Implications of spin-current couplings for $P \pm A$ in inelastic proton scattering

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Nonlocal spin-dependent couplings in the effective nucleon-nucleon interaction are shown to probe current ⊗ spin correlations in inelastic nuclear excitations. Together, these couplings and correlations provide an important dynamical source of polarization-analyzing-power differences observed in inelastic proton scattering. This is illustrated explicitly by schematic calculations for $0^+ \to 1^+$ and $0^+ \to 0^-$ transitions. More realistic distorted-wave impulse approximation calculations have been made for the $0^+ \to 1^+$ transition in ^{90}Zr at $E_x = 8.9$ MeV which support the more transparent schematic considerations. Distorted-wave impulse approximation calculations are also compared with experimental (p,p′) data for the two lowest 1^+ excitations in ^{12}C. For isovector $0^+ \to 1^+$ transitions these nuclear structure spin ⊗ current correlations also enter β decay through the "induced tensor" couplings, and this relationship is used to help identify the nonlocality in the nucleon-nucleon effective interaction.

I. INTRODUCTION

There is an ongoing interest[1,2] in nuclear physics in measuring spin observables such as polarizations (P), analyzing powers (A), and spin-flip probabilities (S) in order to understand nuclear reaction mechanisms and to extract nuclear structure information. In inelastic proton scattering and charge exchange reactions below 50 MeV bombarding energy this effort has met with limited success at the microscopic level.[1] Recent advances[3] in experimental techniques have made possible measurements of a variety of spin observables[4,5] at intermediate energies which clearly require improved theoretical techniques if the full richness of the proton as a probe of nuclear structure is to be realized. Measurements of analyzing powers at intermediate energies have become almost routine;[2] in addition, a number of polarization tranfer measurements have been reported.[5] It has recently been noted[6] that measurements of differences between P and A are especially sensitive to a different aspect of nuclear structure than are most measurements and it is primarily the origin of this difference ($P-A$) on which we focus here. In particular, we wish to make explicit those aspects of the nucleon-nucleon (N-N) coupling and nuclear structure which can give rise to such differences within a single scattering approximation. The $P-A$ topic has been discussed earlier[7–9] with emphasis primarily on natural-parity excitations. More recently Amado[10] and Bleszynski *et al.*[11] have emphasized the role of the reaction Q value in determining $P-A$. Here we emphasize those features of the dynamics which lead to $P \neq A$ and which are relatively insensitive to the Q value. Very recently, Fäldt[12] has shown that higher order terms can give rise to $P \neq A$. The projectile-nucleus coupling necessary for $P \neq A$ is shown to be of essentially the same form (for $0^+ \to 1^+$ transitions) as the induced tensor or "weak electricity" terms in β decay. A very rough estimate is made of the dependence of $P-A$ "effects" on the projectile energy.

II. P AND A FOR THE SCATTERING OF SPIN $\frac{1}{2}$ PARTICLES

The general form of the nonrelativistic (elastic or inelastic) scattering operator for a spin $\frac{1}{2}$ particle is given by[13]

$$M = \alpha + \vec{\beta} \cdot \vec{\sigma}_p , \tag{1}$$

where $\vec{\sigma}_p$ is the Pauli spin operator of the projectile, and $\alpha(\vec{\beta})$ is a scalar (vector) function of the other dynamical variables. In terms of these amplitudes the product of the unpolarized differential cross section σ and the polarization may be written (after taking the trace over the projectile spin matrices) as

$$\sigma \vec{\mathrm{P}} = \frac{\mathrm{Tr}}{(2J_i+1)} (\alpha \vec{\beta}^{\dagger} + \vec{\beta} \alpha^{\dagger} + i \vec{\beta} \times \vec{\beta}^{\dagger}) , \tag{2}$$

where the remaining trace is over the target spin projections and J_i is the initial angular momentum of the target. Similarly, for the analyzing power times σ we find

$$\sigma \vec{\mathrm{A}} = \frac{\mathrm{Tr}}{(2J_i+1)} (\alpha \vec{\beta}^{\dagger} + \vec{\beta} \alpha^{\dagger} - i \vec{\beta} \times \vec{\beta}^{\dagger}) . \tag{3}$$

If the target is unpolarized and the polarization of the residual nucleus is not measured, parity conservation ensures that only those components of $\vec{\mathrm{P}}$ and $\vec{\mathrm{A}}$ normal to the scattering plane are nonzero, giving

$$\sigma P_n = \frac{\mathrm{Tr}}{(2J_i+1)}[\alpha\beta_n^\dagger + \beta_n\alpha^\dagger + i\hat{n}\cdot(\vec{\beta}\times\vec{\beta}^\dagger)] \quad (4)$$

and

$$\sigma A_n = \frac{\mathrm{Tr}}{(2J_i+1)}[\alpha\beta_n^\dagger + \beta_n\alpha^\dagger - i\hat{n}\cdot(\vec{\beta}\times\vec{\beta}^\dagger)] \,, \quad (5)$$

where $\hat{n}$ is a unit vector in the direction $\vec{k}_A \times \vec{k}'_A$ and $\vec{k}_A$ ($\vec{k}'_A$) is the initial (final) relative momentum. If the vector $\vec{\beta}$ is written as

$$\vec{\beta} = \beta_1\hat{e}_1 + \beta_n\hat{n} + \beta_3\hat{e}_3 \quad (6a)$$

and the unit vectors $(\hat{e}_1,\hat{n},\hat{e}_3)$ form an orthogonal right-handed coordinate system, then

$$\hat{n}\cdot(\vec{\beta}\times\vec{\beta}^\dagger) = \beta_3\beta_1^\dagger - \beta_1\beta_3^\dagger \,. \quad (6b)$$

For some purposes[10] it is convenient to take $\hat{e}_1$ and $\hat{e}_3$ in the directions of $\vec{Q}$ and $\vec{q}\,'$, respectively, which are defined by

$$\vec{Q} = \vec{k}_A + \vec{k}'_A, \quad \vec{q}\,' = \vec{k}_A - \vec{k}'_A - \frac{\vec{Q}\cdot(\vec{k}_A - \vec{k}'_A)\vec{Q}}{Q^2} \,, \quad (7)$$

but this choice is not essential. In the adiabatic limit (no energy loss), $\vec{q}\,' \to \vec{q} \equiv \vec{k}_A - \vec{k}'_A$. To better isolate different aspects of nuclear structure and of the reaction mechanism it is useful to work with the combinations $\sigma(P_n \pm A_n)$. In particular,

$$\sigma(P_n + A_n) = \frac{2\,\mathrm{Tr}}{(2J_i+1)}(\alpha\beta_n^\dagger + \beta_n\alpha^\dagger) \equiv \chi_+ = 2(\sigma_{++} - \sigma_{--}) \quad (8)$$

and

$$\sigma(P_n - A_n) = \frac{2\,\mathrm{Tr}}{(2J_i+1)} i(\vec{\beta}\times\vec{\beta}^\dagger)\cdot\hat{n} \equiv \chi_- = 2(\sigma_{-+} - \sigma_{+-}) \,, \quad (9)$$

where the quantities $\sigma_{\nu\nu'}$ represent the differential cross sections for scattering from an initial (ν) to a final (ν') spin projection as measured along the vector $\hat{n}$. Note that χ_+ (χ_-) is linear (quadratic) in the *projectile* spin-dependent amplitudes; in addition χ_+ is sensitive to the component of $\vec{\beta}$ normal to the scattering plane, while χ_- is sensitive to the interference of its in-plane components. Equation (9) shows that in order to study differences between P and A we need to focus on the origin and content of the vector $\vec{\beta}$.

