function, it is difficult to measure them accurately because of the large contribution from the $\Delta(3,3)$ resonance. The free impulse approximation predicts that $S(\vec{q})=1+\mathscr{O}(k_1^2/M^2)$ and $\overline{y_L}=0$ at high $\vec{q}$.

Figure 9 shows $S(\vec{q})$ for $\vec{q}=250$–550 MeV/c. As the data are not known for all ω, the integrations were cut off at $y=0.5(M^*/M)$, though for sufficiently large y the results are relatively insensitive to the choice of cutoff. The size of the error bars results from assuming that systematic errors are dominant. At the highest $\vec{q}$ more than 80% of the sum rule is observed using the free IA analysis. This increases to greater than 90% when M^* and the modified current are employed. Viollier and Walecka[18] estimate that two-body correlations reduce the sum rule by 1–10%. Making the common assumption that the Pauli and Dirac nucleon form factors are equal, although theoretically convenient, results in an artificially small value of the calculated sum rule at high $\vec{q}$. Such an incorrect parametrization was used in earlier evaluations[4,17] of the integrated longitudinal strength of ^{12}C and ^{40}Ca which indicated nearly total agreement with the IA using the free nucleon current. We have used a recent parametrization (fit 8.2 of Ref. 19) for the free nucleon form factors.

The mean value of y has been related by several authors to the presence of various scale breaking processes including exchange forces and two-body correlations in the nuclear ground state. In Fig. 10, $\overline{y_L}$ for both the free and REMA cases is shown. When the free nucleon mass is used $\overline{y_L}$ remains nearly constant at 0.08. The REMA

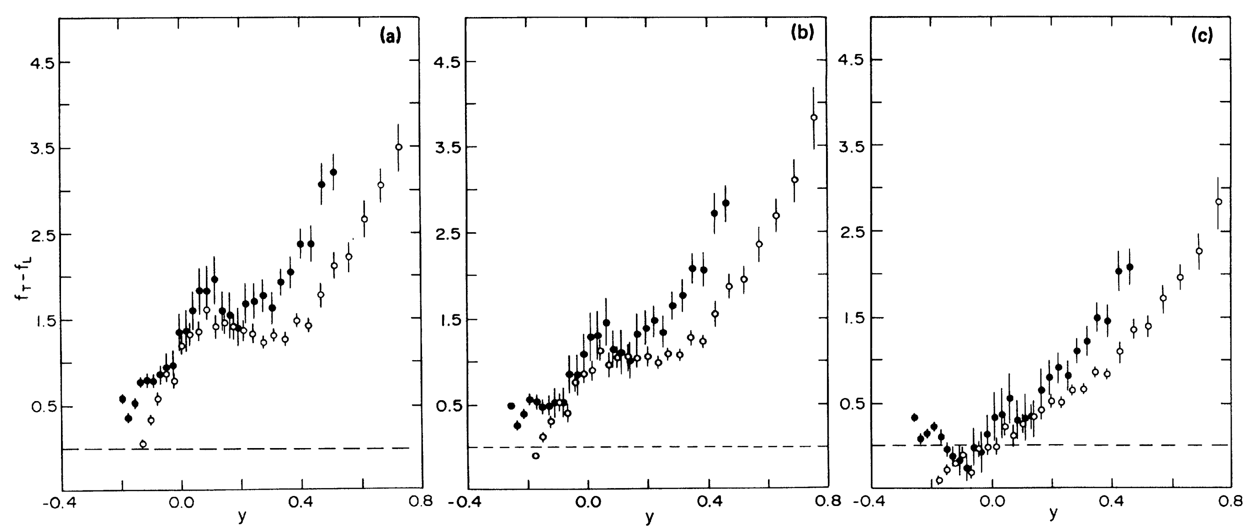

FIG. 8. (a)–(c). The difference f_T-f_L for the same conditions as in Fig. 6(a)–(c). The data difference at $\vec{q}=400$ MeV/c is shown by unfilled circles and at 500 MeV/c by filled circles.

