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^{20}Ne states observed via $^{16}O(\alpha,\alpha_0)^{16}O$ and $^{16}O(\alpha,\alpha_1)^{16}O$

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Resonant energies, widths, strengths, spins, and parities are deduced for a total of 56 compound nuclear levels in ^{20}Ne ($16.4 \leq E_x \leq 21.1$ MeV) using $\alpha+^{16}$O elastic and inelastic scattering data. Writing the reaction amplitude for spinless positive-parity particles as a nonresonant term plus a sum over resonant partial waves permitted fitting the data at up to 20 angles. The procedure works effectively on regions at least up to 1.4 MeV wide and containing up to 15 resonances. Forty-one levels appear in the elastic scattering channel, while 11 are visible in the inelastic scattering to the first excited state in ^{16}O ($E_x = 6.05$ MeV). Four levels appear in both channels. Sixteen of these levels have not been reported previously.

I. INTRODUCTION

The formation of resonances in the compound nucleus ^{20}Ne has generated many investigations using different projectile-target combinations. One of the more widely employed reactions is the scattering of alpha particles by ^{16}O. The work by Mehta, Hunt, and Davis[1] covered the range $12.7 \leq E_x(^{20}\text{Ne}) \leq 20.0$ MeV and they identified 35 levels in this region. Several years later Bergman and Hobbie[2] reported 18 levels in the range $20 \leq E_x \leq 29$ MeV. Using fine energy steps and a new computer program to help identify compound nuclear states, Häusser *et al.*[3] reported 14 levels in the range $16.0 \leq E_x \leq 18.5$ MeV. In 1979, Billen[4] investigated the energy region $14.6 \leq E_x \leq 20.4$ MeV in 8 keV steps. Having simultaneously collected excitation functions at 18 angles and using a program similar to the one used by Häusser *et al.*, he was able to identify 25 levels in ^{20}Ne. However, because of computer limitations he could only analyze energy regions around relatively narrow resonances. With an improved fitting procedure and a larger, faster computer, I have reanalyzed Billen's data using large energy regions which contain many resonances. The use of large fitting regions is important because doing so restricts the amount of fluctuations which the background term can introduce into the results. The modified fitting function more realistically reflects the resonant parameters.

Another method to observe continuum nuclear states uses an excited ^{20}Ne nucleus as the final product in a reaction and observes the decay of that excited state. Young *et al.*[5] and Hindi *et al.*[6] employed the $^{12}C(^{12}C,\alpha)^{20}Ne^*(\alpha)^{16}O$ reaction and identified over 20 levels between 12.1 and 23.4 MeV. Sanders, Martz, and Parker[7] reported 10 levels using the $^{16}O(^{12}C,^{8}Be)^{20}Ne^*(\alpha)^{16}O$ reaction over the range $7.1 \leq E_x \leq 22.9$ MeV. Both Fou *et al.*[8] and Artemov *et al.*[9,10] observe several resonances over the energy ranges 16–18 and 19–23 MeV, respectively, via the $^{16}O(^{6}Li,d)^{20}Ne^*(\alpha)^{16}O$ reaction.

Section II describes the fitting function, its improvements, and the error analysis. Section III presents the results of the analysis.

II. ANALYSIS

The fitting procedure used here has been successfully employed by a number of other investigators.[3,4,11] For a system of spinless positive-parity particles, it expresses the reaction amplitude as a nonresonant term plus a sum over only resonant partial waves. The present version of the fitting routine is the result of a series of modifications to the program PSA described by Billen.[4,12] Reference 13 details the various modifications, but a few are worth mentioning here. Transferrring the program to a VAX 11/780 computer increased the computing speed significantly compared to the Honeywell DDP-124 computer. It also allowed a larger set of data to be analyzed within one fit (and a corresponding increase in the number of resonances which could be included in a fit). Billen's use of the Honeywell computer also dictated analyzing smaller regions of data than I use.

A. Fitting routine

The fitting function used in the program for the differential cross section for the scattering of spin zero particles (where $J=l$) as a function of the center-of-mass scattering angle θ and laboratory bombarding energy E is

$$\frac{d\sigma}{d\Omega}=\frac{1}{k^2}\left|\rho(\theta,E)e^{i\chi(\theta,E)}+\frac{i}{2}\sum_m(2J_m+1)\left[\frac{\Gamma_\alpha}{\Gamma}\right]_m e^{2i\phi_m(\theta)}(e^{2i\beta_m(E)}-1)P_{J_m}(\cos\theta)\right|^2 , \tag{1}$$

where the summation is over all resonances m, and

$$\rho(\theta,E)=\rho_0(\theta)+(E-E_A)\rho_1(\theta) ,$$

$$\chi(\theta,E)=\frac{E-E_A}{E_B-E_A}4\tan^{-1}\chi_1(\theta) ,$$

$$\beta_m(E)=\tan^{-1}(\tfrac{1}{2}\Gamma_m/(E_m-E)) .$$

The background (nonresonant) amplitude $\rho(\theta,E)$ varies linearly with energy at each angle. The energy dependence of the Coulomb amplitude and tails of resonances that are not explicitly included in the fit are lumped into this term's energy dependence. The background phase $\chi(\theta,E)$ is bounded and can change by no more than 2π over the fitting region. (The lowest and highest energies included in the fitting region are E_A and E_B, respectively.) This constraint prevents the background term from mimicking more than one resonantlike oscillation across the fitting region.

The spin J_m is also the angular momentum l of the resonant partial wave, E_m is the resonant energy, and Γ_m is the width of resonance m. The resonant strength $(\Gamma_\alpha/\Gamma)_m$ for resonance m in channel α is treated as an independent parameter. [For exit channels β other than elastic scattering, the strength is the combination $(\Gamma_\alpha\Gamma_\beta)^{1/2}/\Gamma$.] The angular dependence comes from the Legendre polynomial, $P_l(\cos\theta)$. The phase of resonance m with respect to the background term is $\phi_m(\theta)$.

To keep the function well behaved, the program scales the energies and widths so that all of the adjustable parameters are of order unity during the fitting procedure. Billen[12] observed that such a scaling of the parameters yielded a more stable fit.

A test of several fits indicated that including an explicit Coulomb term is unnecessary in the energy range of this data ($16.5\leq E_x\leq 21.0$ MeV). The small change in the Coulomb amplitude over the fitting regions considered here can be easily incorporated in the linear energy dependence of the background term.