The above considerations are quite general. One particular implication of Eq. (9) is that in the distorted wave approximation measurements of $P-A$ should be relatively insensitive[8] to *small* admixtures of $S=1$ transfers (to the target) in predominantly $S=0$ excitations because the relevant interference in Eq. (9) is only between amplitudes characterized by $S=1$ transfer to the projectile. Since the nucleon-nucleon spin-orbit force can transfer $S=0$ to the target and $S=1$ to the projectile, differences between P and A are allowed in such transitions. However, these differences should be small because in lowest order the spin-orbit term produces an amplitude proportional to $\vec{\sigma}_p\cdot\hat{n}$ giving $\vec{\beta}_{so}\sim\hat{n}$ and $\chi_- \sim 0$.

III. SCHEMATIC APPLICATIONS TO $0^+ \to 1^+$ TRANSITIONS

For definiteness and simplicity we consider unnatural-parity transitions of the type $0^+ \to 1^+$ (or $1^+ \to 0^+$) and use the plane-wave impulse approximation (PWIA) to illustrate the underlying physics. Although distortion effects provide an additional source of $P-A$ and should be included in detailed comparisons with observation, they tend to obscure the essential points without altering the primary qualitative features. (See Sec. IV.)

One widely used version[14] of the impulse approximation consists of representing the N-N t matrix by a complex interaction of the form

$$V_{ip} = V_{ip}^C + V_{ip}^{LS}\vec{L}\cdot\vec{S} + V_{ip}^T S_{ip} \,, \quad (10)$$

where C, LS, and T denote the central, spin-orbit, and tensor parts of V_{ip}, respectively, and i (p) denotes a target (projectile) nucleon. The parameters of V_{ip} are adjusted until its antisymmetrized momentum space matrix elements match the free N-N t matrix at each energy of interest. Therefore, when calculating nucleon-nucleus scattering, knockon exchange terms must be calculated explicitly. For many purposes these exchange terms may be included adquately by adding a local pseudopotential[15,16] to V_{ip}; we must, however, go beyond this approximation to understand the microscopic origins of $P-A$. Indeed, it is the nonlocality (or velocity dependence) of the effective coupling between projectile and target nucleons which in the present model gives $P\neq A$ for zero Q value even in the absence of distortion. To show this we restrict ourselves to the central part of the interaction for simplicity. Many of the auxiliary details are given in Ref. 16 and in the Appendix of this paper. Although the exchange terms arising from the tensor force have been shown[6] to be a more important source of $P\neq A$ than are those associated with the central force, the most important features of the mechanism are illustrated more transparently by considering the central force alone.

The direct part of the nucleon-nucleon t matrix which is associated with the $\vec{\sigma}_i\cdot\vec{\sigma}_p$ part of V_{ip}^C for a 1^+ transition $I\to F$ takes the form

$$T_{FI}^S = \langle \vec{k}'_A F \mid V^C \mid \vec{k}_A, I \rangle = V_1^C(q)\left\langle F \left| \sum_i j_0(qr_i)\vec{\sigma}_i \right| I \right\rangle\cdot\vec{\sigma}_p \equiv \vec{B}_S\cdot\vec{\sigma}_p \quad (11)$$

in the PWIA where $V_1^C(q)$ is the Fourier transform of the $\vec{\sigma}_i\cdot\vec{\sigma}_p$ part of V_{ip}, $\vec{q} = \vec{k}_A - \vec{k}'_A$, j_0 is a spherical Bessel function, and isospin indices have been supressed. Inclusion of a zero-range pseudopotential[15] to approximate exchange effects simply adds a constant to $V_1^C(q)$ and does not alter the structure of the term. Henceforth we regard $V_1^C(q)$ as including the zero-range pseudopotential associated with the exchange terms. [Actually Eq. (11) only includes the usually dominant monopole part of V_{ip}^C; for present purposes it is unnecessary to include the more complicated quadrupole contribution. We will return to this point in Sec. IV.]

The new type of nucleon-nucleus amplitude arises when we account explicitly for the nonlocality present in the exchange terms. In the present context nonlocality corresponds to different coordinates for the incident and scattered nucleons. This nonlocality arises from the finite (nonzero) range[16] of V_{ip}^C in the present model. More explicitly, we may imagine expressing V_{ip}^C in terms of its Fourier components $\widetilde{V}^C(k^2)$ in the *exact* exchange integral and then making a Taylor series of $\widetilde{V}^C(k^2)$ about some average momentum transfer which is usually taken[16] to be k_A the incident nucleon-nucleus momentum. This choice is based on the idea that on the average (for $A \gg 1$) the incident nucleon must simply be stopped.

$$\widetilde{V}^C(k^2) \simeq \widetilde{V}^C(k_A^2) + \left[\frac{\partial \widetilde{V}^C}{\partial k^2}\right]_{k^2=k_A^2} (k^2 - k_A^2) \, ;$$

$$\vec{k} \sim \vec{k}_p - \vec{k}_i, \quad \vec{k}_p \sim \vec{k}_A \, . \tag{12}$$

The first term is a constant in k space and gives rise to the zero-range pseudopotential mentioned and included above; the second term corrects for the distribution of momentum transfers due to the motion of the target nucleons and vanishes for a zero range V^C. It is the $\vec{k}_i \cdot \vec{k}_p$ part[16] of the second term which gives rise to the new (current-current) type of coupling. This can be shown[16] to lead to a correction to V_{ip}^C of the form

$$\delta V_{ip}^C = \frac{2}{\pi} \int_0^\infty q^2 dq \sum_{LSJ} (-)^{J+S} 2 \left[\frac{\partial \hat{V}_{ST}}{\partial k^2}\right]_{k^2=k_A^2} q^2 [P_L(i) \otimes \mathscr{O}_S(i)]^J \cdot [P_L(p) \otimes \mathscr{O}_S(p)]^J \, , \tag{13}$$

where

$$P_J(q\vec{r}, \vec{L}) \equiv \frac{j_J(qr)}{qr} [i^{J-1} Y_{J-1}(\hat{r}) \otimes \vec{L}]^J \, , \tag{14}$$

$$\mathscr{O}_0 = 1, \quad \mathscr{O}_1 = \vec{\sigma} \, , \tag{15}$$

and $\hat{V}_{ST}$ is the Fourier transform of that part of V_{ip}^C operative[17] in the exchange terms which transfer spin S and isospin T to the nucleus (or projectile); $\vec{L}$ is the usual orbital angular momentum operator. For $J=1$ transfers,

$$\delta V_{ip}^C \to \frac{1}{2\pi^2} \left[\frac{\partial \hat{V}_{ST}}{\partial k^2}\right]_{k_A^2} \int_0^\infty q^4 dq \frac{j_1(qr_i)}{qr_i} \frac{j_1(qr_p)}{qr_p} \{2\vec{L}(i) \cdot \vec{L}(p) \delta_{S0} + [i\vec{L}(i) \times \vec{\sigma}(i)] \cdot [i\vec{L}(p) \times \vec{\sigma}(p)] \delta_{S1}\} \, . \tag{16}$$

