Impulse approximation versus elementary particle method: π-^{3}He-^{3}H coupling constant

L. Klieb and H. P. C. Rood
Institute for Theoretical Physics, State University of Groningen, Groningen, The Netherlands
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The π-^{3}He-^{3}H form factor $G_\pi(q^2)$ is calculated from a connection between the elementary particle method and the impulse approximation using a realistic wave function with an asymptotic tail. An analytic continuation is used for the timelike region. The π-^{3}He-^{3}H coupling constant $G_\pi(-m_\pi^2)$ is found to be ≈ -1.7, in absolute value higher than the elementary nucleon value $g_\pi = 1.41$ because of the nuclear extension. The form factor is in very good agreement with results of dispersion relations. A determination which led to $G_\pi \approx -1.0$ is shown to be wrong.

[NUCLEAR STRUCTURE ^{3}He; calculated π-^{3}He-^{3}H coupling constant.]

I. INTRODUCTION

This is the first of a series of two papers about a comparison of the impulse approximation (IA) and the elementary particle method (EPM) for the calculation of certain interactions in nuclei. Our investigation was actuated by the many recent contradictory determinations of the π-^{3}He-^{3}H coupling constant $G_\pi \equiv G_\pi(-m_\pi^2)$. [By definition

$$G_\pi \equiv G_{\pi\text{-}^3\text{He-}^3\text{H}}(-m_\pi^2) = \sqrt{2}G_{\pi\text{-}^3\text{He-}^3\text{He}}(-m_\pi^2) .$$

In Refs. 1–5 values for $f^2 = G_\pi^2/8\pi$ are given.] This is the π-^{3}He-^{3}H form factor $G_\pi(q^2)$ as defined within the elementary particle method with the pion on the mass shell.

Several authors have determined a value of G_π which in absolute value is much lower than the elementary pion-nucleon coupling constant

$$g_\pi \equiv g_{\pi\text{pn}}(-m_\pi^2) = 1.41 ,$$

viz.,

$$G_\pi \approx -1.0 . \tag{1.1}$$

This value has been obtained from analyses of pion photoproduction,[1] the charge exchange reaction[2,3] p^{3}H$\to$n^{3}He, elastic scattering[4] ^{3}He^{3}H$\to$^{3}He^{3}H, and partial conservation of the axial vector current (PCAC) with analytic continuation.[5]

On the other hand, there are determinations which lead to values higher in absolute value than g_π,

$$-1.8 \lesssim G_\pi \lesssim -1.5 . \tag{1.2}$$

These values result from application of dispersion relations[6,7] and from extraction from the muon capture rate and PCAC.[8,9]

In this paper we propose a way to calculate G_π, defined in the elementary particle method, with the help of the impulse approximation. In the following paper[10] we consider two-step processes in the impulse approximation and in the elementary particle method and compare them with the value of G_π found here.

The plan of this paper is as follows. First we define in Sec. II our notation. Then in Sec. III we summarize the connection between form factors in the elementary particle method and the impulse approximation as devised by Delorme. In Sec. IV we show how these form factors can be continued into the timelike region and discuss the influence of the asymptotic tail of the wave function. In Sec. V numerical calculations are given, leading to a high value of $G_\pi \approx -1.7$. After a short discussion of possible exchange effects in Sec. VI, a comparison is made with dispersion calculations in Sec. VII. Good agreement is found. In Sec. VIII a recent determination of G_π (Ref. 5) by PCAC + analytic extrapolation of $G_A(q^2)$, which led to a low value of G_π, is shown to be incorrect. In Sec. IX a summary and conclusion are given. Part of this work is based on a doctoral dissertation by one of us.[11]

II. PARTIAL CONSERVATION OF THE AXIAL VECTOR CURRENT

In this section we summarize the relations for the PCAC and nucleon and nuclear parameters. In the elementary particle method the equations for the p-n and ^{3}He-^{3}H doublet are analogous, because both have spin and isospin $\frac{1}{2}$ and even parity. For the $A=3$ form factors capitals will be used and for the nucleon small letters will be used. In the following $|i\rangle = |^3\text{He}\rangle$ ($|\text{p}\rangle$ in the nucleon case) and $|f\rangle = |^3\text{H}\rangle$ ($|\text{n}\rangle$).

The weak vector and axial vector currents are written as

$$\langle f \mid V_\lambda(0) \mid i\rangle = \bar{u}_f[G_V(q^2)\gamma_\lambda + G_M(q^2) i\sigma_{\lambda\mu}q_\mu/2M]u_i , \tag{2.1}$$

$$\langle f \mid A_\lambda(0) \mid i\rangle = \bar{u}_f[G_A(q^2)\gamma_5\gamma_\lambda + G_P(q^2) i\gamma_5 q_\lambda/m_\mu]u_i . \tag{2.2}$$

Here M is the nuclear mass in the EPM (and the nucleon mass in the nucleon case). These conventions conform for the nucleon to Ref. 12 with exception of the sign of the momentum transfer which is defined here as $q = p_f - p_i$.

The PCAC hypothesis states

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$$D(q^2) \equiv -G_A(q^2) + \frac{q^2}{2Mm_\mu} G_P(q^2) = \frac{a_\pi G_\pi(q^2) m_\pi^2}{q^2+m_\pi^2} = \frac{a_\pi G_\pi m_\pi^2}{q^2+m_\pi^2} + I(q^2) \ . \quad (2.3)$$

Here $a_\pi = 0.94$ is the pion decay constant.[13] The pion-nucleus form factor is defined by

$$\langle f \mid j_\pi \mid i \rangle = i G_\pi(q^2) \bar{u}_f \gamma_5 u_i \ . \quad (2.4)$$

The pion-nucleus coupling constant is $G_\pi \equiv G_\pi(-m_\pi^2)$. The function $I(q^2)$ is defined by the last equality in (2.3).