The user assigns trial spins for the resonances and provides initial estimates of the resonant parameters. If the resonance has been reported previously, then the literature usually provides good starting values. If the resonance is new, inspection of the data using the following two criteria often determines the spin: First, there must not be any resonant structure at angles that are zeros of $P_J(\cos\theta)$. Second, there should be some resonant structure at angles that are near maxima of P_J; however, at a given angle, it is possible for nearby resonances to interfere with the given resonance to wash out the structure. This situation is not likely to occur at more than a few angles. Occasionally, several spins will have to be tried in the program to determine the correct one. Starting values for the resonant energy and width can be obtained by using the graphical methods described by Seitz,[14] but frequently, inspection of the data will yield values which are good enough for starting parameters. Large regions are built by adding one resonance at a time and slowly including more data on the ends of a smaller region.

Instead of adjusting all the parameters simultaneously, the program varies the nonresonant parameters (ρ_0, ρ_1, χ_1, and ϕ_m) at each angle while holding the resonant parameters fixed. Then it varies the resonant parameters using all the data. These two steps are repeated until either the percent reduction in chi-square is less than a preset value or the maximum number of iterations is reached. After looking at a plot of the fit to the data, the user decides what steps need to be taken next.

A problem arises when a resonance is near the edge of the fitting region. If a resonance is less than several widths from the edge, the resonant parameters will have an added uncertainty. Consider the following: The amplitude of a resonant term varies with energy through the factor $[\exp(2i\beta_m)-1]$. At $E=E_{\rm res}\pm\Gamma$, this factor is 0.45 its magnitude at $E=E_{\rm res}$, and at $E=E_{\rm res}\pm2\Gamma$, it is still about 0.25. Therefore, the resonance contributes a relatively large amplitude even at $E=E_{\rm res}\pm2\Gamma$. Hence, to obtain accurate resonant parameters, the fitting region should include data distributed within at least $\pm2\Gamma$ of the resonant energy.

Figure 1 illustrates this point with a relatively isolated

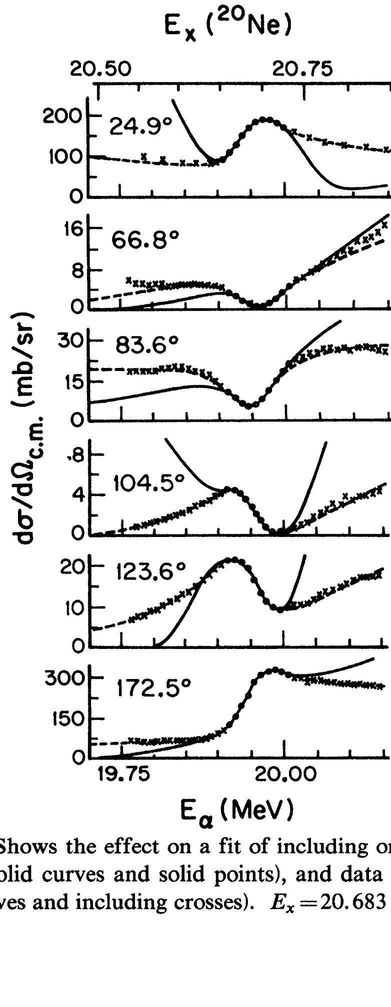

FIG. 1. Shows the effect on a fit of including only data with $E_x\pm\Gamma/2$ (solid curves and solid points), and data with $E_x\pm2\Gamma$ (dashed curves and including crosses). $E_x=20.683$ MeV.

resonance (the 9^- state at $E_x=20.68$ MeV). The solid line is the fit to the data (circles) within $\Gamma/2$ of the resonant energy. The dashed line is the fit to all the data (circles and crosses) within 2Γ of the resonant energy. Both fits started with resonant parameters relatively far from the values obtained when the resonance is included in a large region with other resonances. Even though the fit to the large region has the poorer overall χ^2 per degree of freedom, its values are closer to the results obtained when nearby resonances are included, and it does a better job of fitting the data within at least one width of the resonance.

B. Improvements to the routine

The function in the old program PSA (Refs. 4 and 12) had too many free parameters. One change to the fitting function removed the energy dependence of each resonance's nonresonant phase ϕ_m and placed a single energy dependence in the phase of the background term χ. This eliminated $n\times(m-1)$ parameters from the fit, where n is the number of angles and m is the number of resonances. We can see the effect of this change in the following example.

If we write the cross section of Eq. (1) as $d\sigma/d\Omega=|f(\theta,E)|^2$ and consider a (relatively) isolated resonance, a plot of $f(\theta,E)$ for a particular angle might look like Fig. 2(a). Here, f_b is the amplitude from all terms except the resonance of interest, and for this discussion we assume that it does not change over the range $E_{\rm res}\pm2\Gamma$. The circle represents the locus of points for the resonant amplitude. The angle between f_b and the tangent to the circle is $2\phi-\omega$. The dots along the circle represent the value of $f(\theta,E)$ in steps of $\Delta E=\Gamma/2$. The vectors for $f(\theta,E)$ are shown explicitly for $E=E_{\rm res}$ and $E=E_{\rm res}+\Gamma/2$. Note that the values at $E_{\rm res}\pm\Gamma/2$ are exactly 90° either side of the value at $E=E_{\rm res}$, and the radius of the circle is proportional to the resonant strength.

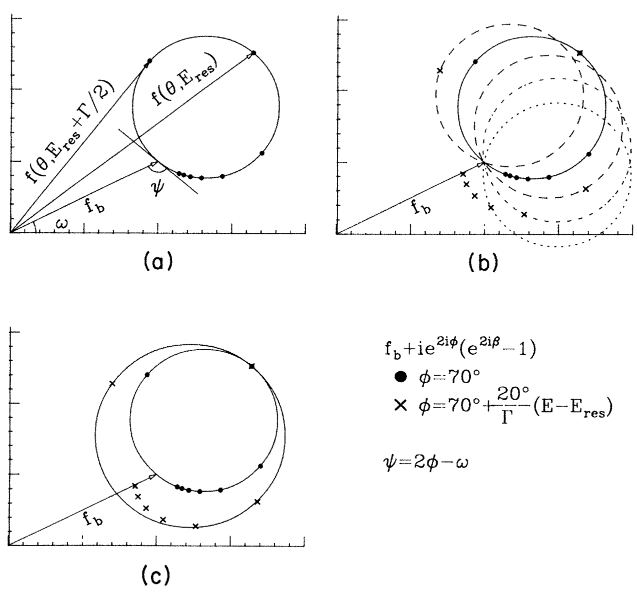

FIG. 2. (a) shows the background amplitude f_b and the resonant circle of a (relatively) isolated state. The resultant amplitude $f(\theta,E)$ corresponds to the points around the circle which are at energy intervals of $\Gamma/2$. Allowing the nonresonant phase $\phi=(\psi+\omega)/2$ to vary with energy rotates the circle about the end of f_b, as shown in (b). Thus, the resultant amplitude $f(\theta,E)$ is the series of crosses instead of dots and hence the resonant parameters describe the large circle in (c) instead of the smaller circle. The result is an incorrect width and resonant strength.