This operator (see the Appendix) clearly samples the current and current $\otimes$ spin densities of the target and contributes terms to the t matrix of the forms

$$T_{FI}^L = \vec{B}_L \cdot \hat{n} \, , \tag{17a}$$

$$T_{FI}^{LS} = (\vec{B}_{LS} \times \hat{n}) \cdot \vec{\sigma}_P \, , \tag{17b}$$

where

$$\vec{B}_L \sim \left\langle F \left| \sum_i \frac{j_1(qr_i)}{qr_i} \vec{L}(i) \right| I \right\rangle \, , \tag{18a}$$

$$\vec{B}_{LS} \sim \left\langle F \left| \sum_i \frac{j_1(qr_i)}{qr_i} [i\vec{L}(i) \times \vec{\sigma}(i)] \right| I \right\rangle \, , \tag{18b}$$

and $\hat{n}$ is a unit vector in the direction $\vec{k}_A \times \vec{k}'_A$. [There are other correction terms to V_{ip}^C which contribute to amplitudes having $(-)^L = \Delta\pi$ and therefore give rise to no new selection rules.] From Eqs. (11), (18a), and (18b) we get for the transition matrix

$$T_{FI} = T_{FI}^S + T_{FI}^L + T_{FI}^{LS}$$
$$= \vec{B}_S \cdot \vec{\sigma}_p + \vec{B}_L \cdot \hat{n} + (\vec{B}_{LS} \times \hat{n}) \cdot \vec{\sigma}_p \, , \tag{19}$$

where $\vec{B}_S$, $\vec{B}_L$, and $\vec{B}_{LS}$ denote target transition "spin" densities analogous to $\vec{\sigma}_p$ for the projectile. In terms of transferred angular momenta (LSJ) $\vec{B}_S \cdot \vec{\sigma}_p \leftrightarrow (011)$, $\vec{B}_L \cdot n \leftrightarrow (101)$, and $(\vec{B}_{LS} \times \hat{n}) \cdot \vec{\sigma}_p \leftrightarrow (111)$. By comparing Eqs. (1) and (19) ($M \to T_{FI}$) we identify

$$\alpha = \vec{B}_L \cdot \hat{n} \, ,$$
$$\vec{\beta} = \vec{B}_S + \vec{B}_{LS} \times \hat{n} \tag{20}$$
$$= \vec{B}_S + B_{LS1} \hat{e}_3 - B_{LS3} \hat{e}_1 \, .$$

In the notation of Ref. 10, $\vec{B}_{LS} = \vec{\epsilon} a_{13} = -\vec{\epsilon} a_{31}$ and $\vec{B}_S = \vec{\epsilon} a_{ii}$ apart from kinematic factors. To evaluate Eq. (9) using this explicit form for $\vec{\beta}$ given by Eq. (20) it is convenient to note the following relationship for the trace over the nuclear substates:

$$\mathrm{Tr}(\vec{A} \cdot \vec{u})(\vec{B}^\dagger \cdot \vec{v}) = \tfrac{1}{3} \langle J_i || A || J_f \rangle \langle J_i || B || J_F \rangle^* \vec{u} \cdot \vec{v} \, . \tag{21}$$

$\vec{A}$ and $\vec{B}$ are vector operators in the nuclear target space like those which generate $\vec{B}_S$, $\vec{B}_L$, and $\vec{B}_{LS}$; $\vec{u}$ and $\vec{v}$ are arbitrary constant vectors; and the reduced matrix elements are as defined in Bohr and Mottleson.[18] Using Eqs. (9), (20), and (21) we soon find [with $\langle B_S \rangle \equiv \vec{B}_s$ in Eq. (11), etc.]

$$\sigma(P_n + A_n) = \frac{4}{3(2J_i+1)} \mathrm{Re} \langle J_i || B_L || J_f \rangle \times \langle J_i || B_S || J_f \rangle^* \, , \tag{22a}$$

$$\sigma(P_n - A_n) = \frac{-8}{3(2J_i+1)} \text{Im}\langle J_i || B_{LS} || J_f \rangle \times \langle J_i || B_S || J_f \rangle^* , \quad (22b)$$

each of which is typically nonzero. Equation (22b) illustrates explicitly a source of $P-A$ which arises from a single (central) part of the interaction. (Note that the standard two-body spin-orbit terms[19] would be of the $\vec{B}_L \cdot \vec{\sigma}_p$ and $\vec{B}_S \cdot \hat{n}$ forms and would alone, or with the static central term only, give rise to $P=A$.)

The interference in Eq. (22a) is between two time-reversal odd operators (B_L and B_S); however, the interference in Eq. (22b) is between time-reversal even (B_{LS}) and time-reversal odd (B_S) operators as should be expected from Refs. 9 and 20. It should also be pointed out that the "frozen nucleus" approximation, as used in Ref. 11 in the Glauber model, manifestly excludes amplitudes such as $\vec{B}_L$ and $\vec{B}_{LS}$ from the outset and hence necessarily gives $P \sim A$.

Although $\vec{B}_S$ and $\vec{B}_{LS}$ both arise from the central part of the force, the imaginary part of $\langle B_{LS} \rangle \langle B_S \rangle^*$ in Eq. (22b) need not vanish. This follows from the fact that those parts of V^C operative in the direct and exchange terms can have different phases since these terms sample quite different parts of the momentum profile of V^C as well as different spin and isospin combinations. (See the Appendix.) If V^C were taken to be real as is done in the ordinary Born approximation, $P=A$. From Eq. (13) we also see that $\vec{B}_{LS}$ depends explicitly on the slope of the interaction in momentum space.

These schematic considerations have been confirmed in PWIA calculations which treat the exchange terms exactly. In particular, the $1p$-shell amplitudes of Cohen and Kurath[21] have been varied[21] for the lowest 1^+ excitations in ^{12}C; these variations have established the strong sensitivity of P, A, and $P-A$ to the $LSJ=111$ spectroscopic amplitude between 100 and 200 MeV bombarding energy when only a complex central interaction is used.

From a nuclear structure viewpoint the observation of $P \neq A$ implies nonvanishing form factors defined in Eqs. (11) and (18b). For transitions within a single shell or at small q these form factors are specified by the matrix elements

$$\langle \vec{\sigma} \rangle \equiv \left\langle F \left|\left| \sum_k \vec{\sigma}(k) \right|\right| I \right\rangle \quad (23a)$$

and

$$\langle i\vec{L} \times \vec{\sigma} \rangle \equiv \left\langle F \left|\left| \sum_k [i\vec{L}(k) \times \vec{\sigma}(k)] \right|\right| I \right\rangle , \quad (23b)$$

along with the oscillator parameter. Recalling that we have suppressed isospin variables for brevity, the matrix element $\langle \vec{\sigma} \rangle$ is essentially the spin part of the electromagnetic $M1$ operator. For isovector excitations where $\vec{\sigma} \rightarrow \vec{\sigma}\,\vec{\tau}$, $\langle \sigma\tau \rangle$ is also proportional to the Gamow-Teller matrix element[18] familiar from β decay. Although less familiar, the matrix element $\langle i\vec{L} \times \vec{\sigma} \rangle$ does arise in β decay where it describes weak electricity or alternately the "induced tensor" term[22] present in the timelike component of the axial current; this is discussed in the Appendix. The $i\vec{L} \times \vec{\sigma}$ component of the transition density also enters the π^- photoproduction process where, however, fourfold variations of its strength are reported[23] to produce only small differences in the calculated cross sections. It is important to stress[24] that measurements of electromagnetic form factors for 1^+ excitations are totally insensitive to this $LSJ=111$ part of the transition density. The single particle matrix elements of $i\vec{L} \times \vec{\sigma}$ are especially simple and we record them here for completeness. In the convention of Ref. 18 these are

$$\langle n'(l'\tfrac{1}{2})j' || (i\vec{L} \times \vec{\sigma}) || n(l\tfrac{1}{2})j \rangle = 0, \quad j=j'$$
$$= \sqrt{j_>(2j+1)(2j'+1)}\,\delta_{ll'} \langle n' | n \rangle, \quad j \neq j' , \quad (24a)$$

where $\langle n \mid n' \rangle$ denotes the radial overlap integral of the two single particle wave functions and $j_>$ is the larger of (j,j'). In terms of $\langle \sigma \rangle$ we have

$$\langle j' || i\vec{L} \times \vec{\sigma} || j \rangle = (j-j') j_> \langle j' || \vec{\sigma} || j \rangle . \quad (24b)$$

One simple but nontrivial implication of Eq. (24a) is that within the present model $P=A$ for $0^+ \rightarrow 1^+$ transitions within a j^n configuration if Q-value effects are not too large; this will be explored in more detail below.