With Primakoff[8] we introduce $\epsilon(q^2)$ by

$$G_P(q^2) = \frac{2Mm_\mu}{q^2+m_\pi^2} G_A(q^2)[1+\epsilon(q^2)] \quad (2.5)$$

so that

$$\epsilon(q^2) = \frac{m_\pi^2}{q^2} \left[1 - \frac{G_\pi(q^2)/G_\pi(0)}{G_A(q^2)/G_A(0)} \right] \quad (2.6)$$

and

$$G_\pi = -\frac{1}{a_\pi} G_A(-m_\pi^2)[1+\epsilon(-m_\pi^2)] \ . \quad (2.7)$$

A nonzero value of $\epsilon(q^2)$ is seen to have its origin in a difference between the momentum dependence of the pion-nucleus and the axial form factors.

The nucleon form factors are rather well known. Because the momentum transfer will be restricted almost everywhere in this paper to $-m_\pi^2 < q^2 < m_\mu^2$, a good approximation is given by a linear fit,

$$g_i(q^2) = g_i(0)(1 - r_i^2 q^2/6) \ , \quad i = V, A, M, \pi \ . \quad (2.8)$$

We have taken $g_V(0) = 1.0$, $g_M(0) = \mu_p - \mu_n = 3.7$, $g_A(0) = -1.260 \pm 0.012$, and $r_V = 0.74 \pm 0.02$ fm, $r_M = 0.83 \pm 0.08$ fm, and $r_A = 0.72 \pm 0.07$ fm (cf. Ref. 13). Further the pion-nucleon coupling constant is[8]

$$g_\pi = g_\pi(-m_\pi^2) = 1.41 \pm 0.02 \ .$$

Hence,

$$g_{\pi pp}^2/4\pi = g_{\pi pn}^2/8\pi = 0.079 \ .$$

From (2.3) it follows that

$$g_\pi(0) = -g_A(0)/a_\pi = 1.33 \pm 0.01$$

so that from (2.8), used at $q^2 = 0$ and $q^2 = -m_\pi^2$, one finds $r_\pi = 0.82 \pm 0.06$ fm. This linear representation for $g_\pi(q^2)$ will be used throughout except in Sec. VII. Also we find

$$\epsilon(q^2) \approx \epsilon(0) = m_\pi^2 (r_\pi^2 - r_A^2)/6 = 0.014 \pm 0.020$$

for $-m_\pi^2 < q^2 < m_\mu^2$, nearly independent of q^2 in the region of q^2 considered. The smallness of ϵ is due to the fact that the nucleon is localized with respect to the scale length m_π^{-1} [cf. Eq. (2.6)].

In the elementary particle method the value of $G_\pi(0)$ which follows from (2.3) is

$$G_\pi(0) = -1.30 \pm 0.01 \ , \quad (2.9)$$

because $G_A(0) = 1.22 \pm 0.01$ can be determined rather directly from ^{3}H beta decay.[8] What concerns us in this paper is the value of $G_\pi \equiv G_\pi(-m_\pi^2)$.

III. A CONNECTION BETWEEN THE ELEMENTARY PARTICLE METHOD AND THE IMPULSE APPROXIMATION

There exist two ways to describe weak, electromagnetic, and strong interactions in nuclei. One is the impulse approximation. Here a one-body operator is applied in the form of an effective Hamiltonian, which reproduces approximately the relativistic nucleon amplitude. Then this effective Hamiltonian is sandwiched between nuclear wave functions, to calculate observables. If wanted, exchange corrections can be added to this basic process.

On the other hand in the elementary particle method the nucleus is not described as an aggregate of nucleons. Only such information like spin, isospin, and parity is used to construct a relativistic amplitude for the whole nucleus.

Delorme[14] devised a connection between the impulse approximation and the elementary particle method using certain multipole developments for the nuclear currents. In this formalism relativistic form factors $G_V(q^2)$, $G_M(q^2)$, $G_A(q^2)$, $G_P(q^2)$, and $G_\pi(q^2)$ for the ^{3}He$\rightarrow$^{3}H transition are expressed in single nucleon form factors and in reduced matrix elements depending on the nuclear wave functions.

The relations are manageable only if one works in the Breit frame, i.e., the frame where $\vec{p}_{He} + \vec{p}_H = 0$, with $\vec{p}_{He}$ ($\vec{p}_H$) the total initial (final) momentum, and includes terms only to zeroth order in $\vec{q}/2M$, where $\vec{q} = \vec{p}_{He} - \vec{p}_H$ is the momentum transfer. Generally, the errors involved in using form factors in other reference frames are small. The nucleon mass will always be approximated by $m = M/3$ when using the Delorme formalism. All these approximations are expected to be accurate within a few percent.

We shall call the approximation for which these "IA" expressions for "EPM" form factors are used, "EPIA."

The connection between form factors and matrix elements in this formalism is, using the conventions of Sec. II for the form factors,

$$G_V(q^2) = \frac{1}{\sqrt{2}} g_V(q^2)[1]^0 \ , \quad (3.1)$$

$$G_M(q^2) = \tfrac{1}{2}\sqrt{6}[g_V(q^2) + g_M(q^2)][\vec{\sigma}]^- - \frac{1}{\sqrt{2}} g_V(q^2)[1]^0 - \frac{3}{2\,|\vec{q}\,|} g_V(q^2)[i\vec{P}]^{1,1} \ , \quad (3.2)$$

$$G_A(q^2) = \frac{1}{\sqrt{6}} g_A(q^2)[\vec{\sigma}]^- \ , \quad (3.3)$$

$$G_P(q^2) = \tfrac{1}{2}\sqrt{6} g_P(q^2)[\vec{\sigma}]^+ - \sqrt{3}\frac{Mm_\mu}{q^2} g_A(q^2)[\vec{\sigma}]^{2,1} \ , \quad (3.4)$$

$$G_\pi(q^2) = \frac{1}{\sqrt{6}} g_\pi(q^2)[\vec{\sigma}]^+ \ , \quad (3.5)$$

with

$$[\vec{\sigma}]^{+}=[\vec{\sigma}]^{0,1}+\sqrt{2}[\vec{\sigma}]^{2,1} ,$$
$$[\vec{\sigma}]^{-}=[\vec{\sigma}]^{0,1}-\frac{1}{\sqrt{2}}[\vec{\sigma}]^{2,1} . \quad (3.6)$$