If we now include an energy dependence to ϕ, the resonant circle will rotate about the end of the vector f_b, as shown by the series of circles in Fig. 2(b). The points in Fig. 2(a) become the crosses shown in Fig. 2(b). In this example, the angle 2ϕ increases by 20° for every $\Gamma/2$ increase in the energy.

Now the crosses nearly form their own circle. Figure 2(c) depicts this new circle along with the original circle from Fig. 2(a). The points within $3\Gamma/2$ of $E_{\rm res}$ all lie on the large circle. (As the energy differs more and more from the resonant energy, nearby resonances will produce changes in f_b, so we should not be too concerned about the points greater than $\sim2\Gamma$ away from $E_{\rm res}$.) Thus, by adding the energy dependence to ϕ we have described a resonance with the width and strength of the large circle with the parameters from the small circle. The net result is that we describe a resonance as having too large a resonant strength and too small a width. Similarly, if ϕ_m decreases as the energy increases, the crosses would form a circle with a smaller radius than the original.

C. Error analysis

Table I lists the resonant parameters of the levels deduced by the fitting procedure and those of previously reported states. The uncertainty in the resonant energies is the quadrature sum of the fit uncertainty from the error analysis and the absolute energy uncertainty. Billen[4,12] assigned an absolute energy uncertainty of ±10 keV(based on preliminary results of the recalibration[15] of the energy-analyzing magnet), whereas the uncertainty should be only $4-6$ keV. Billen assigns an uncertainty of ±15 keV to the two regions ($14.6\le E_\alpha\le16.3$ MeV and $17.1\le E_\alpha\le18.0$ MeV) of data that were taken when there was not an accurate calibration. He shifted these data to match data (at $16.3\le E_\alpha\le17.1$ MeV) taken after the analyzing magnet's recalibration. So for resonances in these two regions, there is an absolute energy uncertainty of ±15 keV. For resonances with $16.3\le E_\alpha\le17.1$ MeV and for resonances above $E_\alpha=18$ MeV, the uncertainty varies from $4.5-5.6$ keV.

The energy spread ΔE of the experiment increases the observed width of the resonances. This produces an experimental width $\Gamma^{\rm lab}_{\rm exp}$ which is approximately related to the natural width $\Gamma^{\rm lab}_{\rm nat}$ of the resonance by the equation $(\Gamma^{\rm lab}_{\rm exp})^2=(\Gamma^{\rm lab}_{\rm nat})^2+\Delta E^2$. The center-of-mass widths listed in the table are then just $(\frac{4}{5})\Gamma^{\rm lab}_{\rm nat}$. The resonant strengths have also been corrected for the finite energy spread by multiplying the extracted values by $(\Gamma^{\rm lab}_{\rm exp}/\Gamma^{\rm lab}_{\rm nat})$, a negligible correction for all but the narrowest resonances. Billen[4] quotes an energy spread of 15 keV for the data below $E_\alpha=18$ MeV and 8 keV for the data above 18 MeV. The energy straggling of the alpha particles in the target gas caused the majority (~13 keV for the data below 18 MeV)

TABLE I. Natural parity levels in ^{20}Ne observed via ^{16}O$(\alpha,\alpha)^{16}$O (this work) and selected levels of ^{20}Ne from previous works.

This work									Previous work			
E_α [a] (MeV±keV)	E_x [a] (MeV±keV)	$\Gamma_{c.m.}$ [a] (keV)	RS[a,b] (%)	γ^2 [c] (keV)	θ^2 [c] (%)	ch[d]	J	other α channels[e]	E_x (MeV±keV)	$\Gamma_{c.m.}$ (keV)	J	Ref.
								0,1 + 2,3	16.433±15	34	0,2,4	3
14.721±15	16.502±12	24± 4	36±3	2.83± 0.53	0.38±0.07	0	6	1 + 2,3,4	16.502±12	25	6	4
									16.509±15	23	2,4,6	3
14.789±18	16.556±15	90±26	16±3	2.74± 0.94	0.37±0.13	0	5					
14.816±15	16.578±12	92± 8	45±3	30.69± 3.36	4.12±0.45	0	7	3	16.52		7	5
									16.577±12	86	7	4
									16.6		7	8
									16.600±15	160	7	6
									16.630±20	190	7	7
14.875±22	16.625±18	80±25	18±4	1.63± 0.62	0.22±0.08	0	3		16.634±14	51	3	4
									16.64	70	(2,3)	16
14.924±20	16.664±16	99±22	23±3	3.10± 0.80	0.42± 0.11	0	4	(3)	16.667±13	79	4	4
									16.672±15	110	0,2	3
									16.68		7	5
									16.7	230	7	9
14.987±18	16.714±15	(25±)[f]	8±3	0.37±	0.05±	0	5	1 + 2,3,4	16.709±14	14	5,(3)	4
									16.717±10	37		6
									16.718±15	10	1,3,7	3
									16.730± 3	2.0	0;$T=2$	20
15.023±33	16.743±26	158±52	10±2	35.95±13.84	4.83±1.86	0	8					
									16.74	80	3	16
									16.8		7	20
15.149±16	16.844±12	16± 8	11±2	0.32± 0.17	0.04±0.02	0	5	1 + 2,3,4	16.846±12	16	5	4
									16.853±15	10	5	3
15.179±25	16.868±20	353±45	28±3	29.26± 4.87	3.93±0.65	0	6		16.87 ±20	320	6	1
									16.9		(6,8)	8
									16.98	100		20
15.430±21[g]	17.068±16[g]	177±29[g]	32±3[g]	7.35± 1.39	0.99±0.19	0	4					
15.535±15	17.152±12	26± 5	22±2	0.98± 0.21	0.13±0.03	0	5	1 + 2,3,4	17.150±12	33	5	4
									17.161±15	37	5,(7)	3
15.607±19[g]	17.210±15[g]	223±26[g]	32±2[g]	9.11± 1.20	1.22±0.16	0	4		17.205±12	142	4	4
									17.259±11	162	7,(9)	6
15.696±20	17.281±16	86±25	16±3	1.46± 0.51	0.20±0.07	0	3	1 + 2,3,4	17.274±13	52	4	4
									17.279±15	32	1,3,4	3
15.710±17	17.292±14	196±18	26±2	86.54± 10.37	11.63±1.39	0	8		17.300±20	220	8	7
									17.301±14	213	8	4

TABLE I. (*Continued*).