Before leaving this section it is worth noting that, for zero Q value, the types of couplings considered so far satisfy the equality[11] $D_{qQ} = -D_{Qq}$ where D_{ij} are the polarization transfer coefficients described in Refs. 4 and 11. This follows readily from Eqs. (20) and (21) and the definition of D_{ij}:

$$D_{ij} = \frac{\text{Tr}(M\sigma_j M^\dagger \sigma_i)}{\text{Tr}MM^\dagger}, \quad \sigma_i = \vec{\sigma} \cdot \hat{e}_i, \text{ etc.} \quad (25)$$

This equality is not preserved, however, when we include the static quadrupole ⊗ spin term in Eqs. (11) and (19). Inclusion of this term gives rise to

$$\delta T^S_{FI} \sim V^C_1(q) \left\langle F \left| \sum_i j_2(qr_i) [Y_2(\hat{r}_i) \otimes \vec{\sigma}_i]^1 \right| I \right\rangle \cdot [Y_2(\hat{q}) \otimes \vec{\sigma}_p]^1 \equiv \sqrt{8\pi} [\vec{Q}_S \otimes Y_2(\hat{q})]^1 \cdot \vec{\sigma}_p , \quad (26)$$

which in PWIA leads to

$$\sigma(D_{qQ}+D_{Qq})=\frac{4\,\mathrm{Re}}{3(2J_i+1)}\langle J_i||B_{LS}||J_f\rangle\langle J_i||Q_S||J_f\rangle^* \quad (27)$$

providing another potential measure of the presence of spin ⊗ current (and quadrupole ⊗ spin) couplings. Inclusion of the tensor force coupling to the nuclear monopole or quadrupole matrix element can, in conjunction with spin ⊗ current, also give $D_{qQ}+D_{Qq}\neq 0$.

IV. MORE REALISTIC CALCULATIONS

A. The ^{90}Zr(p,p′) transition ($E_x=8.9$ MeV) at $E_p=200$ MeV

To illustrate some of the above ideas in a more realistic context we have made DWIA calculations at 200 MeV for the ^{90}Zr(p,p′) reaction to the "giant" 1^+ state at 8.9 MeV of excitation. Figure 1 shows the results for the quantities $\sigma(P\pm A)/2$ which from Eqs. (8) and (9) isolate the in-plane and out-of-plane components of $\vec{\beta}$. When a pure $(g_{9/2}^{-1}g_{7/2})_\nu$ configuration is assumed which admits [see Eq. (24)] a large B_{LS}-type amplitude, $\sigma(P+A)$ is seen to be quite sensitive to turning off either the optical model spin-orbit distortion (dashed line) or the nucleon-nucleon spin orbit force (Δ's) as is implied by Eq. (8). By contrast, $\sigma(P-A)$ is very insensitive to either of these changes. To illustrate the various contributions, calculations were also made by including the full interaction and changing the "excitation" to be of the form $(g_{9/2}^{-1}g_{9/2})_\nu$. From Eq. (24) this configuration forbids ($LSJ=111$) terms, i.e., $\vec{B}_{LS}=0$, and as anticipated $\sigma(P-A)$ becomes very small; setting the Q value to zero in this case reduces $P-A$ to essentially zero. Hence with this interaction[14] $Q\neq 0$ is seen to be a relatively small source of $P-A\neq 0$. By contrast, Fig. 1 shows that $\sigma(P+A)$ is rather insensitive to the above change in configuration, again in accord with the schematic results above.

Coupled with the schematic considerations above, these more realistic results indicate the desirability of isolating the observables $\sigma(P+A)$ and $\sigma(P-A)$. Clearly measurements of $\sigma(P-A)$ provide the possibility of learning about 1^+ "modes" of excitation of the $i\vec{L}\times\vec{\sigma}$-type which complement the information obtainable[2] from (e,e′) measurements. At least one experiment[6] of this type has been reported at 150 MeV and others are underway.

B. 1^+ excitations in ^{12}C at $E_P=150$ MeV

We next consider the excitation of the two lowest 1^+ states in ^{12}C by 150 MeV protons within the DWIA. The data are from Refs. 6 and 21; the t- matrix interaction used is an updated version of that in Ref. 14 based on the SM82 N-N amplitudes of Arndt[25] at 140 MeV. The wave functions are those of Cohen and Kurath[21] (CKWF) which for the $T=1$ state at 15.11 MeV provide a good description of the Gamow-Teller strength, the $M1$ form factor for $q\lesssim 1$ fm^{-1}, and the unpolarized (p,p′) cross section at small q over a wide range of energy. The wave function for the $T=0$ state at 12.71 MeV is much more poorly understood. Above ~400 MeV it provides a reasonable (though not excellent) description of the (p,p′) differential cross section; below 200 MeV it does not work well using either a t-matrix[14] or G-matrix[26] interaction.

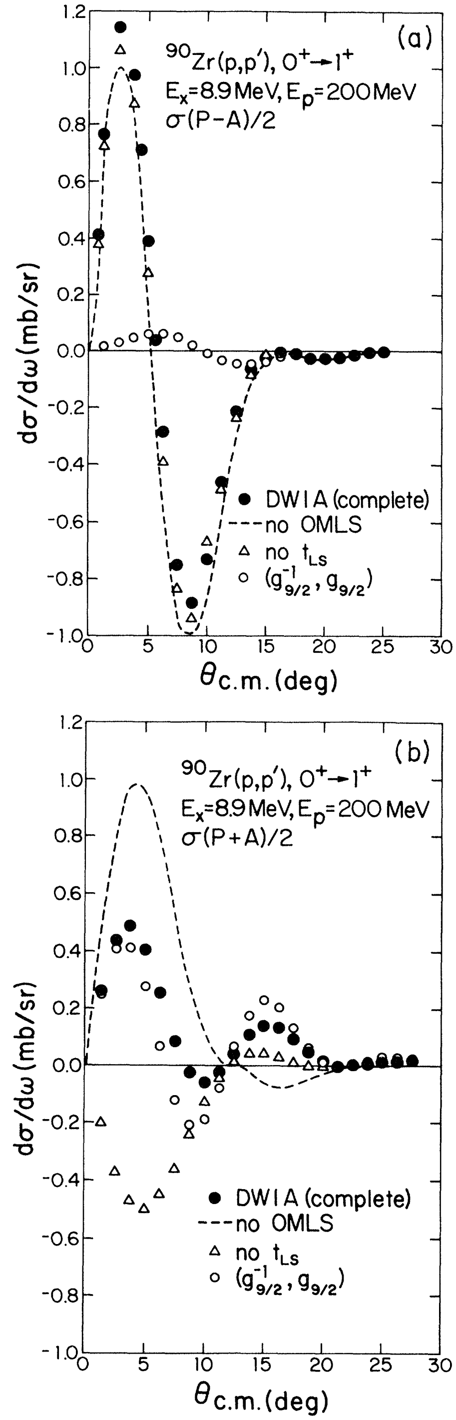

FIG. 1. $\sigma(P\pm A)/2$ for the ^{90}Zr(p,p′) reaction at 200 MeV. The dark points correspond to a complete DWIA calculation. The dashed lines and triangles correspond to turning off the optical model spin-orbit and N-N spin-orbit interactions, respectively. The dark points, dashed lines, and triangles refer to calculations made with the $(g_{9/2}^{-1}g_{7/2})_\nu$ configuration; the open circles refer to a complete calculation using a $(g_{9/2}^{-1}g_{9/2})_\nu$ configuration.