Here

$$[O]^{J}=\langle {}^3\mathrm{H}|| \sum_{i=1}^{3} \tau_{-}(i)\sqrt{4\pi}j_J(\,|\vec{q}\,|\,r(i))Y_J(\hat{r}(i))O(i)||{}^3\mathrm{He}\rangle \quad (3.7)$$

and

$$[O]^{L,J}=\langle {}^3\mathrm{H}|| \sum_{i=1}^{3} \tau_{-}(i)\sqrt{4\pi}j_J(\,|\vec{q}\,|\,r(i))\vec{Y}_{L1J}(\hat{r}(i))\cdot\vec{O}(i)||{}^3\mathrm{He}\rangle \quad (3.8)$$

are reduced matrix elements (Ref. 15, p. 75). Before calculating the reduced matrix element a momentum conserving delta function must be split off in the usual manner (cf. Ref. 11, p. 70). The charge lowering operator is $\tau_{-}=(\tau_1-i\tau_2)/2$.

The nucleon form factors used by Delorme in Ref. 14 are related to ours as $g_V^D(q^2)=g_V(q^2)$ and

$$g_M^D(q^2)=g_M(q^2)+g_V(q^2) ,$$

and the nuclear form factors as $F_V(q^2)=G_V(q^2)/2M$ and

$$F_M(q^2)=[G_V(q^2)+G_M(q^2)]/2M .$$

In EPIA the PCAC relation for the nucleus [Eq. (2.3)] is automatically satisfied if PCAC for the nucleon holds, and the approximation $M/m=3$ is used.

IV. EXTRAPOLATION TO THE TIMELIKE REGION

The formalism of Sec. III to calculate relativistic form factors by expressing them in reduced matrix elements of nuclear wave functions (EPIA) allows also for an analytic continuation into the timelike region of the momentum transfer q^2. In the Breit frame $q^2=\vec{q}^{\,2}$, as $q_0=0$. An extrapolation to $q^2<0$ can therefore be accomplished by replacing in the radial integrals in Eqs. (3.7) and (3.8) the absolute value of the momentum transfer $|\vec{q}\,|$ by $i\,|\vec{q}\,|$.

It is well known from the theory of dispersion relations that the relativistic nuclear form factors develop singularities in the timelike region caused by anomalous thresholds. In this case the two lowest thresholds come from ^{3}He$\rightarrow$dp and ^{3}He$\rightarrow$d*p [d* is the singlet (i.e., $J=0,T=1$)] breakup (see Sec. VI). Therefore one may expect that for momentum transfers near the anomalous thresholds form factors cannot be reliably calculated when using a wave function which is different from zero in only a finite region of space. The form factors of such a wave function have no singularity. Thus in impulse approximation singularities come from the long range asymptotic tail of the wave function. Form factors will be calculated both with and without inclusion of such a tail in the wave function.

The form of the asymptotic wave function used is [cf. Ref. 16, Eq. (19)]

$$\psi_{\mathrm{asympt}}(\vec{y},\vec{p})=C\sqrt{\alpha}3^{1/4}[\exp(-\alpha\sqrt{3}y/2)/y] \times Y_{00}(\hat{y})\phi_S^d(p)Y_{00}(\hat{p})\chi\eta/\sqrt{2} . \quad (4.1)$$

Here

$$\vec{y}=[\vec{r}(2)+\vec{r}(3)-2\vec{r}(1)]/\sqrt{3} ,$$
$$y=|\vec{y}\,| ,$$
$$\vec{p}=[\vec{k}(2)-\vec{k}(3)]/2 ,$$

with $\vec{k}(i)$ and $\vec{r}(i)$ being the momentum and position of particle i. Particle 1 is the spectator. The quantity ϕ_S^d is the S state radial wave function of particles 2 and 3 in momentum space, normalized as

$$\int_0^\infty |\phi_S^d(p)|^2p^2dp=1 ,$$

and χ and η the spin and isospin wave function belonging to it. Further,

$$\alpha=[4m_{\mathrm{nucleon}}B/3]^{1/2} ,$$

with B the binding energy of particle 1 in the nucleus, and C the asymptotic normalization of the S component of the wave function. This form applies only to these components in the wave function for which the orbital angular momentum L of the 2–3 pair and l of the spectator particle are both zero. (Therefore also total angular momentum $\mathscr{L}=0$.) This suffices as the two components with $\mathscr{L}=L=l=0$ and $S=0,T=1$, respectively, $S=1,T=0$, make up for roughly 90% of the total probability.

The singularity in the form factors resulting from these asymptotic tails can be found from the integral (for $k^2<0$)

$$\int |\psi_{\mathrm{asympt}}(\vec{y},\vec{p})|^2e^{i\vec{k}\cdot\vec{y}/\sqrt{3}}y^2dy\,p^2dp\,d\hat{y}\,d\hat{p}=C^2\alpha\sqrt{3}\tfrac{1}{2}\int_0^\infty e^{-\alpha\sqrt{3}y}j_0(i\,|\vec{k}\,|y/\sqrt{3})dy$$
$$=C^2\frac{3\alpha}{2|\vec{k}\,|}\tfrac{1}{2}\ln\left[\frac{\alpha\sqrt{3}+|\vec{k}\,|/\sqrt{3}}{\alpha\sqrt{3}-|\vec{k}\,|/\sqrt{3}}\right] , \quad (4.2)$$

which has the form of the singularity in dispersion theory. (cf. the expression given in Ref. 6 for $k^2>0$.) The cut starts at

$$k^2=-9\alpha^2=-12m_{\mathrm{nucleon}}B$$

just as found in a dispersion relation for the cut originating from an anomalous threshold. This is in accordance with the work of Blankenbecler and Cook.[17]

The numerical calculation is based on the $A=3$ wave function calculated by Brandenburg *et al.*,[18] as used in Ref. 19. As the potential in this calculation the Reid soft core[20] potential is used, constrained to the $^1S_0$ and $^3S_1$-$^3D_1$ channels. The wave function has been Fourier-Bessel tranformed for the spectator particle to coordinate space.

