Impulse approximation Dirac theory of inelastic proton nucleus collective excitations

E. Rost, J. R. Shepard, and E. R. Siciliano
Nuclear Physics Laboratory, University of Colorado, Boulder, Colorado 80309

J. A. McNeil
Department of Physics, Villanova University, Villanova, Pennsylvania 19085
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The Dirac impulse approximation formulation of intermediate energy, proton nucleus scattering is extended to include the inelastic excitation of collective degrees of freedom. In this approach, the deformed spin-orbit interaction appears naturally and is found to have a large effect on the inelastic spin observables. The agreement with experimental data for the collective inelastic observables is comparable to that obtained for the corresponding elastic ones.

I. INTRODUCTION

It has recently been shown[1] that intermediate-energy proton elastic scattering is well described in a Dirac optical potential framework where the interaction strengths are obtained via the impulse approximation from NN amplitudes.[2] Especially impressive agreement has been found for the spin-dependent observables.[3]

In this paper we extend the Dirac impulse approximation framework to include the inelastic excitation of collective degrees of freedom. In this extension the nuclear density (*not* the potential) is generalized and the folded first-order collective term becomes the transition density. An advantage of such an approach is that the effective spin-orbit piece of the transition potential generated from the NN amplitudes is treated on the same footing as the effective central interaction, thereby eliminating the explicit calculation of a deformed spin-orbit interaction.

Previous works have emphasized the close connection (data-to-data relations) between elastic and collective inelastic scattering.[4] Recently the connection has been shown to persist in Dirac eikonal approaches to collective excitation.[5] Given the excellent agreement with elastic scattering, good fits to inelastic scattering are thus to be expected and indeed are found. However, full partial-wave calculations are needed to extract collective strengths and as a first step in extending the calculations to regions where the data-to-data formulae or the eikonal approximation break down. One obvious application will be to the excitation of unnatural parity states, where detailed microscopic nuclear properties can be studied. Such a microscopic formulation of inelastic scattering in the Dirac impulse approximation is already underway.

In the next section we derive the expressions for the scattering observables in the Dirac impulse approximation collective model. In the following section the formulation is compared to the standard nonrelativistic one and the deformed spin-orbit interactions arising in the two approaches are compared. In Sec. IV comparison to the published data for the 800 MeV ^{54}Fe(p,p′) reaction is made.

II. GENERAL FORMULATION

We begin with the prescription for the elastic optical potential of Ref. 2 which is

$$\langle 0 \mid U(r) \mid 0 \rangle = -\frac{4\pi i k}{m} \int d\vec{r}\,'[\, F_{Sp}(\vec{r}-\vec{r}\,')\rho_{Sp}(\vec{r}\,')+\gamma_0 F_{Vp}(\vec{r}-\vec{r}\,')\rho_{Vp}(\vec{r}\,') + F_{Sn}(\vec{r}-\vec{r}\,')\rho_{Sn}(\vec{r}\,')+\gamma_0 F_{Vn}(\vec{r}-\vec{r}\,')\rho_{Vn}(\vec{r}\,')\,] \,, \tag{1}$$

in units of $\hbar=c=1$ where S (V) denotes scalar (vector) and p (n) denotes proton (neutron). Here k is the laboratory momentum of the proton and γ_0 denotes the 0th component of the Dirac γ matrices. The NN invariant amplitudes F are obtained in the impulse approximation from the free NN interactions as determined by phase shift analysis[6] of NN data. (We have ignored the very small tensor contributions to the optical potential.) In practice the amplitudes are parametrized in the following momentum-space form by a sum of Yukawa forms,

$$\tilde{F}_i(q)=\sum_j A_{ij}(1+q^2/\mu_{ij}^2)^{-1}, \quad i=Sp,Vp,Sn,Vn \,, \tag{2}$$

so that we have in configuration space

$$F_i(s)=(4\pi)^{-1}\sum_j \mu_{ij}^3 A_{ij}\exp(-\mu_{ij}s)/(\mu_{ij}s) \,. \tag{3}$$