At this point it is important to reemphasize that measurements of the transverse electromagnetic form factors for these transitions provide information on the nuclear structure spin *or* current matrix elements[27] associated with $\vec{B}_S$ and $\vec{B}_L$ and their quadrupole counterparts. [See Eqs. (11), (18), and (27).] For isovector $0^+ \rightarrow 1^+$ transitions, β-decay *rate* measurements are also sensitive primarily[28] to the nuclear structure associated with $\vec{B}_S$. However, recent measurements of alignment correlation coefficients ($\alpha_\pm$) from the β decay of mirror nuclei in the mass-12 system yield[29]

$$F_E^{(1)}/F_A = -\tfrac{3}{4}(\alpha_+ + \alpha_-)$$
$$= \frac{3.8\pm0.5}{2m} \simeq \frac{\langle(-i\vec{L}\times\vec{\sigma})\vec{\tau}\rangle}{\langle\vec{\sigma}\,\vec{\tau}\rangle}\frac{1}{2m}\,, \qquad (28)$$

where m is the nucleon mass, $F_E^{(1)}$ is the first class contribution to the form factor corresponding to weak electricity (or the induced tensor term), and F_A is the axial vector form factor. The CKWF yield[21]

$$\frac{\langle 0^+ \,|\, (-i\vec{L}\times\vec{\sigma})\vec{\tau} \,|\, 1^+\rangle}{\langle\vec{\sigma}\,\vec{\tau}\rangle} = 3.72 \qquad (29)$$

for this ratio (denoted by d/AC in Ref. 28) is in excellent agreement with the β-decay results. Therefore, for the (p,p′) excitation of the 15.11 MeV state in ^{12}C, those aspects of nuclear structure most important for calculating $\sigma(P-A)$ may be regarded as known. Since this is the only nucleus for which such detailed information is available for transitions to the ground state, ($\vec{p}$,p′) and ($\vec{p}$,n) measurements of $P-A$ provide a *potential* technique for extracting information on $\langle i\vec{L}\times\vec{\sigma}\rangle$. For nonisovector transitions neither electromagnetic nor β-decay measurements determine $\langle i\vec{L}\times\vec{\sigma}\rangle$.

For the two transitions considered here it is instructive to examine the transition densities in an *LS* representation as has been done by Lee and Kurath.[21] For each *LSJ* transfer their $1-p$ shell spectroscopic amplitudes A_{LSJ} are given in Table I; the A_{LSJ} for a pure $(p_{3/2}^{-1}, p_{1/2})$ excitation are listed for comparison. The relative sizes of the A_{LSJ} illustrate why these two transitions are especially sensitive to the inclusion of the $LSJ=111$ ($\leftrightarrow \vec{B}_{LS}$) terms.

Measurements of $P-A$ may also be used to place gross constraints or limitations on the N-N effective interaction. For example, where significant values of $\sigma(P-A)$ are observed, the purely real $\pi+\rho$ model[30] of the effective isovector force is inadequate. Although some deficiencies in the $\pi+\rho$ model are to be expected, such measurements place limitations on the deviations.

TABLE I. Transition density matrix elements in an *LSJ* transfer representation.

LSJ	A_{LSJ} (15.11 MeV)	A_{LSJ} (12.71 MeV)	A_{LSJ} $(p_{3/2}^{-1} p_{1/2})$
011	0.160	0.152	−0.385
211	0.096	0.049	−0.215
111	0.515	0.537	0.500
101	−0.023	−0.093	0.289

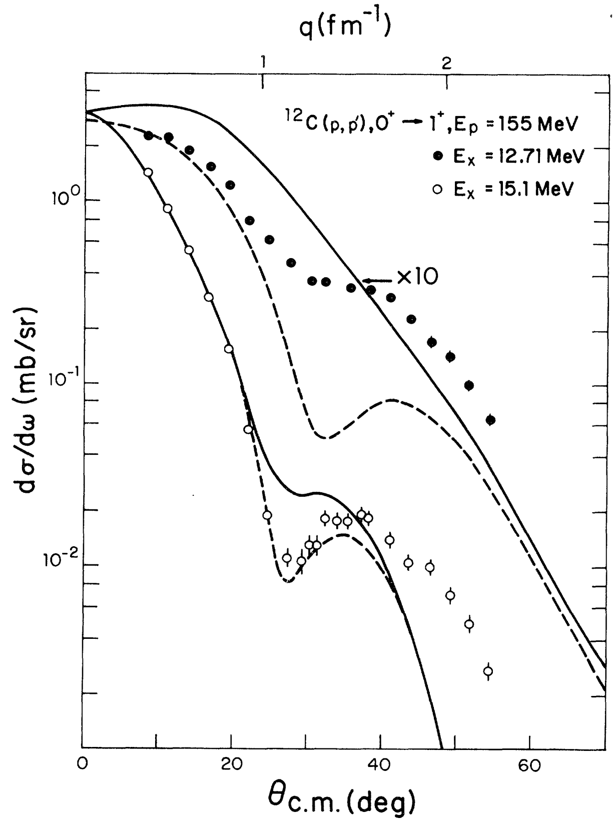

FIG. 2. Comparison of DWIA results (at 150 MeV) with experimental data of Ref. 21. The calculated and measured cross sections for the 12.71 MeV state have been multiplied by ten. The solid (dashed) curve corresponds to using the CKWF (MCKWF).

The calculated and experimental results for $\sigma(P\pm A)$, for these two 1^+ transitions are shown in Figs. 2 and 3. Positive (negative) experimental values of $\sigma(P\pm A)$ are denoted by solid (open) circles. The experimental and calculated unpolarized differential cross sections are shown in Fig. 2 to place our overall level of understanding of these excitations in perspective. The calculations using the CKWF are denoted by the full curves. Calculations for these transitions using modified CKWF (MCKWF) constructed by deleting the A_{111} amplitudes are shown as dashed curves. Removal of this amplitude sets $\langle i\vec{L}\times\vec{\sigma}\rangle=0$ without affecting the calculated transverse form factor; the β-decay *rate* is also negligibly altered.[28] It should be emphasized that our purpose here is to establish trends and to explore sensitivities, not to obtain precise fits to the data.

1. The 15.11 MeV (T=1) excitation

For this transition the unpolarized cross section is well described in the DWIA out to 20°–25° which corresponds to $q \sim 1$ fm^{-1}. By using the CKWF the calculated $\sigma(P+A)$ has the wrong sign forward of $\sim 13°$ and is too small by a factor of ~ 2. Beyond $\sim 15°$ there is qualitative agreement between theory and experiment. The calculated values of $\sigma(P+A)$ are negligibly different when the MCKWF are used, which is in accord with the schematic