The component with spin $S=0, T=1$ for the 2—3 pair, corresponding to the "singlet deuteron" in dispersion relation (Sec. VI), is about as important in the wave function as the $S=1, T=0$ "normal deuteron" state. Therefore we have included also the asymptotic tail of this state, assuming as in Ref. 21 that its strength C is the same as for the $S=1, T=0$ state. The constant C has been calculated for the wave function employed here in Ref. 21. The experimental nuclear binding energy $E_B=8.5$ MeV is not well reproduced by this wave function ($E_B^{\text{calc}}=6.7$ MeV). This has consequences for the value of $B=E_B-E_{\text{d}}$, where $E_{\text{d}}=2.225$ MeV is the deuteron binding energy (well reproduced by the Reid soft core potential) or the singlet deuteron binding energy ($E_{\text{d}*}=-0.07$ MeV).

Because of the antisymmetry of the wave function under particle exchange, calculations may be made for the spectator particle only and an explicit form for $\phi_S^d(p)$ is not needed.

V. NUMERICAL RESULTS

The calculated nuclear form factors may be fitted for $-m_\pi^2<q^2<m_\mu^2$ to within 1% accuracy by double pole expressions

$$G_i(q^2)=G_i(0)/(1+\beta_i q^2)^2\ ,\quad i=V,A,M,\pi\ . \tag{5.1}$$

The small $[i\vec{\text{P}}]^{1,1}$ terms [with $\vec{\text{P}}(i)=\vec{\text{p}}_f(i)+\vec{\text{p}}_i(i)$] have been neglected. The results are the following (the asymptotic tail is not included):

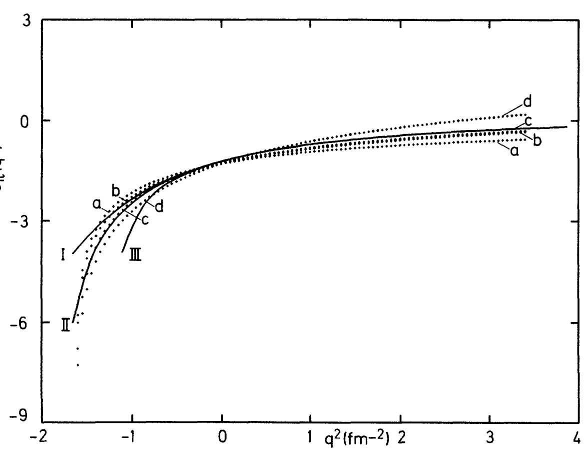

FIG. 1. Results for $G_\pi(q^2)$ for the cases (see text) (a)—(d) dispersion relations, (a) $g_\pi(s)$ constant, $s_f=-(3m_\pi)^2$; (b) $g_\pi(s)$ constant, $s_f=-(4m_\pi)^2$; (c) $g_\pi(s)$ double pole, $s_f=-(3m_\pi)^2$; (d) $g_\pi(s)$ double pole, $s_f=-(4m_\pi)^2$; I-III EPIA: I: EPIA without asymptotic tail; II: asymptotic tail with experimental binding energy; III: asymptotic tail with Reid soft core binding energy.

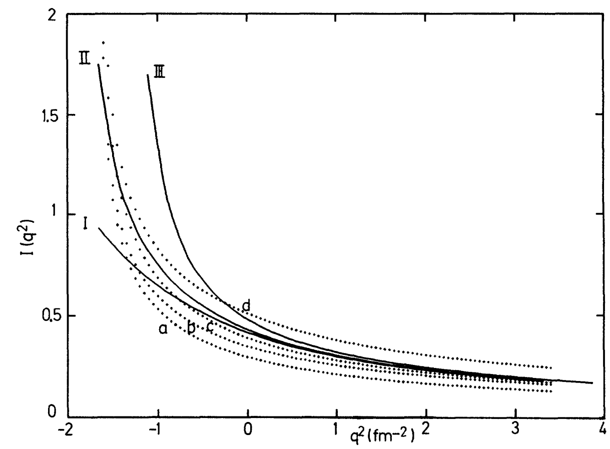

FIG. 2. Results for $I(q^2)$ [Eq. (2.3)]. The cases are the same as in Fig. 1.

$$G_V(0)=1.0\ ,\quad \beta_V=0.41\ \text{fm}^2\ , \tag{5.2}$$

$$G_M(0)=-14.0\ ,\quad \beta_M=0.30\ \text{fm}^2\ , \tag{5.3}$$

$$G_A(0)=1.16\ ,\quad \beta_A=0.36\ \text{fm}^2\ , \tag{5.4}$$

$$G_\pi(0)=-1.23\ ,\quad \beta_\pi=0.30\ \text{fm}^2\ . \tag{5.5}$$

These double pole fits have been made to values at $q^2=0$ and $q^2=m_\mu^2$.

A reasonable fit for the pseudoscalar form factor is obtained with Eq. (2.5) as

$$\epsilon\approx-0.05\ ,\quad -m_\pi^2<q^2<m_\mu^2\ . \tag{5.6}$$

In Figs. 1 and 2 the results for $G_\pi(q^2)$ and $I(q^2)$ (2.3) are shown, both with and without the asymptotic tail in the wave function included.

The following points merit attention:

(i) The value of the π-^{3}He-^{3}H coupling constant is with the extrapolation of Sec. IV;

$$G_\pi=-1.68 \tag{5.7}$$

when the asymptotic tail is *not* included. It is

$$G_\pi=-1.69 \tag{5.8}$$

if the experimental binding energy is used in the asymptotic tail. If one uses the value calculated from the Reid soft core potential one finds $G_\pi=-1.74$. The double pole form (5.1), (5.5) leads to $G_\pi=-1.70$. All these values are essentially the same.