This work									Previous work			
E_α[a] (MeV±keV)	E_x[a] (MeV±keV)	$\Gamma_{c.m.}$[a] (keV)	RS[a,b] (%)	γ^2[c] (keV)	θ^2[c] (%)	ch[d]	J	other α channels[e]	E_x (MeV±keV)	$\Gamma_{c.m.}$ (keV)	J	Ref.
15.878±18[g]	17.427±14[g]	219±25[g]	24±1[g]	356.50±43.32	47.91±5.82	0	9		17.35	35	9	9
									17.372		9	18
									17.385	10		3
									17.394±14	241	9	4
									17.40		9	17
									17.40		9	5
									17.40		8,7	8
									17.50	200	(0)	16
16.017±16	17.538±13	86± 9	45±3	9.72± 1.21	1.31±0.16	0	6	1 + 2,3,4	17.54 ±20	184		1
									17.541±15	136	6	3
									17.55	19	(2; $T=1$)	20
16.099±17	17.603±14	136±16	36±3	7.82± 1.13	1.05±0.15	0	5	4				
16.302±23	17.765±18	(124±)[f]	13±3	1.94±	0.26±	0	4		17.65	100	4	16
									17.751±15	36	4,(0)	3
									17.75	200	3	16
16.405±17[g]	17.848±13[g]	200±30[g]	38±3[g]	11.71± 1.99	1.57±0.27	0	5		17.77 ±20	184		1
									17.91		(0)	20
									18.001±15	<10	7	3
16.622± 6	18.022± 5	34± 7	34±4	1.74± 0.41	0.23±0.06	0	5	1 + 2,3,4	18.021±15	45	2,5,6	3
									18.024± 8	35	5	4
16.695±30	18.080±24	142±54	20±5	3.31± 1.51	0.44±0.20	0	4		18.09 ±20	144	(6)	1
									18.09		(2)	19
16.748± 6	18.122± 4	29± 6	46±6	5.67± 1.39	0.76±0.19	0	7	1 + 2,3,4	18.06		7	5
									18.112±15	33	7	3
									18.120± 8	29	7	4
									18.130± 8	33	odd	19
									18.153±10		7	6
									18.20	200	(2,4)	16
16.949±13[g]	18.283±10[g]	188±27[g]	32±2[g]	12.92± 2.02	1.74±0.27	0	6	4	18.32 ±20	240	(6)	1
17.129±24	18.427±19	185±41	19±2	13.64± 3.35	1.83±0.45	0	7	(1 + 2),3,4	18.4			5
									18.427	9.5	2; $T=2$	20
17.210±21	18.491±17	132±26	21±3	3.92	0.53	0	5	3				
									18.538± 7	138	8	6
17.368±23	18.617±18	185±30	24±3	41.25± 8.45	5.54±1.14	0	8	4				
17.524±29	18.742±24	141±46	17±4	4.72± 1.90	0.63±0.26	0	6	(1 + 2)	18.69 ±20		(6)	1
17.522±24	18.764±19	141±31	22±3	10.95± 2.83	1.47±0.38	0	7		18.7		7	10
									18.7			5

TABLE I. (*Continued*).

	This work								Previous work			
E_α[a] (MeV±keV)	E_x[a] (MeV±keV)	$\Gamma_{c.m.}$[a] (keV)	RS[a,b] (%)	γ^{2}[c] (keV)	θ^{2}[c] (%)	ch[d]	J	other α channels[e]	E_x (MeV±keV)	$\Gamma_{c.m.}$ (keV)	J	Ref.
17.793±29[g]	18.957±23[g]	196±62[g]	15±2[g]	23.89± 8.20	3.21±1.10	0	8					
17.906±18[g]	19.048±15[g]	(87±)[f,g]	18±3[g]	2.07±	0.28±	0	5	1 + 2				
17.999±23[g]	19.122±18[g]	184±29[g]	23±2[g]	39.03± 8.14	5.25±1.09	1	6[h]	(2)4,5	19.113±10	149	6	4
18.048±18[g]	19.161±14[g]	236±38[g]	38±4[g]	16.45± 3.17	2.21±0.43	0	6		19.17 ±20	200	(6)	1
18.198±17	19.281±13	137±32	12±2			1	6	(5)	19.322± 9	123	6	4
18.216±30[g]	19.295±24[g]	426±62[g]	36±3[g]	47.23± 7.92	6.35±1.06	0	7		19.4	320	7	10
									19.41 ±20	280	6	1
18.397±11	19.440± 9	131±13	38±1			1	6	3,4	19.437±10	102	6	4
18.514±29	19.533±23	249±55	27±4	11.63± 3.09	1.56±0.42	0	6	2,3				
18.563±25[i]	19.573±20[i]	142±49[i]	9±2[i]			1	7[i]		19.577±11	50	7	4
18.662±23	19.652±18	139±35	14±2			1	6		19.648±10	89	6	4
18.757±28	19.727±23	328±56	23±2	46.50± 8.91	6.25±1.20	0	8	(2),3				
18.900±48[g]	19.842±38[g]	353±116[g]	18±3[g]	10.50± 3.87	1.41±0.52	0	6		19.85 ±20	280	6	1
18.949±52	19.881±42	(121±)[f]	8±3	2.60±	0.35±	0	7		19.9	320	7	10
18.918±11[g]	19.856± 9[g]	172±24[g]	26±2[g]			1	5		19.914±12	203	5	4
19.083±39[g]	19.988±31[g]	128±93[g]	11±4[g]	1.39± 1.13	0.19±0.15	0	4	2,(5)				
19.128±16	20.024±13	81±32	10±4			1	6	4				
19.227±28	20.103±22	187±33	29±3			1	7		20.130±17	156	7	4
19.304±47[g]	20.165±37[g]	281±92[g]	18±4[g]	7.98± 3.16	1.07±0.42	0	6	3	20.15	250	7	2
19.464±19	20.293±15	252±36	28±3			1	7	5	20.317±12	203	7	4
19.521±22	20.338±18	189±40	26±3			1	5					
19.524±16	20.341±13	133±33	25±4	8.12± 2.40	1.09±0.32	0	7	3	20.4	200	7	2
19.618±39	20.416±31	212±84	14±3	4.53± 2.04	0.61±0.27	0	6		20.4	360	6	2
19.651±32	20.442±25	366±54	32±3			1	6		20.433±16	346	6	4
19.679±35	20.464±28	280±69	20±3	6.41± 1.85	0.86±0.25	0	5	2				
									20.478±11	250	(8)	6
19.952± 8	20.683± 6	78±11	33±3	33.17± 5.57	4.46±0.75	0	9[h]	2,3	20.67	100	9	5
19.952±43	20.683±34	(78±)[f]	4±2	22.51±	3.03±	1	9		20.683± 9	75	9	4
									20.70	120	9	2
									20.704±11	120		6
20.025±34	20.741±27	198±59	13±2			1	7[h]	4	20.782±11	122	7	4
20.064±40[g]	20.772±32[g]	286±66[g]	21±3[g]	13.49± 3.66	1.81±0.49	0	7	4				
20.095±32	20.797±26	171±54	11±2			1	5					

TABLE I. (*Continued*).