The choice of Yukawa forms in the development of amplitudes is very convenient in carrying out the folding in Eq. (1). Thus for a given term

$$\int d\vec{r}' F_i(\vec{r}-\vec{r}')\rho_i(\vec{r}')=\sum_j \mu_{ij}A_{ij}\left[\exp(-\mu_{ij}r)\int_0^r dr'\rho(r')\sinh(\mu_{ij}r')+\sinh(\mu_{ij}r)\int_r^\infty dr'\rho(r')\exp(-\mu_{ij}r')\right] . \quad (4)$$

Values for A_{ij} and μ_{ij} were obtained by performing a least-squares fit to the forward amplitudes ($\theta<60°$). Five values of j were sufficient to give excellent fits and the parameters are given in Table I for 800 MeV proton scattering.

In order to introduce collective degrees of freedom we write Eq. (1) in schematic form

$$U(r,R)=t\rho(r,R) , \quad (5)$$

where t is independent of R and sums are suppressed. The nuclear radius is now generalized,

$$R(\theta')=R_0\left[1+\sum_\lambda \beta_\lambda Y_0^\lambda(\theta')\right] , \quad (6)$$

to first order in the deformations β_λ where θ' is measured from body-fixed axes. Expanding the density about $R=R_0$ yields

$$\rho(r,R)=\rho(r,R_0)+R_0\sum\beta_\lambda Y_0^\lambda(\theta')\left[\frac{\partial\rho}{\partial R}\right]_{R=R_0} \equiv\rho_0+\Delta\rho , \quad (7)$$

which may be rotated to space-fixed axes

$$\Delta\rho=R_0\sum_{\lambda M}\beta_\lambda D^{\lambda*}_{M0}(\alpha\beta\gamma)\left[\frac{\partial\rho}{\partial R}\right]_{R=R_0}Y_M^{\lambda*}(\hat{r}) , \quad (8)$$

where $(\alpha\beta\gamma)$ denote the Euler angles of the nuclear principle axes relative to a space-fixed frame and which represent the nuclear coordinates in a rotational model. In this model the initial and final nuclear states in the ground-state band are described in terms of $D^{J*}_{M0}(\alpha\beta\gamma)$. Integrating over the nuclear coordinates then forces $\lambda=J$ and gives for the transition density

$$\langle J\mid\Delta\rho\mid 0\rangle=\sum_M R_0\beta_J(2J+1)^{-1/2}\left[\frac{\partial\rho}{\partial R}\right]_{R=R_0}Y_M^{J*}(\hat{r}) . \quad (9)$$

Ignoring noncentral parts of the NN interaction we can perform the "$t\rho$" folding which yields schematically for an off-diagonal matrix element

$$\langle J\mid U(r,R)\mid 0\rangle=G_J(r)\sum_M Y_M^{J*}(\hat{r}) , \quad (10)$$

where the form factor $G_J(r)$ is

$$G_J(r)=-\frac{\beta_J R_0}{(2J+1)^{1/2}}\frac{4\pi ik}{m}\sum_i\int d\vec{r}' F_i(\vec{r}-\vec{r}')\times\frac{\partial}{\partial R}\rho_i(r',R) . \quad (11)$$

Equation (11) has assumed equal deformations for scalar, vector, proton, and neutron parts for notational convenience. This restriction is easily removed. Referring to Eq. (1) the structure of $G(r)$ is evidently

$$G(r)=G_S(R)+\gamma_0 G_V(r) . \quad (12)$$

Distorted waves are generated from the Dirac equation using the elastic (or "diagonal") optical potential. These are solved by making a partial wave expansion employing standard techniques.[7] For outgoing boundary conditions, the Dirac distorted wave is written as a four-component column vector

$$\langle\vec{r}\mid\Psi^{(+)}_{\vec{k}s}\rangle=\begin{bmatrix}u_{\vec{k}}(\vec{r})\chi_s^{1/2}\\ w_{\vec{k}}(\vec{r})\chi_s^{1/2}\end{bmatrix} , \quad (13)$$

where χ_s is a Pauli spinor of projection s. The DWIA amplitude is thus

TABLE I. Relativistic amplitudes [$(\mathrm{GeV}/c)^{-2}$] at $T_{\mathrm{lab}}=800$ MeV.