It is important to note that $|G_\pi(0)|$ is lower than $g_\pi(0)$ because of the presence of the D state in the wave function. In fact,

$$G_\pi(0)=-g_\pi(0)[p(S)-\tfrac{1}{3}p(D)+\tfrac{1}{3}p(S')]\ ,$$

where $p(S)$, $p(D)$, and $p(S')$ are the probability of the (leading) S, D, and S' states. For the wave function employed here $p(D)\approx 8.6\%$ and $p(S')\approx 1.7\%$. With only a symmetric S state present $G_\pi(0)$ would have the value $-g_\pi(0)$.

In contrast, $|G_\pi(-m_\pi^2)|$ is higher than $g_\pi(-m_\pi^2)$ because of the form factor of the nucleus, coming from its spatial extension, which multiplies $g_\pi(q^2)$ in Eq. (3.5).

(ii) The asymptotic tail of the wave function is, as seen from Figs. 1 and 2, negligible in the region of the momentum transfer considered $(-m_\pi^2 < q^2 < m_\mu^2)$. This happens because the anomalous threshold is far enough. This justifies our preliminary calculation in Ref. 22, where the asymptotic tail is not included.

(iii) The function $\epsilon(q^2)$ varies very little with q^2 $[-0.070 < \epsilon(q^2) < -0.056$ for $-m_\pi^2 < q^2 < m_\mu^2]$. It has been suggested that in EPIA ϵ would be zero and that only exchange could bring about an appreciable deviation.[8] The argument behind this is that the nuclear part of G_P and G_A would be the same and that the EPM form factors are proportional to the corresponding nucleon form factors. These assumptions do not hold exactly [Eqs. (2.5), (3.3), and (3.4)].

(iv) Usually in EPM, one takes the same momentum dependence for $G_M(q^2)$ and $G_A(q^2)$ and motivates this by the impulse approximation. The correspondence of the EPM form factors to the impulse approximation as formulated in EPIA shows that this is not entirely true, because of the extra convective terms in $G_M(q^2)$ (3.2) and the different single nucleon form factors (2.8) when compared with $G_A(q^2)$ (3.3). More serious objections against this assumption will be given in Sec. VI.

(v) The result for the partial muon capture rate $\mu^- + {}^3\mathrm{He} \rightarrow {}^3\mathrm{H} + \nu_\mu$, is, using EPIA,

$$\Lambda_{\mu c} = 1258 \text{ s}^{-1} \text{ (EPIA)} . \tag{5.9}$$

This is 0.7% less than the result calculated in "standard" impulse approximation with the same wave function: $\Lambda_{\mu c} = 1268$ s^{-1} (IA). [The difference with the IA value found in Ref. 23 comes from a slightly different definition of g_P and a slightly different value of $g_A(0)$.] This small difference between IA and EPIA is due to the truncations in the formalism of Delorme as discussed in Sec. III.

VI. EXCHANGE EFFECTS

In this section the information available from experiment on the EPM form factors is summarized. Comparison with the results of the calculation of the EPIA form factors gives an indication of the magnitude of exchange effects.

From electron scattering[24,25] via conserved vector current (CVC) it is found that

$$G_V(0) = 1.0 , \quad \beta_V = 0.35 \pm 0.04 \text{ fm}^2 , \tag{6.1}$$

$$G_M(0) = \mu^{\text{He}} - \mu^{\text{H}} - 1 = -16.3 , \quad \beta_M = 0.25 \pm 0.04 \text{ fm}^2 , \tag{6.2}$$

using the double pole parametrization (5.1). (However, there is no "deep reason" for such a double pole parametrization. A single pole fit, for instance, does not appreciably alter our conclusions.) Further, from the β decay of ^{3}H, it follows[8] that

$$G_A(0) = 1.22 \pm 0.01 . \tag{6.3}$$

These are the only EPM weak form factors for the $A = 3$ doublet that may be considered to be firmly established. Comparing these data with the EPIA values (5.2)–(5.4) one concludes the following:

(i) The renormalization at $q^2 = 0$ is $+16\%$ for $G_M(0)$ and $+5\%$ for $G_A(0)$. This reflects the different influence of exchange on vector and axial vector current (supported by theoretical calculations).

(ii) There is perhaps an exchange effect on the q^2 behavior of the vector form factor, although there is a fairly large uncertainty in the electromagnetic form factors for low q^2 (2–3 % at $q^2 \approx m_\mu^2$). Another possibility is that perhaps even a "realistic" wave function is not realistic enough.

Because of (i) the usual assumption of a similar behavior of $G_A(q^2)$ and $G_M(q^2)$ for $-m_\pi^2 < q^2 < m_\mu^2$ is questionable: no reliable calculation on the q^2 dependence of the axial vector form factors is available (cf. Sec. VIII). So there exists an appreciable uncertainty in the predicted values of $G_A(q^2)$ for $q^2 \neq 0$. At the momentum transfer in ordinary muon capture we consider

$$G_A(0.96 m_\mu^2) = 1.07 \pm 0.05 \tag{6.4}$$

a reasonable guess, assuming $\beta_A = 0.25 \pm 0.10$ fm^2. Figure 3 shows the dependence of $G_P(0.96 m_\mu^2)$ on $G_A(0.96 m_\mu^2)$ imposed by the experimental capture rate. We take this as

$$\Lambda_{\mu c} = 1529 \pm 37 \text{ s}^{-1} \tag{6.5}$$

in accordance with Ref. 26. The uncertainty in $G_A(0.96 m_\mu^2)$ in addition to the experimental uncertainty leads to values of $G_P(0.96 m_\mu^2)$ and $\epsilon(0.96 m_\mu^2)$ with a large error,

$$G_P(0.96 m_\mu^2) = 22^{+24}_{-15} ; \quad \epsilon(0.96 m_\mu^2) = 0.06^{+1.06}_{-0.72} . \tag{6.6}$$