This work									Previous work			
E_α [a] (MeV±keV)	E_x [a] (MeV±keV)	$\Gamma_{c.m.}$ [a] (keV)	RS[a,b] (%)	γ^2 [c] (keV)	θ^2 [c] (%)	ch[d]	J	other α channels[e]	E_x (MeV±keV)	$\Gamma_{c.m.}$ (keV)	J	Ref.
									20.85			2
									20.89 ±30			6
[20.260±32	20.929±25	302±61	21±2	100.77±26.06	13.54±3.50	1	7[h]]		20.920±12	181	7	4
20.320±41[g]]	20.977±33[g]	312±77[g]	23±3[g]	15.53± 4.34	2.09±0.58	0	7]		21.05	200	7	2
									21.05 ±20	140		6
20.423± 8[j]	21.059± 6[j]	60± 6[j]	46±3[j]	30.69± 3.66	4.12±0.49	0	9	3	21.080±30	100	9	7
									21.10	80	9	2

[a]The uncertainties of the deduced resonant parameters were determined by the method described in Sec. II. This method does not take into account the correlations between the various resonant and nonresonant parameters. The errors assigned to the resonant energy include the absolute energy uncertainty added in quadrature with the fit uncertainty.

[b]RS (resonant strength)$=\Gamma_{\alpha_0}/\Gamma$ for resonances in elastic channel

$$=(\Gamma_{\alpha_0}/\Gamma_{\alpha_1})^{1/2}/\Gamma \text{ for resonances in inelastic channel } (E_x=6.05 \text{ MeV}) .$$

[c]The value of γ^2 is calculated using the formula

$$\gamma^2=\Gamma_{c.m.}(\Gamma_\alpha/\Gamma)(F_l^2+G_l^2)/2kr ,$$

where F_l and G_l are the regular and irregular Coulomb wave functions of angular momentum l and

$$r=(1.25 \text{ fm})(4^{1/3}+16^{1/3}) .$$

The error in γ^2 and θ^2 are the quadrature sum of the errors in the width Γ and the resonant strength Γ_α/Γ. For resonances in channel 1, values of γ^2 and θ^2 are given only when the resonance is observed in both the elastic and inelastic channels. The value of θ^2 is calculated using the reduced width γ^2 and its Wigner limit γ^2_{WL}.

$$\theta^2=\gamma^2/\gamma^2_{WL}=\gamma^2/(3\hbar^2/2\mu r^2) .$$

[d]ch (channel) =0 elastic channel

=1 inelastic channel, $E_x(^{16}O)=6.05$ MeV .

[e]Resonantlike structure of about the same width is visible in these other (α,α_i) channels of Billen's data. Where the structure is relatively weak, the channel is listed in parentheses.
[f]For the data within 2Γ of the resonant energy, the error analysis program failed to find a value between $\Gamma/2$ and 2Γ that would double the χ^2.
[g]The resonance was nearer than 2Γ to the edge of the fitting region, so parameters are somewhat less well determined than for other resonances. See discussion of effect of fitting region size on resonant parameters in Sec. II.
[h]Probably the same state.
[i]An acceptable χ^2_ν occurs without this weak state, but including it gives an 8.5% improvement in χ^2_ν.
[j]Level parameters are only approximate, since resonance is outside the range of data.

of the energy spread. Lower target gas pressures above 18 MeV account for the improved energy spread.

The uncertainties in the resonant parameters were calculated using the same technique Billen[4] used. The value of a parameter was varied until the χ^2 of the data points within 2Γ of the resonance doubled, unless the resonance was $<2\Gamma$ from the edge of the fitted region. Only data used in the fit to the resonance is included in the error analysis. If the resonance is very weak ($\Gamma\alpha/\Gamma<0.1$) and/or the fit is poor in the region of the resonance, the value of the parameter may double or halve without doubling χ^2. In this case, I do not quote an uncertainty but enclose the value in parentheses. Also, if the resonance is near the edge of a fitting region (within 2Γ) both the parameter and uncertainty are somewhat ill determined. These values are noted in Table I.

Remember that the amplitude of a resonance changes most rapidly near the resonant energy and consider two sets of data: one consisting of all points within $E_{\text{res}}\pm2\Gamma$, and the other a smaller range (say $E_{\text{res}}\pm\Gamma$). A change in a resonant parameter will produce a larger change in the χ^2 of the points near E_{res} than in the points farther away. Thus, the χ^2 for the set of points near E_{res} will double at a smaller change than the χ^2 for the larger set of points. The smaller data range will therefore give a smaller error than the large set.

The errors I assign to the resonant energy are generally close to Billen's, but my uncertainties for the width and resonant strength tend to be appreciably larger than his values. The difference arises primarily because the size of the fitting region I use is larger. As noted above, one needs an energy range of about 2Γ on each side of the resonant energy E_{res} to obtain reliable parameters and a good estimate of the uncertainties for the resonance. Billen's fits rarely included all of the data points within $E_{\text{res}}\pm2\Gamma$ (especially for broad levels), so that his uncertainties are (in general) determined by a set of points that are closer to E_{res} than mine. Thus, his smaller set of data yielded an erroneously smaller uncertainty than mine.

III. RESULTS

The cross sections were divided into eight energy regions (A thru H) for analysis. The discussion will focus separately on each region. The first six regions are elastic scattering data and the last two regions are inelastic data. Since Billen has adequately discussed previous work on some of these levels, I will confine my remarks to new levels, to comparisons with his results, and to other recent work. The density of ^{20}Ne states is sufficiently high that it is very difficult to fit a resonance unless one explicitly includes nearby resonances. In general, on each end of the eight regions discussed below, there is a broad resonance taken from another fitting region, whose resonant parameters were held fixed during the fit.

A. Region A: $16.475\le E_x\le17.168$ MeV, Fig. 3, $\chi_\nu^2=2.53$

This energy region was studied in detail by Billen.[4] He employed five resonances to fit the energy range from 16.44 to 16.76 MeV and a single resonance for the range 16.79 to 16.90 MeV, but he did not fit the rest of this region. I find that two more resonances are needed below 16.9 MeV.