$$F_{ij}(q)=A_{ij}\left[1+\frac{q^2}{\mu_{ij}^2}\right]^{-1}$$

i		$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
A_{Sp}	Re	0.11	−0.89	−0.75	−0.52	−0.38
	Im	1.35	−4.15	−9.95	−2.52	4.43
A_{Vp}	Re	−0.34	2.20	1.64	0.56	−0.14
	Im	−0.68	0.01	5.54	2.40	−1.34
A_{Sn}	Re	0.18	−2.42	−1.38	−0.37	0.16
	Im	1.56	−3.78	−11.65	−2.48	6.22
A_{Vn}	Re	−0.26	2.86	1.61	0.35	−0.34
	Im	−0.84	1.12	6.57	2.07	−2.65
μ_{ij} (GeV)		0.125	0.350	0.550	0.900	1.500

$$\langle \vec{k}'s' \mid T_{J0} \mid \vec{k}s \rangle = \langle \psi^{(-)}_{\vec{k}'s'} \mid \gamma_0 \langle J \mid U \mid 0 \rangle \mid \psi^{(+)}_{\vec{k}s} \rangle$$
$$= \sum_M \{ \langle u_{\vec{k}'s'}(\vec{r})\chi^{1/2}_{s'} \mid [G_V(r)+G_S(r)]Y^{J*}_M(\hat{r}) \mid u_{\vec{k}s}(\vec{r})\chi^{1/2}_s \rangle$$
$$+ \langle w_{\vec{k}'s'}(\vec{r})\chi^{1/2}_{s'} \mid [G_V(r)-G_S(r)]Y^{J*}_M(\hat{r}) \mid w_{\vec{k}s}(\vec{r})\chi^{1/2}_s \rangle \}$$
$$= T_1 + T_2 \, . \tag{14}$$

It is seen that T_1 corresponds to the usual nonrelativistic DWIA term involving upper components and roughly the derivative of the *central* potential (sum of vector and scalar). The second term can be related to a *spin-orbit* interaction term since it originates with the small components of the projectile and its strength is the difference of the vector and scalar terms. Using

$$w_{\vec{k}s}(\vec{r})\chi^{1/2}_s = -\frac{i\vec{\sigma}\cdot\vec{\nabla}u_{\vec{k}s}(\vec{r})\chi^{1/2}_s}{E+m+V_s(r)-V_V(r)} \tag{15}$$

from the Dirac equation and an eikonal approximation it is possible to write T_2 in terms of upper components where similarities in form to the usual spin-orbit interaction are apparent.[5] However, in the present case there is an altered radial dependence from the conventional $(1/r)(d/dr)V_{so}(r)$ used in a Schrödinger equation approach which amounts to an enhancement in the region of the nuclear interior.

In the above formulation the nuclear properties are all contained in the four densities $\rho_i(r)$. Previous calculations[1,3] of Dirac impulse approximation elastic scattering have used Hartree-Fock[8] or other bound state models to generate the densities for detailed comparison to data.

These procedures are quite cumbersome, however, and we wish to use a simpler prescription which retains most of the important physics of the more fundamental approaches. Our prescription utilizes phenomenological electron scattering charge densities $\rho_{ch}(r)$ which fix the proton vector densities. We assume that these densities are parametrized as three-parameter Fermi functions, i.e.,

$$\rho_{ch}(r) = \frac{1+\alpha_{ch}\left[\frac{r}{R_{ch}}\right]^2}{1+\exp\left[\frac{(r-R_{ch})}{a_{ch}}\right]} \, . \tag{16}$$

We then unfold the proton charge form factor from $\rho_{ch}(r)$ by replacing the phenomenological diffuseness value in Eq. (16) by

$$a_{pt} = [a^2_{ch} - (0.215)^2]^{1/2} \, .$$

This gives the proton vector point density, $\rho_{Vp}(r)$.