This error is so large that any speculation on an extrapolation to $q^2 = -m_\pi^2$, which is needed to determine G_π, is useless. The conclusion can only be that with the present uncertainty about the value of $G_A(0.96 m_\mu^2)$ the muon capture rate is not suitable for constraining the pseudoscalar part of the weak interaction in ^{3}He. True, Primakoff[9] obtains a better determined value from the muon capture

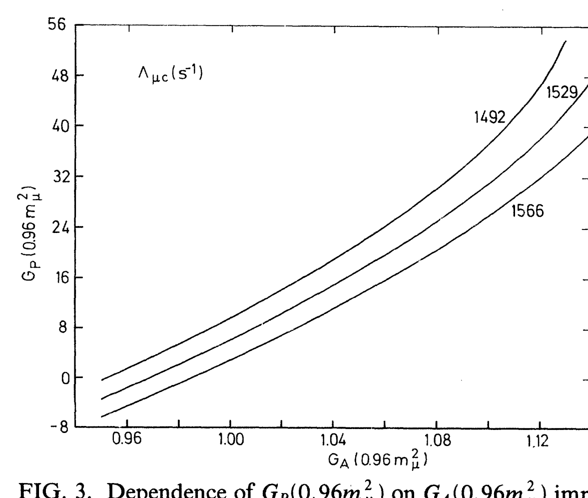

FIG. 3. Dependence of $G_P(0.96 m_\mu^2)$ on $G_A(0.96 m_\mu^2)$ imposed by muon capture rates of 1529 s^{-1}, 1566 s^{-1}, and 1492 s^{-1}.

rate [$\epsilon(m_\mu^2)=0.05\pm0.25$], but only because in our opinion he is too optimistic in assessing an error in $G_A(m_\mu^2)$.

VII. DISPERSION RELATIONS

As an alternative to the approach of Secs. III—V, one may employ dispersion relations. Arguments against the use of the forward dispersion relations for pion-nucleus scattering (successful in the nucleon case) have been given by Dumbrajs.[4] Another method is to use a dispersion relation for the divergence of the axial current $D(q^2)$ (2.3) as first done by Jarlskog and Yndurain.[6] If the pion pole is separated the following expression for $I(q^2)$ results:

$$I(q^2)=\frac{1}{\pi}\int_{s_f}^{s_0}\frac{\mathrm{Im}D(s')}{s'-q^2}ds' \ . \tag{7.1}$$

The lowest contribution [Fig. 4(a)] to $\mathrm{Im}D(s)$ comes from the breakup $^3\mathrm{He}\rightarrow\mathrm{dp}$ starting at the anomalous threshold $s_0=-(1.8m_\pi)^2$ and is given by

$$\mathrm{Im}D(s)_{\mathrm{dp}}=\tfrac{2}{3}\pi a_\pi r_t g_\pi(s)\frac{m_\pi^2 m_\mathrm{d}^2}{(m_\pi^2+s)[-s(4M^2+s)]^{1/2}}\times\left[1+\frac{s_0-2s}{12m^2}\right] . \tag{7.2}$$

(Here we take the "pseudoscalar" coupling constant f_2 [Eq. (4.3) from Ref. 6] equal to zero.) Following Kopeliovich[7] the contribution of the $\mathrm{d}^*=\mathrm{pn}$ singlet ($J=0,T=1$) state in the continuum is included by assuming all strength in a resonance just above threshold,

$$\mathrm{Im}D(s)_{\mathrm{d^*p}}=\tfrac{2}{3}\pi a_\pi r_s g_\pi(s)\frac{m_\pi^2 m_{\mathrm{d}^*}^2}{(m_\pi^2+s)[-s(4M^2+s)]^{1/2}} . \tag{7.3}$$

This anomalous cut starts at $s_0^*=-(2.1m_\pi)^2$. The latest values for couplings[27] are used, $r_t=\sqrt{r_{t'}r_{t''}}=0.46$, $r_s=0.37$, where $r_{t'}$ is the He-d-p coupling constant, $r_{t''}$ is the H-d-n coupling constant, and r_s is the He-d*-p coupling constant. The H-d*-n coupling constant is unknown but assumed to be the same as the He-d*-p coupling. The

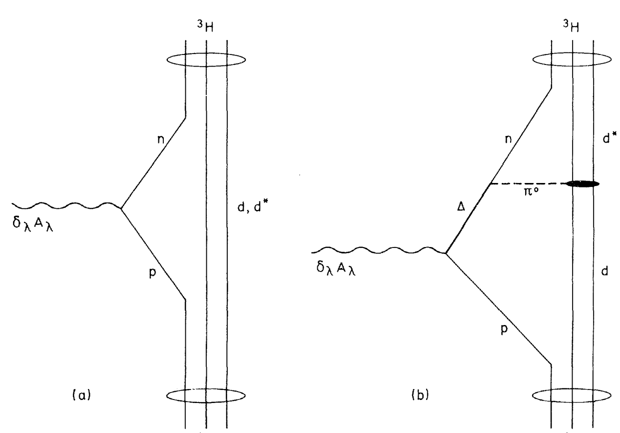

FIG. 4. Diagrams contributing to the imaginary part of the divergence of the axial vector current.

contributions of the normal three pion cut starting at $s'=-(3m_\pi)^2$ and of other processes have been disregarded.

In Fig. 2 the function $I(q^2)$ calculated in this way for some typical cases,

$$g_\pi(s)=g_\pi(-m_\pi^2)=1.41$$

and

$$g_\pi(s)=1.33/(1+0.029s/m_\pi^2)^2 ,$$

is compared with the result in EPIA. The linear representation (2.8) is not realistic here because of the extrapolation needed over a wide range; the case of a constant $g_\pi(q^2)$ is given to show that results are rather independent of the assumptions. As lower limits $s_f=-(3m_\pi)^2$ and $s_f=-(4m_\pi)^2$ have been chosen, but a lower limit[7] of $-(2m)^2$, m being the nucleon mass, does not make a qualitative difference. However, in the latter case results become more sensitive to assumptions for the pion-nucleon form factor $g_\pi(s)$. It may be noted that $I(q^2)$ is rather well described by a pole located at the anomalous threshold. This was used by us in Ref. 22.