From data at seven angles between 61° and 178°, Häusser *et al.*[3] report a level at $E_x=16.433$ MeV to which they assign $\Gamma_{\text{c.m.}}=34$ keV and $J^\pi=(0, 2, \text{or } 4)^+$. I tried including this level (with four different spins: 0, 2, 4, and 6) in some early fits to the region 16.395–16.650 MeV. Poor fits (χ^2 per degree of freedom, $\chi_\nu^2>6$) suggest that there are tails of broad, lower-energy resonances in the fitting region which interfere with attempts to fit this region. Because of this inability to fit this lower level, I have restricted my fits to energies above 16.47 MeV.

The Fig. 3 fit includes all ten resonances of region A and two resonances from region B whose parameters were held fixed during the fitting procedure.

The first level in this region is a $J^\pi=6^+$ at $E_x=16.502$ MeV. This state had the same energy and almost the same width as Billen reports, but is only 76% as strong. It had also been reported by Häusser *et al.*, but they could not resolve the spin assignment.

The next level is a 5^- at $E_x=16.556$ MeV and has not previously been reported. This weak resonance was added after several attempts using the levels at 16.502 and 16.578 MeV failed to fit the data between 16.53 and 16.61 MeV. The fits were particularly poor at $\theta_{\text{c.m.}}=136.8°$ which is only 1.1° from a zero of $P_7(\cos\theta)$ but near a $P_5(\cos\theta)$ maximum. As can be seen from Fig. 3, the 7^- level dominates this energy range, but could not describe the structure at 136.8° by itself.

The 7^- level at $E_x=16.578$ MeV has almost the same parameters Billen found. A 7^- level has been observed via ^{12}C(^{12}C,α)^{20}Ne(α)^{16}O by Young *et al.*[5] at $E_x=16.52$ MeV which is ~60 keV lower in energy than I observe it. From the three observed α-particle decays (to the ^{16}O ground state, the first doublet at 6.05–6.13 MeV, and the second doublet at 6.92–7.12 MeV) Young *et al.* find the ground state branching ratio is 0.72 ± 0.03. This value is significantly larger than my extracted 0.45 ± 0.03. However, Hindi *et al.*,[6] using the same reaction as Young, report a 7^- level at 16.600 ± 0.015 MeV ($\Gamma=160\pm30$ keV) which, while nearly twice as wide as I determine, almost overlaps in resonant strength, namely 0.60 ± 0.10. Presumably the same level is also observed by Sanders *et al.*[7] via the ^{16}O(^{12}C,^{8}Be)^{20}Ne(α)^{16}O reaction, but at $E_x=16.63\pm0.02$ MeV. They assign a spin of 7^-, but the width of 190 keV and strength of 0.90 ± 0.10 are a factor of 2 larger than I determine. As Billen points out[4] such branching would produce very large excursions in the elastic scattering data. Since these large excursions are not observed, the Sanders *et al.* value must be in error. Fou *et al.*[8] assign a spin of 7^- to a resonance at 16.6 MeV that they obesrve via the ^{16}O(^{6}Li,d)^{20}Ne(α)^{16}O reaction. Their Fig. 2 indicates comparable strength to ^{16}O excited states and hence is consistent with our lower strengths.

The next two levels in this region are the weak 3^- resonance at $E_x=16.625$ MeV and the 4^+ level at 16.665 MeV. Billen and I agree within errors on the parameters for these weak levels, but I favor larger widths for both states. Gorodetzky *et al.*[16] report a possible 2^+ or 3^- resonance at 16.64 MeV with comparable width via either ^{19}F(p,^{8}Be)^{12}C or ^{16}O(α,^{8}Be)^{12}C. This may be the 3^- I ob-

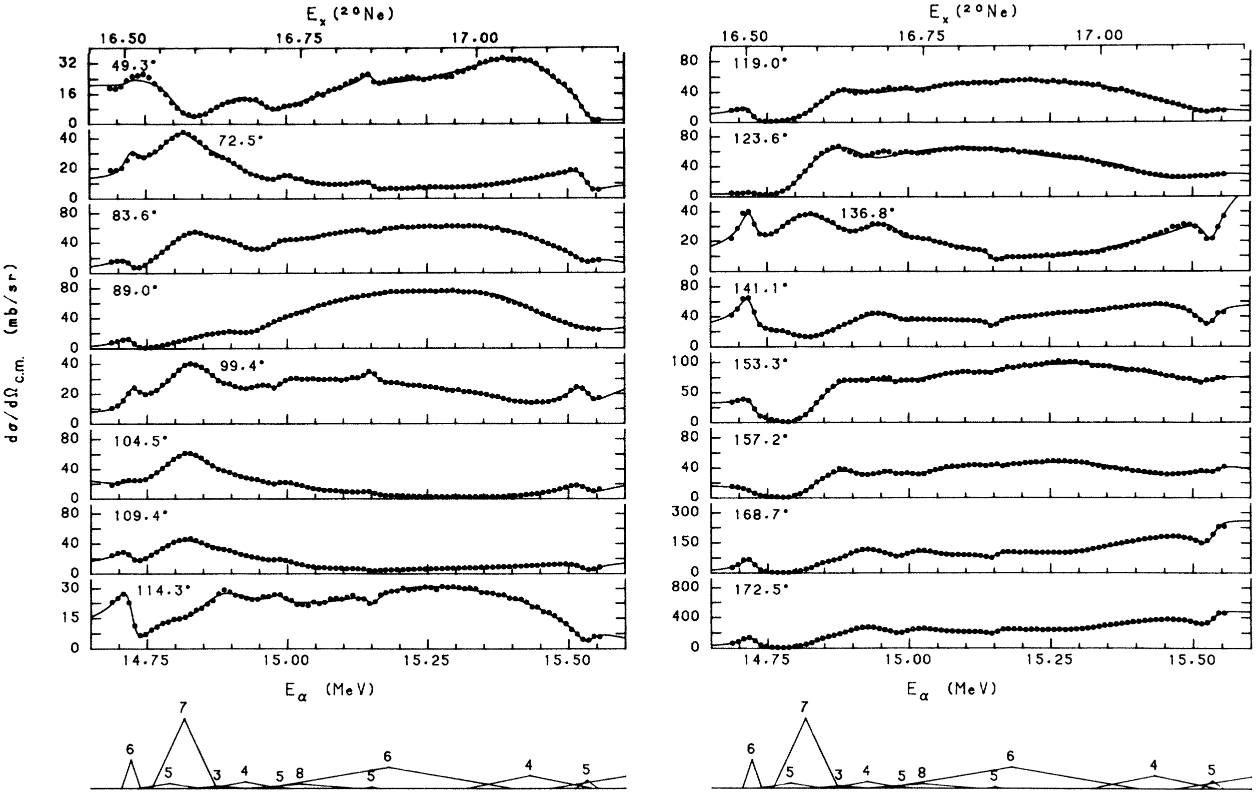

FIG. 3. The region A fit to the elastic data ($\chi_\nu^2=2.53$) includes the 11 resonances shown as triangles below the E_α scale and the tail of a 4^+ resonance from region B. For resonance parameters see Table I. The height of the triangles is proportional to $(2J+1)\Gamma_{\alpha_0}/\Gamma$ and the base width equals Γ.