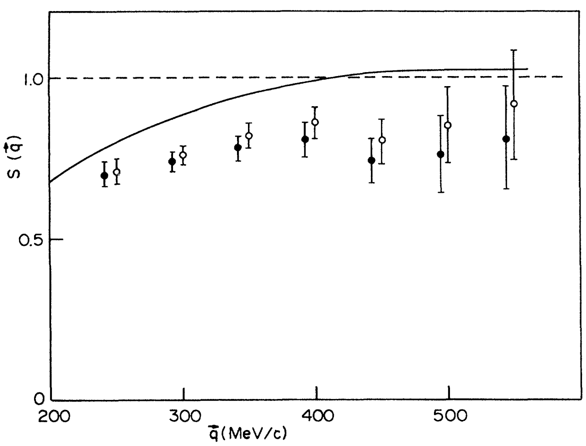

FIG. 9. The integral of the longitudinal scaling function versus $\vec{q}$. The results for $M^*=M$ (filled circles), for $M^*=0.82M$ with the modified current (unfilled circles) are shown. The IA prediction using the Hartree-Fock momentum distribution (Ref. 8) and assuming $M^*=0.82M$ is shown by the solid curve. The calculation is not particularly sensitive to the value of M^*.

causes $\overline{y_L}$ to decrease with increasing $\vec{q}$, paralleling the IA predictions and healing to zero for $\vec{q}$ greater than 500 MeV/c. This is further confirmation that the complications in the reaction process are being successfully accounted for by the REMA. Theoretical predictions for $\overline{y_L}$ from Tornow *et al.*[20] and Rosenfelder[21] are also shown in Fig. 10. Rosenfelder used Cohen-Kurath correlation functions with the two-body effective interaction from Gillet,[22] while Tornow *et al.* used the Reid soft-core interaction and correlated wave functions from Ref. 23. A qualitative estimate by Suzuki[24] based on ρ and π exchange is also shown. All these authors used the nonrelativistic form of the scaling variable. Tornow's calculation is in good agreement with the data treated with free kinematics except at the lowest momentum transfers

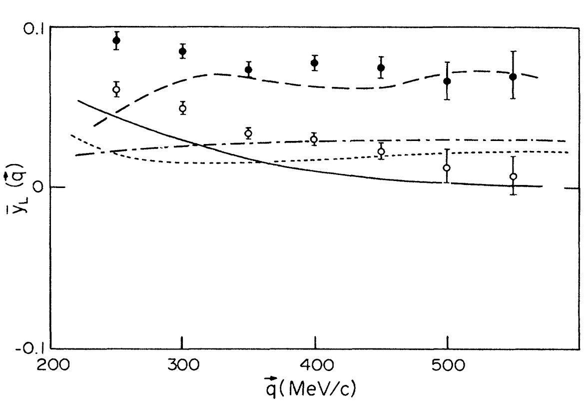

FIG. 10. Mean value of the relativistic scaling variable for the longitudinal response function versus $\vec{q}$. The data assuming $M^*=M$ are shown as filled circles and with $M^*=0.82M$ as unfilled circles. The curves show the nonrelativistic calculations of Rosenfelder (Ref. 21) (dotted), Suzuki (Ref. 24) (dash-dot), and Tornow *et al.* (Ref. 20) (dashed). The IA prediction for $M^*=0.82M$ is shown by the solid curve.

where it underestimates the mean shift. Rosenfelder's results fall below this same data treatment for all $\vec{q}$.

Nonrelativistically, a shift in $\bar{y}$ requires either the introduction of nonlocal interactions or of an energy-dependent potential.[6] These two alternate approaches are operationally quite similar since a nonlocal interaction can be replaced by an equivalent energy-dependent potential. However, in the first case the local medium properties dominate the modifications since the nonlocal effects are conceived as originating in short range meson exchanges, while in the latter the energy-dependent potential can also be associated with the final state interaction. For example, one can attempt to identify the energy shift directly with the mean separation energy of the knocked-out nucleon. The difficulty with this latter interpretation is that the time scale of the electron-nucleus interaction is much shorter than the evolution time of the nuclear system. The Dirac phenomenological approach presents a third alternative in that the static mean field provides most of the observed shift. These various approaches need to be reconciled with each other,[25] and a consistent framework for treating the various components of the reaction mechanism must be developed.