The results of the dispersion relation agree very well with the results of EPIA. This holds especially when in EPIA the long range asymptotic tail of the wave function is included. But also without such an inclusion up to the pion pole the agreement is very good. This is what one expects. Indeed, the diagrams included in the dispersion relation are precisely the "diagrams" of the impulse approximation. The d (d*) exchange corresponds to the $S=1,T=0$ ($S=0,T=1$) part of the wave function.

In both dispersion relation and EPIA the typical "exchange" diagrams [Fig. 4(b)] are not included. In principle, extension of the dispersion calculations with exchange diagrams would be nice but is complicated and involves sweeping assumptions on several couplings. (Cf. the work by Gross[28] on the deuteron form factor.)

An important feature of this dispersion relation approach, which was overlooked by Jarlskog and Yndurain, is the possibility of determining $G_\pi(-m_\pi^2)$ *without making any continuation to the pion pole,* neither for $G_A(q^2)$ nor for $G_\pi(q^2)$. So, taking $q^2=0$ in Eq. (2.3), we obtain

$$D(0)=-G_A(0)=a_\pi G_\pi(-m_\pi^2)+I(0) , \tag{7.4}$$

which is the Goldberger-Treiman relation in the nuclear case. This relation allows for a direct determination, now that $I(0)$ has been calculated. The value which follows from the work by Jarlskog and Yndurain is $I(0)=0.21$, and therefore $G_\pi=-1.52$ for the diagram taken into account by Jarlskog and Yndurain. In the four different cases mentioned by us the values of $I(0)$ fall into the range $I(0)=0.30$—0.51.

The dispersion relation not only fixes $I(0)$ but also $I(q^2)$ for values of q^2 different from zero. It is therefore also possible to calculate $G_\pi(q^2)$ in this dispersion approach over its whole range [from the last equality in Eq. (2.3)], again *without making any assumption* on the behavior of $G_A(q^2)$ for $q^2\neq0$ (Fig. 1). A reasonable parametrization with a double pole is $\beta_\pi=0.25$ fm^2, which leads to $G_\pi(-m_\pi^2)\approx-1.7$. This result is about the same as in EPIA. As already emphasized, this agreement is due to the fact that the same physical input is involved.

It is also possible to calculate $\epsilon(q^2)$ with the help of Eq. (2.6), but then we are forced to make assumptions on the behavior of $G_A(q^2)$ for $q^2 \neq 0$. (Cf. Sec. VIII.) So $G_P(q^2)$ does not follow in a model independent way. With $G_\pi = -1.7$ and the double pole expression for $G_A(q^2)$ with $\beta_A = \beta_M$ [Eq. (6.2)] one finds $\epsilon = 0.02$, nearly independent of q^2 for $-m_\pi^2 < q^2 < m_\mu^2$. The slight difference with the EPIA value, Eq. (5.6), is due to the different behavior of $G_A(q^2)$ in the two cases.

VIII. PCAC AND ANALYTIC CONTINUATION

Dumbrajs[5] has attempted to show that there is no real conflict between the (in absolute value) high Primakoff value of $G_\pi \approx -1.7$ and the low value $G_\pi \approx -1.0$ obtained by pion photoproduction[1] and charge exchange $p^3H \rightarrow n^3He$ and elastic scattering $^3He^3H \rightarrow {}^3He^3H$.[2-4] He contends that the low value is correct by obtaining another extrapolation for $G_A(q^2)$. We will show now that his argumentation is incorrect.

Dumbrajs starts out with the (correct) equation

$$G_\pi(-m_\pi^2)/G_\pi(0) = [1+\epsilon(-m_\pi^2)]G_A(-m_\pi^2)/G_A(0) , \quad (8.1)$$

which is a variant of (2.7). Primakoff in Ref. 8 also used this equality. With Primakoff, Dumbrajs assumes (cf. Sec. VI) that

$$G_A(q^2)/G_A(0) = G_M(q^2)/G_M(0) \quad (8.2)$$

for all q^2. Experimental data on the magnetic form factor by Collard *et al.*[24] are used, together with a special mapping method, for analytic continuation of $G_M(q^2)$ to $q^2 = -m_\pi^2$. The behavior of $G_M(q^2)$ found (shown in Fig. 5) for $q^2 < 0$ drastically deviates from the double pole extrapolation by Primakoff [Eq. (6.2)]. From the figure it is seen that in Dumbrajs's extrapolation there is a maximum in $|G_M(q^2)|$ around $q^2 = -0.25\ \text{fm}^{-2}$. At still lower q^2, $G_M(q^2)$ even changes its sign and thus approaches the anomalous threshold with the sign opposite to the Primakoff extrapolation.

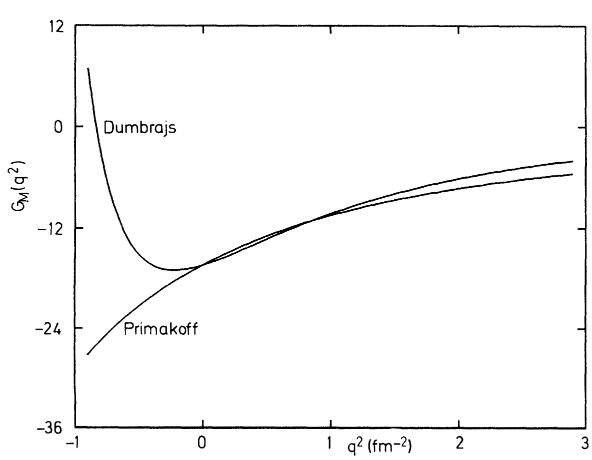

FIG. 5. The form factor $G_M(q^2)$ according to the double pole fit of Primakoff and the fit of Dumbrajs.