The scalar densities are related to the vector densities in a microscopic approach by

$$\rho_S(r) = \rho_V(r) - 2\sum_k |\omega_k(\vec{r})|^2 \, , \tag{17}$$

where ω_k denotes the lower-component, bound-state wave function for the kth filled orbital. Typical bound state lower components have normalizations of about 2.5% using bound state potentials comparable with those for the real part of the low-energy, p-nucleus Dirac optical potential. We ignore any geometry changes and set $\rho_{Sp} = 0.95\rho_{Vp}$. Finally the neutron-proton geometry differences are ignored giving $\rho_{Sn}=\rho_{Sp}$ and $\rho_{Vn}=\rho_{Vp}$. In this manner we can calculate the elastic scattering for any target without adjustable parameters and inelastic scattering with a single strength parameter β_J.

The structure of the distorted-wave calculations is basically similar to standard nonrelativistic DWBA calculations with spin-orbit effects.[9] However, there are some differences in detail which are given in the Appendix.

III. DEFORMED SPIN-ORBIT INTERACTION

In the previous section we have shown that the DWIA collective transition amplitude is composed of two terms, T_1 and T_2, which are related to the Schrödinger theory central and spin-orbit interaction, respectively. For a treatment comparable to that developed in Sec. II, one would write the transition terms as[10]

$$\Delta U^c = \beta^c R \frac{\partial U^c}{\partial R} \, , \tag{18}$$

$$\Delta U^{so} = \beta^{so}\vec{\sigma}\cdot\vec{\nabla}\left[R(\theta')\frac{\partial}{\partial R}U^{so}\right]\frac{1}{i}\vec{\nabla} \, , \tag{19}$$

with central and spin-orbit deformation parameters β^c and β^{so}. In some low-energy applications,[11] fits to analyzing power data required $\beta^{so}\simeq 1.5\beta^c$, although $\beta^{so}\approx\beta^c$ seemed to be adequate for 800 MeV (p,p') data.[12] In the Dirac approach one would employ scalar and vector deformation parameters in the form factor of Eq. (11), although Eq. (17) suggests that their difference should be small. However, the enhancement of the nuclear interior in the spin-orbit interaction, which propagates to the T_2 transition term [cf. Eq. (15) and the discussion following], provides natural enhancement over the central T_1 term.

In Fig. 1 we show calculations for the 800 MeV ^{54}Fe(p,p')^{54}Fe* reaction exciting 2^+ and 3^- levels.[13] Calculations using both T_1 and T_2 terms are compared with those using only the central T_1 term. The deformed spin-orbit piece is seen to be substantial in its effect on the inelastic analyzing powers. This feature has been observed previously in the work of Liljestrand *et al.*[12] using a Schrödinger-based approach. The Dirac-based approach in Fig. 1 accentuates this difference probably because of the interior spin-orbit enhancement to which we have often referred.