V. SUMMARY

Substantial deviations from the IA are observed in the separated ^{12}C data when analyzed in a relativistic scaling formulation using a free nucleon current. On the other hand, $\vec{q}$ independence of the individual response functions sets in at lower momentum transfers than might be expected. We have shown that the data can be reconciled with an effective one-body current in the relativistic effective mass approximation. This indicates that a Dirac phenomenological approach may form an appropriate framework for further theoretical developments including higher order processes in the reaction mechanism. The separated response functions demonstrate considerably greater sensitivity to the dynamics of the quasielastic process than has been hitherto apparent from the unseparated data. y scaling has been shown to provide a powerful tool for studying the systematics of the quasielastic region.

Data of the same quality and dynamical range as the ^{12}C results should soon be available from ^{56}Fe and isotopes of calcium.[26] These will provide insight into the A dependence of the media effects. Studies of the energetics of nucleon knockout by coincidence (e,e'p) measurements will be useful in discriminating between one-, two-, and many-body contributions to the reaction process. A longitudinal-transverse separation of the coincidence data will be especially helpful in determining if the one-nucleon current is indeed being modified by the medium as indicated by our analysis. A program of coincidence reaction measurements to be performed at the MIT-Bates Electron Linear Accelerator has been initiated to explore questions related to the microscopic character of the nuclear electromagnetic current.[27]

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APPENDIX

Assuming one can treat nucleons as Dirac particles, the nucleon electromagnetic current operator can be written as

$$\mathscr{J}_\mu(q)=F_1(q^2,M,q\cdot k)\gamma_\mu+\frac{\kappa}{2M}F_2(q^2,M,q\cdot k)\sigma_{\mu\nu}q^\nu\ , \tag{A1}$$

where F_1 and F_2 are dimensionless structure functions; M is the nucleon mass; κ is the anomalous magnetic moment; $\sigma_{\mu\nu}=i/2[\gamma_\mu,\gamma_\nu]$, γ_μ are the Dirac matrices; and k is the initial nucleon four-momentum. For $k'^2=k^2=M^2$, one can replace $q\cdot k$ by $-q^2/2$. Isospin indices have been suppressed. The quasifree scattering approximation consists of treating the nucleus as an incoherent ensemble of noninteracting fermions with a spectral distribution $\mathscr{S}(\vec{k},k_0)$ and treating the reaction as an impulsive transfer of four-momentum q in the one photon exchange approximation. If the nucleons are on the mass shell one obtains a scalar occupation density $n(\vec{k})$ defined by

$$\mathscr{S}(\vec{k},k_0)=2Mn(\vec{k})\delta(k_0^2-\vec{k}^2-M^2)\ ;$$
$$\int \mathscr{S}(k,k_0)d^4k=\int\left[\frac{M}{k_0}\right]n(\vec{k})d\vec{k}=A=Z+N\ . \tag{A2}$$

The invariant cross section for quasielastic scattering can then be written as

$$d\sigma=\frac{4\alpha^2}{q^4}\frac{(P_T^2)^{1/2}}{[(s\cdot P_T)^2-s^2P_T^2]^{1/2}}\frac{d\vec{p}}{p_0}\frac{d\vec{k}}{k_0}\frac{d\vec{k}\,'}{k_0'}\,||\mathscr{M}||(1-\mathscr{P})n(\vec{k})\delta^4(s+k-p-k')\ , \tag{A3}$$