Dumbrajs[5] has suggested that it is the cut singularity from the breakup $^3He \rightarrow dp$ which causes this bend. But also the form factor $G_M(q^2)$, if fitted with a single pole instead of the double pole of Eq. (6.2), is approximated rather well as

$$G_M(q^2) \approx G_M(0)/[1+q^2/(2m_\pi)^2]$$

and is therefore a fair representation of the cut starting at about $q^2 \approx -(1.8m_\pi)^2$. Furthermore, the expression for $G_M(q^2)$ [and $G_A(q^2)$, too] also has this cut in EPIA, which depends on the long range asymptotic tail (Sec. IV). Therefore $^3He \rightarrow dp$ breakup cannot explain the bend. In the case of the extrapolation found by Dumbrajs very strong exchange processes have to be present. The cuts of the associated diagrams start at a higher threshold value than cuts of the $^3He \rightarrow dp$ and $^3He \rightarrow d^*p$ breakup which were discussed in Sec. VII and supposed to be the leading contributions. Therefore some scepticism about the reality of the behavior found for $G_M(q^2)$ in Ref. 5 seems justified.

It was seen in Sec. VI that the momentum dependence of $G_A(q^2)$ is somewhat poorly determined. The assumed equality of $G_M(q^2)/G_M(0)$ and $G_A(q^2)/G_A(0)$ is (approximately) valid in the impulse approximation only. If these very strong exchange processes are present there are no reasons why $G_M(q^2)$ and $G_A(q^2)$ should have approximately the same momentum dependence any longer.

As will be shown now, the matter of extrapolation is irrelevant for the rest of the argument, since the second step in Ref. 5 is not justified. In order to calculate $G_\pi(-m_\pi^2)$ from Eq. (8.1), Dumbrajs takes the value $\epsilon(-m_\pi^2) = -0.05$ from the work by Primakoff.[8] This value, however, was obtained there from the Jarlskog and Yndurain calculation for $I(q^2)$, with use made of the double pole extrapolation for $G_A(q^2)$. In that case ϵ turns out to be about constant, indeed, for $-m_\pi^2 < q^2 < m_\mu^2$. In Sec. VII it was shown that the calculation by Jarlskog and Yndurain fixes $G_\pi(-m_\pi^2)$ from the knowledge of $I(0)$ and $G_A(0)$ only and *independently of any extrapolation for* $G_A(q^2)$ at a high value. No different continuation can alter this. [In fact, the view may be taken that a dispersion relation for $G_A(q^2)$ can also be written down with the input of the Jarlskog and Yndurain calculation. This will then also fix $G_A(q^2)$ for all q^2. It can be expected that in that case $G_A(q^2)$ will have the behavior found also in the impulse approximation, as the input is about the same.]

The extrapolation by Dumbrajs would also mean that $\epsilon(q^2)$ is not constant any longer. For instance, in his parametrization $G_A(-m_\pi^2) = 1.13$ and therefore $\epsilon(-m_\pi^2) = +0.27$ using the value $G_\pi = -1.52$ following from the Jarlskog and Yndurain calculation or $\epsilon(-m_\pi^2) = 0.43$ from the value $G_\pi = -1.7$. In both cases $\epsilon(m_\mu^2) \approx -0.05$.

As said above, the extrapolation of $G_A(q^2)$ by Dumbrajs, if realistic, implies large exchange effects. In that case the calculation by Jarlskog and Yndurain would also be questionable. As a possibility to determine $G_\pi(-m_\pi^2)$ one might assume that $\epsilon(q^2)$ is still constant, as a first approximation. In Sec. VI the value

$$\epsilon(0.96m_\mu^2)=0.06^{+1.06}_{-0.72}$$

was found from the muon capture rate. The enormous uncertainty in this also leads to badly determined values for G_π, both for the Dumbrajs and Primakoff extrapolations of $G_A(q^2)$:

$$G_\pi(-m_\pi^2)=-1.27^{+0.86}_{-1.38}$$

$[G_A(-m_\pi^2)=1.13]$, in the Dumbrajs scheme, and

$$G_\pi(-m_\pi^2)=-1.80^{+1.22}_{-1.97}$$

for $G_A(-m_\pi^2)=1.59$ as follows from the Primakoff extrapolation. Primakoff[9] in this way has obtained $G_\pi=-1.77\pm0.45$. The smaller, but still appreciable, uncertainty is caused by his overoptimistic estimate of the error in $\epsilon(0.96m_\mu^2)$ from the muon capture rate (Sec. VI).

IX. CONCLUSIONS

The value of the concept of a pion-nucleus coupling constant is rather limited. The form factor always plays a role. With such a loosely bound system as a nucleus it is very hard to find a way to define and measure G_π. Still, in this work we showed that a (in absolute value) high value of $G_\pi\approx-1.7$ follows in a natural way from the connection between the definition of $G_\pi(-m_\pi^2)$ (within the elementary particle method) and the impulse approximation. We discussed how to make the necessary analytic continuation and the influence of the asymptotic tail. The value of G_π found is supported by and consistent with dispersion relations which have the same physical input. Also arguments are given why exchange is not thought to have such a catastrophical influence that this can explain the low values found by some authors.

One determination leading to a low G_π has been disproved in the present work. In an earlier paper[22] we showed already that the low values of G_π from charge exchange reaction[2,3] $p^3H\rightarrow n^3He$ and elastic scattering[4] $^3He^3H\rightarrow{}^3He^3H$ are implausible. A low value of G_π is also found in an EPM analysis of pion photoproduction.[1] The situation there is difficult and in the companion paper[10] we show that such a two-step process needs a very careful analysis. Therefore the low value of G_π found in this process can also not be considered as trustworthy.

As far as is known to us these were all analyses of G_π leading to a low value. There is no convincing evidence for such a low value. Contrary, all evidence points to a value of G_π which is about -1.7.

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