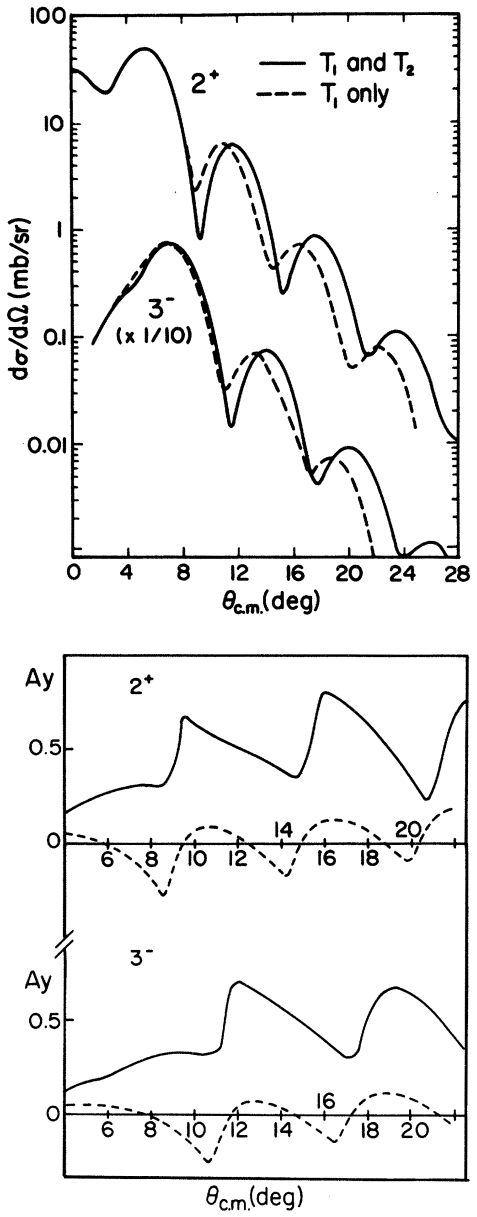

FIG. 1. Dirac impulse approximation calculations for the excitation of the 2^+ (1.408) and the 3^- (4.872) levels of ^{54}Fe by 800 MeV protons. The dashed curves are calculated with only the T_1 term in Eq. (14) which is analogous to omitting the deformed spin-orbit interaction term.

The deformed spin-orbit term in Eq. (19) is actually quite cumbersome to apply directly and an approximation (the Oak Ridge term[13]) is often used for calculational convenience. In our formulation no calculational approximations of this type are required. Thus one result of our work is to suggest a simpler and more unified treatment of the deformed spin-orbit interaction for collective (p,p') transitions.

IV. COMPARISON TO DATA

It is not our goal in this paper to perform detailed analyses of experimental data although such analyses will be interesting once the Los Alamos Meson Physics Facility

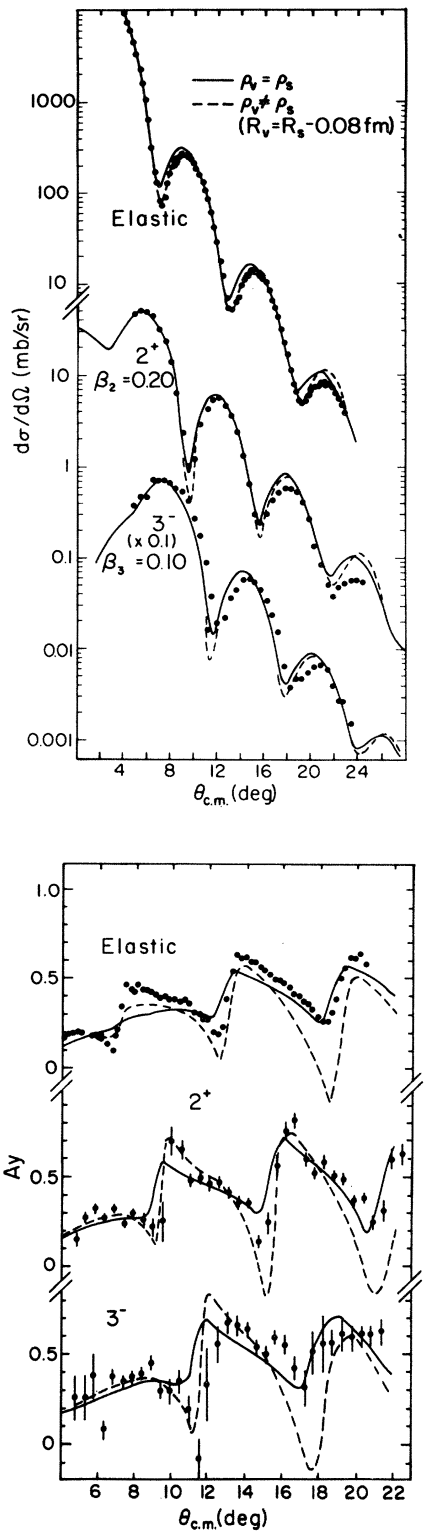

FIG. 2. Experimental and theoretical cross sections and analyzing powers for the excitation of the ground, the 2^+ (1.408), and the 3^- (4.872) states of ^{54}Fe by the ^{54}Fe(p,p')^{54}Fe* reaction at 800 MeV. The dashed curves were obtained by changing the scalar and vector densities as discussed in the text. The data are taken from Ref. 14.