serve at 16.625 MeV. Häusser *et al.* have reported a 110-keV wide 0^+ or 2^+ resonance at 16.672 MeV. Billen could not obtain good fits in this region unless this resonance was a 4^+. I agree with his assignment and also point out that at 16.67 MeV there is a definite dip in the poorly fit data at 123.6° [near a zero of $P_2(\cos\theta)$] and a lack of structure at 72.5°, 109.4°, and 153.3° [near zeros of $P_4(\cos\theta)$].

Artemov *et al.*[9] report via ^{16}O(^{6}Li,d)^{20}Ne a broad (230±100 keV) 7^- level at 16.7 MeV with $\Gamma_{\alpha_0}/\Gamma\sim1$. Such a state is inconsistent with the Billen data and the present analysis. Young *et al.*[5] via ^{12}C(^{12}C,α)^{20}Ne observe a 7^- level at 16.68 MeV but their state decays predominantly to the 6.05−6.13 and 6.92−7.12 MeV doublets in ^{16}O. Consistent with my analysis, they see almost no decay to the ground state of ^{16}O.

Next, Billen had a narrow very weak 5^- (or possibly 3^-) level at $E_x=16.709\pm0.014$ MeV. I tried each J value and obtained slightly better fits with the 5^-, especially at 123.6° and 157.2° which are near zeros of P_5, and at 141.1° which is near a zero of P_3. Hindi *et al.* claim a 37±10 keV wide level at 16.717±0.010 MeV but make no spin assignment. Häusser *et al.* also report this level but could not make a firm spin assignment.

The seventh level in this region is a weak 8^+ state at 16.743 MeV. This weak, broad level was not included in Billen's fitting region. Although Young *et al.* searched for an 8^+ state in this region, they could only identify the 7^- level at 16.68 MeV, discussed above, whose ground state branch of 0.05±0.02 makes it too weak for us to see. However, they noted a broad structure under the alphas associated with the narrow 16.68 MeV state. I suggest this broad structure corresponds to the new, broad, weak 8^+ resonance.

The next level is another narrow 5^- state at 16.844 MeV observed by Billen, and also by Häusser *et al.* We obtain approximately the same resonant parameters.

From analysis of the angular distribution of their elastic α scattering, Mehta, Hunt, and Davis[1] suggested a 320-keV wide 6^+ level at 16.87 MeV. With only minor changes in width and resonant energy, I obtained a significant χ^2 reduction when I included this level. Although the resonant strength is only 0.28, the $(2J+1)$ factor causes it to dominate much of this region. Fou *et al.*[8] want both 6^+ and 8^+ states in this energy range to account for their angular correlation data from ^{16}O(^{6}Li,d)^{20}Ne(α)^{16}O. The final state in this region is a strong 4^+ level at 17.068 MeV which has not been reported previously.

B. Region B: $17.120\le E_x\le17.520$ MeV, Fig. 4, $\chi_\nu^2=1.69$

Billen[4] and I agree that five resonances are required for a fit to this region, but we disagree on the spin of one of the resonances. Besides searching on the five resonances of region B, the program included one resonance each

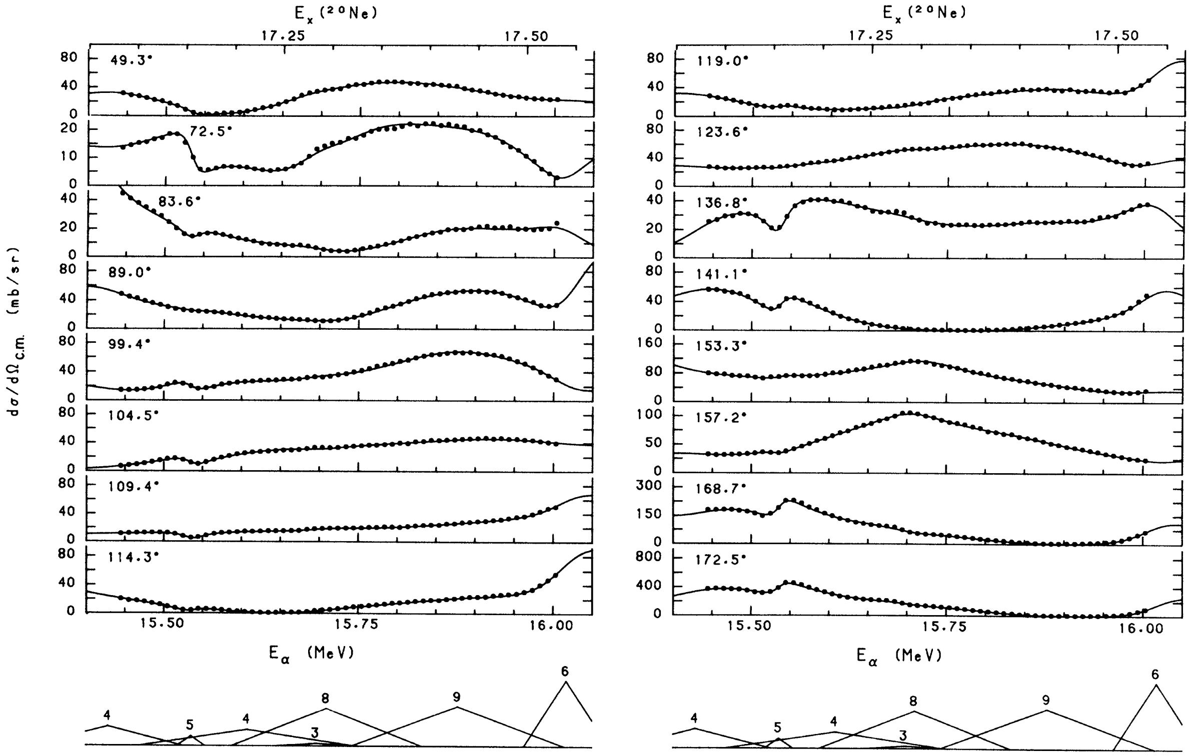

FIG. 4. The region B fit to the elastic data ($\chi_\nu^2 = 1.69$) includes the seven resonances shown as triangles below the E_α scale with the height of the triangle proportional to $(2J+1)\Gamma_{\alpha_0}/\Gamma$ and the base width equal to Γ. For level parameters see Table I.

from regions A and C whose resonant parameters were held fixed.