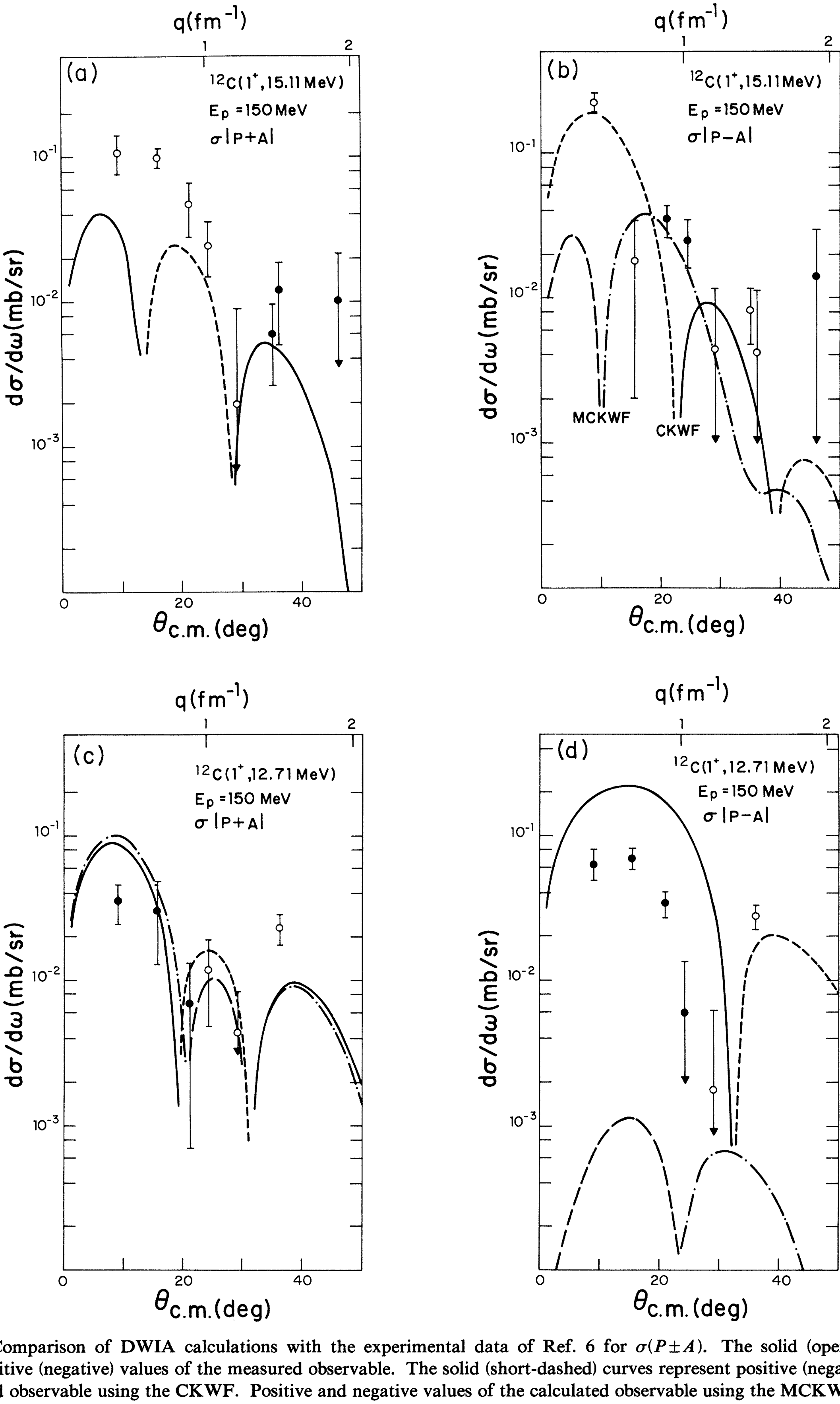

FIG. 3. Comparison of DWIA calculations with the experimental data of Ref. 6 for $\sigma(P\pm A)$. The solid (open) data points represent positive (negative) values of the measured observable. The solid (short-dashed) curves represent positive (negative) values of the calculated observable using the CKWF. Positive and negative values of the calculated observable using the MCKWF are denoted by long-dashed dot (—·—) and long-dashed (——) curves, respectively.

model above [see Eq. (22a)]. We expect the combination $\sigma(P+A)$ to be especially sensitive to the optical model and N-N spin-orbit interactions as was illustrated for the ^{90}Zr transition. The calculated values of $\sigma(P-A)$ are only in good agreement with the experimental datum near 9° when the CKWF are used; the MCKWF yield $\sigma(P-A)$, too small at small q by at least an order of magnitude. Recall that the β-decay information on $\langle i\vec{L}\times\vec{\sigma}\rangle$ is most relevant for small q; moreover it is the small q region where the unpolarized differential cross section is best described. Beyond 10°–15° neither set of wave functions describes the $\sigma(P-A)$ data well overall. The MCKWF do, however, seem to describe σ and $\sigma(P-A)$ between 20° and 25° which suggests that the nonlocal part of the N-N interaction is not correct in detail in this region of larger momentum transfer. It would clearly be desirable to have more forward angle data on $\sigma(P\pm A)$.

2. *The 12.71 MeV (T=0) excitation*

Neither set of wave functions describes the differential cross section well at this energy. Unlike the isovector transition, there is a very substantial change in the magnitude and shape of $d\sigma/d\omega$ when A_{111} is set to zero (MCKWF). This arises from the fact that this transition is dominated by the exchange terms in the present model, presumably due to an absence of a simple meson to exchange in this channel. The effect on $\sigma(P+A)$ of changing the wave functions is rather small (see Fig. 3) but not completely negligible as is the case for the 15.11 MeV transition. Moreover, the calculations are in qualitative agreement with experiment for $\sigma(P+A)$. It is useful here to note that the isoscalar N-N spin-orbit force is relatively large and well understood;[14] the isovector N-N spin-orbit force is believed to be small but is poorly understood. Apart from being too large by a factor of ~ 3, the calculated $\sigma(P-A)$ for this transition has roughly the correct shape and phase when the CKWF are used. From Fig. 3 results obtained with the MCKWF ($A_{111}=0$) disagree completely with the data suggesting that significant spin $\otimes$ current correlations are present in the wave functions ($A_{111}\neq 0$) and that the spin-dependent part of the effective interaction is significantly nonlocal, though apparently not in the same way as prescribed by the free t-matrix interaction.

V. P AND A FOR $0^+\to 0^-$ TRANSITIONS

P and A are particularly interesting for $0^+\to 0^-$ transitions where only the longitudinal part of the static N-N coupling contributes.[30] The general form of the nucleon-nucleus scattering operator for this case is

$$M=A\vec{\sigma}\cdot\vec{q}+B\vec{\sigma}\cdot\vec{Q}\ , \tag{30}$$

where q and Q are defined earlier. Comparison of Eqs. (1) and (30) gives

$$\alpha=0,\ \ \vec{\beta}=A\vec{q}+B\vec{Q} \tag{31}$$

for this case, which from Eqs. (8) and (9) gives $P_n=-A_n$ and

$$\sigma(P_n-A_n)=-2\sigma A_n=-8\,\mathrm{Im}(AB^*)k_Ak_A'\sin\theta\ , \tag{32}$$

where θ is the center-of-mass scattering angle.

In the static approximation in which the N-N coupling is local and any exchange terms are included by a zero-range pseudopotential, the B term $\to 0$ and $\vec{P}=0=\vec{A}$. When corrections[16] are made to the treatment of exchange, however, there appear current couplings analogous to those for $0^+\to 1^+$ transitions which take the form (see the Appendix)

$$\delta V_{ip}^C\sim(\vec{\sigma}_i\cdot\vec{p}_i)(\vec{\sigma}_p\cdot\vec{p}_p) \tag{33}$$

and contribute to the B term in Eq. (31). Actual PWIA and DWIA calculations[30] and unpublished experimental data[31] indicate analyzing powers significantly different from 0, again indicating the importance of the spin $\otimes$ current terms in the effective N-N interaction.

It is interesting to note (see the Appendix) that for isovector $0^+\to 0^-$ transitions, the A and B terms are the inelastic scattering counterparts of the spacelike and timelike couplings[32] which enter the corresponding β-decay calculations. Hence measurements of nonzero A_n for such states is a positive signature of appreciable couplings of both of these types in the N-N interaction as well as sizable nuclear matrix elements for these types of operators. A study of the analyzing power for this class of transitions at a number of bombarding energies could provide important information on the nonlocality of the effective interaction.

VI. ENERGY DEPENDENCE

It is important to understand the energy dependence of these current $\otimes$ spin couplings to know where their consequences may best be studied. Here we make a very rough estimate of this dependence by assuming that V_{ip}^C may be represented by an energy independent Gaussian interaction of mean square radius $\langle r^2\rangle$ and that the direct momentum transfer is of the order of the Fermi momentum k_F. In this case the ratio of the current $\otimes$ spin amplitude to the direct amplitude should go roughly as

$$\frac{T_{FI}^{LS}}{T_{FI}^{S}}\simeq\langle r^2\rangle[k_F\exp(k_F^2\langle r^2\rangle/6)[k_A\exp(-k_A^2\langle r^2\rangle/6)]\ , \tag{34}$$

so that the effects of the current $\otimes$ spin couplings should decrease with increasing energy at intermediate energies at a rate dependent on $\langle r^2\rangle$. [Eq. (34) assumes an even-state interaction; a more quantitative estimate should also include the ratio of $\hat{V}_{ST}$ to V_{ST} (see Sec. III) as well as explicit consideration of the tensor force.] A decrease in $P-A$ with increasing energy has been observed.[5,6] Although the simple Gaussian interaction is known to be oversimplified, it illustrates the salient features.