(LAMPF) $(\vec{p},p')$ data are available. We consider here only the $^{54}Fe(\vec{p},p')^{54}Fe^*$ reaction for which data have been published.[14] In Fig. 2 we compare the cross sections and analyzing powers with calculations using as input only the electron scattering densities and free NN amplitudes as outlined above. It is seen that excellent agreement is obtained with the cross sections and fair agreement with the inelastic analyzing powers. In detail, however, our analyzing power calculations do not have sufficient structure at forward angles, especially for the elastic scattering. It seems likely that this discrepancy is due to the approximate description of the nuclear densities. Indeed, for elastic scattering, slight differences among the geometries are known[1] to have appreciable influence on the calculated spin observables.

To illustrate this, the dashed curves in Fig. 2 were calculated by arbitrarily adding 0.04 fm to the scalar radius and subtracting 0.04 fm from the vector radius. As discussed in Ref. 1 such small changes are needed to fit the sharp details of the elastic scattering spin observables and this also appears to be the case for the inelastic transitions.

Overall, we observe agreement with data which is comparable for both elastic and inelastic transitions. The single extracted strength parameters, β_2 and β_3, agree well with other measurements[12,15] of these quantities.

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APPENDIX

The proton-nucleus scattering is described by the Dirac equation which includes a scalar and vector potential, $V_S(r)$ and $V_V(r)$, respectively. Since the potential is central we expand the four-component wave function in partial waves

$$\begin{bmatrix} u_{\vec{k}}(\vec{r})\chi_s^{1/2} \\ w_{\vec{k}}(\vec{r})\chi_s^{1/2} \end{bmatrix} = 4\pi \left[\frac{E+m}{2m}\right]^{1/2} \frac{1}{r} \sum_{\kappa\mu} (l\tfrac{1}{2}\,\mu-s\;s;j\mu)\exp(i\delta_\kappa^C) i^l Y^{l*}_{\mu-s}(\hat{k}) \begin{Bmatrix} g_\kappa(r)[Y^l(\hat{r})\chi^{1/2}]^j_\mu \\ if_\kappa(r)[Y^{\tilde{l}}(\hat{r})\chi^{1/2}]^j_\mu \end{Bmatrix}, \tag{A1}$$

where κ is the Dirac kappa label which specifies l and j

$$\kappa = \begin{cases} -l-1 \\ l \end{cases} \text{ for } j = \begin{cases} l+\frac{1}{2} \\ l-\frac{1}{2} \end{cases} \tag{A2}$$

and $\tilde{l}$ is the "other" l with the same j. The normalization is chosen to asymptotically match to a plane wave (in the absence of Coulomb effects)

$$\langle \vec{r} \mid \Psi^{(+)}_{\vec{k}s} \rangle \to \left[\frac{E+m}{2m}\right]^{1/2} \begin{bmatrix} \chi_s^{1/2} \\ \dfrac{\vec{\sigma}\cdot\vec{k}}{E+m}\chi_s^{1/2} \end{bmatrix} e^{i\vec{k}\cdot\vec{r}}. \tag{A3}$$

Substituting Eq. (A1) into the Dirac equation yields coupled first-order radial equations for g_κ and f_κ or a second-order equation in g_κ (or f_κ). Using standard techniques the $\vec{r}\cdot\vec{\nabla}$ or Darwin term may be removed by a transformation yielding the equation

$$\left\{\frac{1}{2E}\left[-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}-k^2\right]+V_{\rm eff}(r)-(1+\kappa)V_{\rm so}(r)\right\}B^{-1/2}(r)g_\kappa(r)=0, \tag{A4}$$

where

$$V_{\rm eff}(r)=\frac{1}{2E}\left\{(V_S+V_V)(V_S-V_V+E+m)+(V_S-V_V)(-E+m)+\left[-\frac{1}{r}\frac{B'}{B}-\frac{1}{2}\frac{B''}{B}+\frac{3}{4}\left[\frac{B'}{B}\right]^2\right]\right\},$$

$$V_{\rm so}(r)=-\frac{1}{2Er}\frac{B'}{B}, \tag{A5}$$

and

$$B(r)=1+\frac{V_S-V_V}{E+m}.$$

We note, that for large r, $B(r)\to 1$ so that except for algebraic unpleasantness Eq. (A4) is of the same structure as the radial Schrödinger equation. This similarity is even greater if one ignores the quadratic Coulomb part of $V_{\rm eff}(r)$ and the Coulomb contributions to $B(r)$. With these approximations the asymptotic Coulomb functions take on the simpler nonrelativistic forms which are far more convenient to handle numerically and δ_κ^C in Eq. (A1) becomes the Coulomb phase σ_l of Ref. 17. Tests of these approximations for intermediate-energy proton scattering show them to be very accurate. Equation (A4) can now be solved numerically with a Numerov integration method[16]