where α is the fine structure constant, units are $\hbar=c=1$, and $\mathscr{P}(\vec{k},\vec{k}+\vec{q})$ denotes the Pauli-blocking operator which antisymmetrizes the fermion wave functions. Here one can also define a nonrelativistic occupation density $\tilde{n}(\vec{k})=M/k_0 n(\vec{k})$ which has the usual normalization encountered in the literature. The metric is such that a four-vector $a=(a_0,\vec{a})$ has a norm $a^2=a_0^2-\vec{a}^2$. In the above equation, s and p denote the initial and final electron four-momenta, k and k' denote the initial and final nucleon four-momenta, $q=s-p=(\omega,\vec{q})$ is the four-momentum transferred to the nuclear system, and P_T is the initial four-momentum of the nuclear system. The invariant matrix element of the transition is given by

$$||\mathscr{M}||=T_2(q^2,M^2)\left[(k\cdot s)(k\cdot p)-\frac{(s\cdot p)}{2}M^2\right]+M^2(s\cdot p)T_1(q^2,M^2)\ ; \tag{A4}$$

$$T_{1(\mathrm{p,n})}=|F_1+\kappa F_2|^2_{(\mathrm{p,n})}\ , \tag{A5a}$$

$$T_{2(\mathrm{p,n})}=\left[|F_1|^2-\frac{\kappa q^2}{4M^2}|F_2|^2\right]_{(\mathrm{p,n})}\ . \tag{A5b}$$

It is useful to define the mean nucleon Sachs form factors in the medium

$$\tilde{G}_E^2(q^2,M^2)=\frac{Z}{A}\left|F_{1\mathrm{p}}+\frac{\kappa_\mathrm{p}q^2}{4M^2}F_{2\mathrm{p}}\right|^2+\frac{N}{A}\left|F_{1\mathrm{n}}+\frac{\kappa_\mathrm{n}q^2}{4M^2}F_{2\mathrm{n}}\right|^2 \tag{A6a}$$

and

$$\tilde{G}_M^2(q^2,M^2)=\frac{Z}{A}|F_{1\mathrm{p}}+\kappa_\mathrm{p}F_{2\mathrm{p}}|^2+\frac{N}{A}|F_{1\mathrm{n}}+\kappa_\mathrm{n}F_{2\mathrm{n}}|^2\ . \tag{A6b}$$

For a self-conjugate ($Z=N$) nucleus like ^{12}C, one can assume as a good approximation that the proton and neutron occupation densities are the same. If one further assumes for simplicity that the occupation densities are spherically symmetric, then the inclusive electron scattering cross section can be reduced to

$$\frac{d\sigma}{d\Omega_e dp_0}=\sigma_{\mathrm{MOTT}}\frac{\pi M}{|\vec{q}|}\int_{k_{\mathrm{min}}^2}^{\infty}d\vec{k}^2(1-\mathscr{P})n(\vec{k})\left[\frac{M}{k_0}\right]\left\{\left[\frac{-q^2}{|\vec{q}|^2}\tilde{T}-\frac{q^2}{2M^2}\tilde{G}_M^2\tan^2\frac{\theta}{2}\right]\right.$$
$$\left.+\frac{k_\perp^2}{M^2}\left[\frac{\tilde{T}}{[1-q^2/(4M^2)]}\left[\frac{-3q^2}{2|\vec{q}|^2}+\tan^2\frac{\theta}{2}\right]\right]\right\}, \tag{A7a}$$

where

$$\tilde{T}=\left[\tilde{G}_E^2-\frac{q^2}{4M^2}\tilde{G}_M^2\right], \tag{A7b}$$

$$\sigma_{\text{MOTT}}=\frac{\alpha^2}{4s_0{}^2}\frac{\cos^2\theta/2}{\sin^4\theta/2}\ ;\ \cos\theta=(\hat{p}\cdot\hat{s}), \tag{A7c}$$

$$\epsilon_{\min}=[M^2+k_{\min}^2]^{1/2}=M\nu-\frac{\omega}{2}, \tag{A7d}$$

$$\frac{k_\perp^2}{M^2}=\left[\frac{(\epsilon+\omega/2)^2}{(\epsilon_{\min}+\omega/2)^2}-1\right]\left[1-\frac{q^2}{4M^2}\right]=\frac{|\vec{k}\times\hat{q}|^2}{M^2}, \tag{A7e}$$

$$\nu=\left[\frac{|\vec{q}|^2}{-q^2}\left[1-\frac{q^2}{4M^2}\right]\right]^{1/2}, \tag{A7f}$$

and

$$k_{\min}=My=-\frac{|\vec{q}|}{2}\left[1-\frac{2\omega}{|\vec{q}|^2}M\nu\right]. \tag{A7g}$$