VII. SUMMARY

A number of the ideas described here regarding the $P-A$ problem have been discussed more formally by other authors.[7–12] Since there has been considerable confusion on this topic it seemed appropriate to make some of the relevant ideas more concrete and to indicate explicitly the new aspects of nuclear structure which enter in lowest order.

In particular, we have shown both schematically and with explicit DWIA calculations that current ⊗ spin couplings are important for understanding measurements of $\vec{P}$ and $\vec{A}$ for unnatural-parity transitions in (p,p′) and (p,n) scattering and have indicated explicitly for $0^+\to 1^+$ and $0^+\to 0^-$ transitions how one source of this type of nonlocal coupling, the exchange terms, gives rise to $P\neq A$. Other sources of nonlocality in the spin-dependent parts of the N-N interaction can also be important in this respect. One of these (explicit energy dependence) has been discussed by Walker;[33] multistep processes[12] and explicit velocity dependence[34] are two other possible sources.

Some theoretical advantages for considering the observables $\sigma(P\pm A)$ have been suggested and illustrated both schematically and by a comparison of DWIA calculations with data for the excitation of the two lowest 1^+ states in ^{12}C by 150 MeV protons. For isovector 1^+ transitions we have related the measurement of $\sigma(P-A)$ to weak electricity present in β decay through the matrix element $\langle i\vec{L}\times\vec{\sigma}\rangle$. Although a quantitative theoretical understanding of the sources of nonlocality in the spin-dependent part of the effective N-N interaction is currently unavailable, measurements of $\sigma(P-A)$ provide a convincing signature of their presence and should prove quite helpful in discriminating between different N-N interactions having differing nonlocal behaviors. Once this aspect of the N-N interaction is understood (or calibrated), the (p,p') reaction may be used as a more quantitative probe of the poorly understood but very interesting current ⊗ spin modes of excitation in nuclei.

ACKNOWLEDGMENTS

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APPENDIX

In this appendix we show that in the nonrelativistic impulse approximation several of the isovector knockon exchange terms have velocity-independent and velocity-dependent counterparts in the weak-interaction current. In the PWIA the knockon exchange contribution[17] to the nucleon-nucleus t matrix is given by

$$T=\int d\vec{r}_1\int d\vec{r}_2 e^{-i\vec{k}'\cdot\vec{r}_2}\chi_{v'}^{\dagger}(2)\psi_f^*(\vec{r}_1)\hat{V}(\vec{r}_{12},\vec{\sigma})\times\psi_i(\vec{r}_2)\chi_v(2)e^{i\vec{k}\cdot\vec{r}_1}\ , \tag{A1}$$

where we consider a typical single particle transition $i\to f$; the $\chi_{v,v'}$ represent the initial and final state spins of the projectile, $\hat{V}=-VP^{\sigma}P^{\tau}$ is the interaction appropriate for exchange, and isospin indices have been suppressed for brevity. The spin dependence of $\hat{V}$ is denoted schematically by $\vec{\sigma}$. It is convenient to change to relative and center-of-mass coordinates defined by

$$\vec{r}=\vec{r}_{12}=\vec{r}_1-\vec{r}_2\ ,\quad 2\vec{R}=\vec{r}_1+\vec{r}_2\ . \tag{A2}$$

If $\hat{V}$ is not *intrinsically* velocity dependent this readily gives

$$T=\int d\vec{R}\int d\vec{r}e^{i\vec{K}\cdot\vec{r}}e^{i\vec{q}\cdot\vec{R}}\psi_f^*(\vec{R}+\vec{r}/2)\hat{V}(\vec{r},\vec{\sigma})\times\psi_i(\vec{R}-\vec{r}/2)\ , \tag{A3}$$

where

$$\vec{K}=\frac{\vec{k}+\vec{k}'}{2},\quad \vec{q}=\vec{k}-\vec{k}' \tag{A4}$$

If the interaction is of short range we may use

$$\psi(\vec{R}\pm\vec{r}/2)\simeq\psi(\vec{R})\pm\frac{\vec{r}}{2}\cdot\nabla_R\psi(\vec{R}) \tag{A5}$$

to write $T\simeq T^{(0)}+T^{(1)}$ where

$$T^{(0)}=\int d\vec{R}\,\psi_f^*(R)\langle\chi_{v'}|e^{i\vec{q}\cdot\vec{R}}\tilde{V}(\vec{K},\vec{\sigma})|\chi_v\rangle\psi_i(\vec{R})\ , \tag{A6}$$

and

$$T^{(1)}=\tfrac{1}{2}\int d\vec{R}\,\psi_f^*(\vec{R})\langle\chi_{v'}|e^{i\vec{q}\cdot\vec{R}}[\overleftarrow{\nabla}_R\cdot\vec{u}(\vec{K},\vec{\sigma})-\vec{u}(\vec{K},\vec{\sigma})\cdot\vec{\nabla}_R]|\chi_v\rangle\psi_i(\vec{R})\ , \tag{A7}$$

where

$$\tilde{V}(\vec{K},\vec{\sigma})\equiv\int d\vec{r}e^{i\vec{K}\cdot\vec{r}}\hat{V}(\vec{r},\vec{\sigma})\ , \tag{A8}$$

$$\vec{u}(\vec{K},\vec{\sigma})\equiv\int d\vec{r}e^{i\vec{K}\cdot\vec{r}}\vec{r}\hat{V}(\vec{r},\vec{\sigma})=-i\vec{\nabla}_K\tilde{V}(\vec{K},\vec{\sigma})\ , \tag{A9}$$

and $\overleftarrow{\nabla}_R$ acts only on $\psi_f^*(R)$. Integrating the $\overleftarrow{\nabla}_R$ term by parts gives

$$T^{(1)}=\tfrac{1}{2}\int d\vec{R}\,\psi_f^*(R)\langle\chi_{v'}|e^{i\vec{q}\cdot\vec{R}}(-i\vec{q}-2\vec{\nabla}_R)\cdot\vec{u}(\vec{K},\vec{\sigma})|\chi_v\rangle\psi_i(\vec{R})\ . \tag{A10}$$

Using the anticommutator relationship

$$\{e^{i\vec{q}\cdot\vec{R}},\nabla_R\}=e^{i\vec{q}\cdot\vec{R}}(i\vec{q}+2\nabla_R) \tag{A11}$$

when acting on a wave function to the right, we obtain

$$T^{(1)}=-\tfrac{1}{2}\int d\vec{R}\,\psi_f^*(\vec{R})\times\langle\chi_{v'}|\{e^{i\vec{q}\cdot\vec{R}},\vec{u}(\vec{K},\vec{\sigma})\cdot\nabla_R\}|\chi_v\rangle\psi_i(\vec{R})\ . \tag{A12}$$

The generalizations of Eqs. (A6) and (A12) for $T^{(0)}$ and $T^{(1)}$ to include arbitrary configurations in the initial and final states are the following:

$$T_{FI}^{(0)}=\left\langle \chi_{\nu'}\Psi_F \left| \sum_i e^{i\vec{q}\cdot\vec{r}_i}\hat{V}(\vec{K};\vec{\sigma}_i,\vec{\sigma}_p)\right|\Psi_I\chi_\nu\right\rangle \quad \text{(A6')}$$

and

$$T_{FI}^{(1)}=-\tfrac{1}{2}\left\langle \chi_{\nu'}\Psi_F \left| \sum_i \{e^{i\vec{q}\cdot\vec{r}_i},\vec{u}(\vec{K},\vec{\sigma}_i,\vec{\sigma}_p)\cdot\vec{\nabla}_i\}\right|\Psi_I\chi_\nu\right\rangle . \quad \text{(A12')}$$