to integrate to a matching radius R_M where V_S and V_V are negligible so that

$$A_\kappa g_\kappa(r)=\{F_l(kr)+C_\kappa[G_l(kr)+iF_l(kr)]\},\quad r\geq R_M \tag{A6}$$

and from which the normalizations A_κ and scattering amplitudes

$$C_\kappa=(2i)^{-1}[\exp(2i\delta_\kappa)-1]$$

are determined.

It is possible to obtain the lower component functions $f_\kappa(r)$ from $g_\kappa(r)$ by using

$$f_\kappa(r)=(E+m+V_S-V_V)^{-1}\left[\frac{d}{dr}g_\kappa+\frac{\kappa}{r}g_\kappa\right]. \tag{A7}$$

However, the functions g_κ and f_κ oscillate rapidly for the energies of interest and thus Eq. (A7) is numerically inconvenient to apply directly. We employ the procedure of using the equivalent of Eq. (A4) for lower components

$$\left\{\frac{1}{2E}\left[-\frac{d^2}{dr^2}+\frac{\tilde{l}(\tilde{l}+1)}{r^2}-k^2\right]+\tilde{V}_{\rm eff}(r)-\kappa\tilde{V}_{\rm so}(r)\right\}\tilde{B}^{-1/2}(r)f_\kappa(r)=0\,, \tag{A8}$$

where $\tilde{V}_{\rm eff},\tilde{V}_{\rm so}$ are functions of $\tilde{B}$ instead of B and

$$\tilde{B}(r)=1-\frac{V_S+V_V}{E-m}\,. \tag{A9}$$

Using Eq. (A7) asymptotically and Coulomb function recursion relations, we can readily determine the appropriate lower component matching conditions, which are

$$\tilde{A}_\kappa g_\kappa(r)=\{\tilde{F}_l(kr)+C_\kappa[\tilde{G}_l(kr)+i\tilde{F}_l(kr)]\},\quad r\geq R_M \tag{A10}$$

with

$$\tilde{F}_l=\frac{k}{E+m}\times\begin{cases}\dfrac{\eta}{l+1}F_l-\left[1+\left[\dfrac{\eta}{l+1}\right]^2\right]^{1/2}F_{l+1}\\[2ex] -\dfrac{\eta}{l}F_l+\left[1+\left[\dfrac{\eta}{l}\right]^2\right]^{1/2}F_{l-1}\end{cases},\quad j=\begin{cases}l+\frac{1}{2}\\ l-\frac{1}{2}\end{cases} \tag{A11}$$

where $\eta=(Ze^2E)/k$ is the Coulomb parameter. The same equation holds as well for $\tilde{G}_l$ in terms of $G_l,G_{l\pm1}$. The scattering amplitudes C_κ in Eq. (A10) must be the same as those obtained in Eq. (A6), which provides a check on the numerical accuracy.

A minor detail concerns the phases for the "incoming" final-state distorted wave in the transition matrix element. These can be straightforwardly determined using a Green's function approach[18] and yield the l'-dependent phases $i^{-l'}\exp(i\sigma_{l'})$ for the final incoming distorted wave.

Following standard references,[19] the differential cross section is given by

$$\frac{d\sigma}{d\Omega}=\left[\frac{m}{2\pi}\right]^2\frac{k_f}{k_i}\,|T_{fi}|^2\,, \tag{A12}$$

where T_{fi} is the T-matrix element with distorted waves normalized as in Eq. (A1). This differs from the Schrödinger normalization by the factor

$$(E_i+m)(E_f+m)/(4\mu_i\mu_f)\,,$$

where μ_i (μ_f) is the generalized reduced mass for the incident (final) channel [usually taken to be the reduced energy $E_1E_2/(E_1+E_2)$]. For the intermediate energy cases considered in this paper, the factor is close to unity.

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