If one ignores a small term proportional to $k_\perp^2/M^2$, where $k_\perp$ is the component of $\vec{k}$ perpendicular to the momentum transfer direction $\hat{q}$, and if Pauli-blocking effects are negligible, then the reaction scales as a function of the single dimensionless variable y, i.e.,

$$\frac{d\sigma}{d\Omega_e dp_0}=A\frac{\sigma_{\text{ep}}}{|\vec{q}|}f(y), \tag{A8a}$$

where

$$\sigma_{\text{ep}}=\sigma_{\text{MOTT}}\left[\frac{-q^2}{|\vec{q}|^2}\tilde{T}-\frac{q^2}{2M^2}\tilde{G}_M^2\tan^2\frac{\theta}{2}\right]. \tag{A8b}$$

Equation (A8) can be interpreted as an operational definition of $f(y)$. The cross section in terms of longitudinal and transverse photon components in the laboratory frame is

$$\frac{d\sigma}{d\Omega_e dp_0}=\sigma_{\text{MOTT}}\left[\frac{q^4}{|\vec{q}|^4}R_L+\left[\tan^2\frac{\theta}{2}-\frac{q^2}{2|\vec{q}|^2}\right]R_T\right]. \tag{A9}$$

Assuming quasifree scattering dominates the reaction mechanism at large $\vec{q}$ the integrated structure functions should satisfy the sum rule

$$\int f(y)dy=1+\mathcal{O}\left[\frac{k_\perp^2}{M^2}\right], \tag{A10}$$

which, however, is invalid for the transverse scattering due to the large contributions from the $\Delta(3,3)$ isobaric resonance channel. The cross section is maximum at $y=0$ which corresponds to $\omega=-q^2/2M$ and vanishes at the photon point $q^2=0$ as $y\rightarrow+\infty$.

The above formulation neglects energy-momentum conservation on the total nuclear system and thus cannot be expected to reproduce the details of the structure at low excitation energies where energetic constraints on the final nuclear system play a major role. Pauli blocking effects which are most important for small excitation energies (≤ 30 MeV) can be neglected at high energy transfers. A simple approximation for Pauli blocking contributions can be derived by assuming that the scattering is linear in occupation density. Since an impulsive momentum boost relates the momentum distribution at $\vec{k}$ and $\vec{k}+\vec{q}$, the Pauli principle can be satisfied by antisymmetrizing the density at $\vec{k}$ and $\vec{k}+\vec{q}$

$$(1-\mathcal{P})n(\vec{k})=\begin{cases}n(\vec{k})-n(\vec{k}+\vec{q}) & \omega\geq 0\\ 0 & \omega<0\end{cases}, \tag{A11}$$

where

$$|\vec{k}+\vec{q}|=[(\epsilon+\omega)^2-M^2]^{1/2},\qquad \epsilon=(k^2+M^2)^{1/2}. \tag{A12}$$

The restriction to $\omega\geq 0$ reflects the fact that the initial nuclear system is in its ground state. Nuclear recoil has been ignored. The above prescription is equivalent to assuming that the nucleus can be treated as a superposition of Fermi gas momentum distributions. A similar result is obtained by Rosenfelder[10] based on the analytic properties of the response functions.

Although the above results are well known in general,[10,28,29] the above formulation is new in the sense that the description in terms of the Sachs form factors makes the appropriate definition of the relativistic scaling functions transparent. We have also corrected a persistent though small error in the normalization of the occupation density.

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