For the central part of the interaction

$$\hat{V}(\vec{K};\vec{\sigma}_i,\vec{\sigma}_p)=\hat{V}_0(K^2)+\hat{V}_1(K^2)\vec{\sigma}_i\cdot\vec{\sigma}_p \quad \text{(A13)}$$

and

$$\vec{u}(\vec{K},\vec{\sigma}_i,\vec{\sigma}_p)=-2i\vec{K}[\hat{V}'_0(K^2)+\hat{V}'_1(K^2)\vec{\sigma}_i\cdot\vec{\sigma}_p] ;$$

$$\hat{V}'_s\equiv\frac{\partial\hat{V}_s(K^2)}{\partial K^2} . \quad \text{(A14)}$$

If K^2 is further approximated by k^2 the term $T^{(0)}$ represents the standard short-range approximation described in Sec. III which samples only the static transition density; for the central part of the force the leading (monopole) term is essentially the knockon exchange contribution to $\vec{B}_S$ of Eq. (11). Several of the isovector parts of the operators appearing in Eqs. (A6') and (A12') have their counterparts in the weak vector and axial-vector currents responsible for β decay. For example, the spin-independent and spin-dependent parts of the nuclear operators in $T^{(0)}$ are proportional to the leading terms in the timelike component of the vector (Fermi) current and the spacelike component of the axial-vector (Gamow-Teller) current, respectively.[22] The correspondence between the operators responsible for $T^{(1)}$ and those present in β decay is less familiar. To make this connection we note that the dominant terms of the spacelike part of the vector current and the timelike component of the axial vector current are[22]

$$\vec{V}^{(\pm)}(\vec{r}_i)=-it_\pm(i)\left[\frac{g_v}{2m}\{e^{i\vec{q}\cdot\vec{r}_i},\nabla_i\} - \frac{g_m}{2m}e^{i\vec{q}\cdot\vec{r}_i}\vec{\sigma}_i\times\vec{q}\right] , \quad \text{(A15)}$$

$$A_0^{(\pm)}(\vec{r}_i)=\frac{ig_A}{2m}t_\pm(i)\{e^{i\vec{q}\cdot\vec{r}_i},\sigma_i\cdot\vec{\nabla}_i\} =\frac{-g_A}{2m}t_\pm(i)\{e^{i\vec{q}\cdot\vec{r}_i},\vec{\sigma}_i\cdot\vec{p}_i\} , \quad \text{(A16)}$$

where $t_\pm$ are the isospin raising and lowering operators, m is the average nucleon mass, and g_v and g_m are the usual[22] vector and weak magnetism form factors. [The sign change in $\vec{q}$ relative to Ref. 22 (Holstein) corresponds to definitions of momentum transfer $\vec{q}$ which are opposite in sign.] From Eqs. (A12') and (A14) we see that the spin-independent part of the operator responsible for $T^{(1)}$ may be readily associated with the g_v term in $\vec{V}(\vec{r}_i)$; the g_m (weak magnetism) part of $\vec{V}(\vec{r}_i)$ is a static transverse correction term of order q/m. The timelike component of the axial current A_0 corresponds most closely to the spin dependent part of $T^{(1)}$ where from Eq. (A12') and (A14) the operator acting on each nucleon is

$$\mathcal{O}_{S=1}^{(1)}(i)=-V'_1\{e^{i\vec{q}\cdot\vec{r}_i},\vec{K}\cdot\vec{p}_i\vec{\sigma}_p\cdot\vec{\sigma}_i\},\quad \vec{p}_i=-i\nabla_i . \quad \text{(A17)}$$

Although the couplings in Eqs. (A16) and (A17) are similar, they are not identical because of the additional recouplings of the form $[\vec{p}_i\otimes\vec{\sigma}_i]$,[1,2] implied by Eq. (A17). In the long wavelength (small momentum transfer) approximation where $e^{i\vec{q}\cdot\vec{r}}\sim 1+i\vec{q}\cdot\vec{r}$ a closer correspondence can be derived; for proton scattering we also neglect terms involving $\vec{q}\cdot\vec{K}\propto Q$ value. In this approximation, the parity changing and unchanging parts of $\mathcal{O}^{(1)}$ may be written as

$$\mathcal{O}^{(1)}\simeq-2V'_1\sum_{J=0,1,2}(-)^J(\vec{K}\otimes\vec{\sigma}_p)^J\cdot(\vec{p}_i\otimes\vec{\sigma}_i)^J,\quad \Delta\pi=\text{yes}$$

$$\simeq+2iV'_1\sum_J(-)^J\left\{\frac{i}{\sqrt{2}}[(\vec{q}\otimes\vec{K})^1\otimes\vec{\sigma}_p]^J\cdot(\vec{L}_i\otimes\vec{\sigma}_i)^J+[(\vec{q}\otimes\vec{K})^2\otimes\vec{\sigma}_p]^J\cdot[(\vec{r}_i\otimes\vec{p}_i)^2\otimes\vec{\sigma}_i]^J\right\},\quad \Delta\pi=\text{no} . \quad \text{(A18)}$$

For 1^+ excitations the relevant part of $\mathcal{O}^{(1)}$ is

$$\mathcal{O}^{(1)}(1^+)\rightarrow 2iV'_1\{-\tfrac{1}{2}[(\vec{q}\otimes\vec{K})^1\otimes\vec{\sigma}_p]^1\cdot(\vec{L}_i\times\vec{\sigma}_i)+[(\vec{q}\otimes\vec{K})^2\otimes\vec{\sigma}_p]^1\cdot[(\vec{r}_i\otimes\vec{p}_i)^2\otimes\vec{\sigma}_i]^1\} . \quad \text{(A19)}$$

An analogous small-q approximation for $A_0(\vec{r}_i)$ gives

$$A_0^{(\pm)}(\vec{r}_i)\simeq\frac{g_A}{2m}t_\pm(i)\left\{-2(\vec{\sigma}_i\cdot\vec{p}_i)-\frac{iq^2}{3}(\vec{\sigma}_i\cdot\vec{r}_i)-\sqrt{(2/3)}q^2(\vec{r}_i\otimes\vec{\sigma}_i)^2\cdot C_2(\hat{q})-\vec{q}\cdot[\vec{\sigma}_i+2i\vec{r}_i(\vec{\sigma}_i\cdot\vec{p}_i)]\right\} , \quad \text{(A20)}$$

where $C_2(\hat{q})$ is the normalized spherical harmonic,[35] the first three terms contribute to parity changing (0^- or 2^-) excitations (see Sec. V), and the term in square brackets contributes to positive parity excitations. Using Eqs. (21) and (22) of Horie and Sasaki[36] the term in square brackets may, for transitions within a single l shell, be rewritten as

$$\vec{\sigma}+2i\vec{r}(\vec{\sigma}\cdot\vec{p})=\{\vec{\sigma}-\sqrt{10}[C_2(\hat{r})\otimes\vec{\sigma}]^1\}\left[1+\tfrac{2}{3}r\frac{\partial}{\partial r}\right] -i\vec{L}\times\vec{\sigma} . \quad \text{(A21)}$$

If, in addition, the participating radial wave functions are equal the first term does not contribute due to a vanishing radial integral and we are left with

$$A_0^{(\pm)}(\vec{r}_i)=\frac{g_A}{2m}\vec{q}\cdot(i\vec{L}_i\times\vec{\sigma}_i)t_\pm(i) . \quad \text{(A22)}$$

Under these same conditions the rank-two part of Eq. (A19) does not contribute so that

$$\mathscr{O}^{(1)}(1^+)\rightarrow\frac{V_1'}{2}[(\vec{k}\times\vec{k}\,')\times\vec{\sigma}_p]\cdot(i\vec{L}_i\times\vec{\sigma}_i) . \quad \text{(A19')}$$

Recalling that $\hat{n}\sim\vec{k}\times\vec{k}\,'$ we see that $\mathscr{O}^{(1)}(1^+)$ in (A19′) is essentially T^{LS} of Eq. (17b) and in the nuclear subspace corresponds (in the small-q limit) to the timelike component of the axial current in β decay.

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