The first resonance in region B occurs in the overlap region of A and B and is a narrow 5^- level at 17.152 MeV. In fact, in region A, the program also searched on this level and the resonant parameters were consistent, but the parameters determined from the region B fit were held constant for the final fit of region A. I find a narrower and somewhat weaker level than Billen did. Häusser *et al.*[3] also saw this resonance.

At $E_x = 17.210$ MeV is another 4^+ level. Its resonant parameters were held constant when it was included in fits of region A. Billen and I both observed this level, but I obtain a much wider and weaker state. Hindi *et al.*[6] via ^{12}C(^{12}C,α)^{20}Ne report a $\Gamma = 162 \pm 20$ keV 7^- (or possibly 9^-) level at 17.259 MeV decaying mainly to ^{16}O excited states but with a ground state branching ratio of 0.15 ± 0.02. I see no evidence for such ground state transitions in this region. The triple correlation data of Ref. 6, however, is not well fit ($\chi_\nu^2 = 3.9$) and perhaps their ground state branch arises from the 4^+ state.

The next level is at 17.281 MeV. Häusser *et al.* assigned it a width of 32 keV and a spin of either 1^-, 3^-, or 4^+. When Billen analyzed this region, he settled upon a 4^+ level with a width of 52 ± 10 keV. In fitting this region, I soon found that an odd-J level was needed. Both 5^- and 3^- states were tried, and the 3^- gave the better fit. However, the width (86 ± 25 keV), while overlapping Billen's 52 ± 10 keV, is much larger than the 32 keV reported by Häusser. My much broader 4^+ at 17.210 MeV may have been what Billen observed as two 4^+ levels. Häusser *et al.*'s data had about three times the resolution of Billen's data, and shows a sharp enough resonance to account for the difference in width.

The fourth resonance is an 8^+ level at 17.292 MeV. This state was also reported by Sanders *et al.*[7] via ^{16}O(^{12}C,^{8}Be)^{20}Ne whose width and resonant energy for the level agree well with both Billen's and my results, but their strength (0.40 ± 0.10) is somewhat larger than ours (0.26 ± 0.02).

The fifth resonance in region B is the broad (219 ± 25 keV) 9^- level at 17.427 MeV. Although Fifield *et al.*[17] established the existence of a 9^- state at $E_x = 17.4$ MeV by a triple angular correlation using the ^{12}C(^{12}C,α)^{20}Ne(α)^{16}O reaction, this level cannot be the one we see because the decay was $>99\%$ to ^{16}O* (6.13 MeV) and $<1\%$ to the ground state; also their width appears to be much narrower. For this state Medsker *et al.*[18] quote $E_x = 17.372$ MeV and their Fig. 1 shows a level no wider than their (unstated) resolution. However, Artemov *et al.*[9,10] via the ^{16}O(^{6}Li,d)^{20}Ne(α)^{16}O reaction make a 9^- assignment to a state at $E_x = 17.35 \pm 0.1$ MeV which has a 40% alpha decay to the ^{16}O ground state. Such a level could correspond to ours except that they quote a width of 35 ± 15 keV which they footnote as "estimate according to magnitude of the excitation cross section." Their actual data (Fig. 2 of Ref. 10) would suggest a much broader state. Fou *et al.*,[8] via the same reaction as Artemov *et al.*, see strong alpha decay of an ~ 17.4 MeV ^{20}Ne state to the ground state of ^{16}O. They find a width of about 210 keV for the state but note that the angular correlations require at least two levels of opposite parity.

Their fit employed an 8^+ and 7^- but was not satisfactory for $55° < \theta_{c.m.} < 100°$. Although they show that a pure 9^- does not fit either, it is not clear if they tried an $8^+,9^-$ combination which our scattering results would suggest (the 8^+ from the neighboring 17.292 MeV level). I tried an $8^+,7^-$ combination for Billen's (α,α_0) data, but the fit was much poorer than the $8^+,9^-$ combination.

C. Region C: $17.343 \le E_x \le 17.924$ MeV, Fig. 5, $\chi_\nu^2=2.49$; $17.788 \le E_x \le 18.298$ MeV, Fig. 6, $\chi_\nu^2=6.26$

This region proved most difficult to fit. Even with the region split in two, fits with low χ_ν^2 were difficult to obtain. Billen fit only the two narrow resonances that appear in the upper portion at 18.02 and 18.12 MeV. The fit (Fig. 5) of the lower portion of region C overlaps the upper portion of region B and includes the three upper resonances of region B and four resonances of region C.

Even though the broad 4^+ and 8^+ resonances (at 17.21 and 17.29 MeV) from region B lie outside the fitting region, they are wide enough to affect the excitation functions far into this region. Their resonant parameters were held constant during the fitting procedure. The 9^- level ($E_x=17.427$ MeV) from region B is in the overlap region and so its resonant parameters were allowed to vary. The resultant resonant strength was slightly larger (0.28 ± 0.02 compared to 0.24 ± 0.01), but the other parameters were well within the assigned uncertainties. Table I quotes the results from region B.

The first resonance in region C is a 6^+ level at 17.538 MeV and was assigned 6^+ by Häusser *et al.*[3] They reported a width of 136 keV compared to my value of 86 ± 9 keV. Mehta, Hunt, and Davis[1] first saw this level and by visual inspection assigned an even larger width.

The 5^- level at 17.603 MeV strongly overlaps the 6^+ level and has not been reported previously. Its presence is quite evident from inspection of the excitation functions (especially at angles that are near zeros of P_6, e.g., 49.3°). The failure of Häusser *et al.* and Mehta *et al.* to realize that overlapping levels were involved probably accounts for their larger width assignment to the 6^+ state.

A weak ($\Gamma_{\alpha_0}/\Gamma=0.13$) 4^+ level at 17.765 MeV was an attempt to verify the resonance reported by Häusser *et al.*, as 4^+ or (0^+) at $E_x=17.751$ MeV and a width of 36 keV. The fitting routine broadened this level to 124 keV, which makes it difficult to confirm whether this level is the one reported by Häusser *et al.* Two sets of Billen's data overlap in the energy region near this resonance. The poor match of the data sets at 72.5°, 49.3°, and 123.6° may wash out any sharp structure from a narrow resonance. However, the 89.0° excitation function has a structure which is narrow enough to be the resonance cited by Häusser *et al.* There may be both a narrow and a broad 4^+ resonance at this energy. Gorodetzky *et al.*[16] via ^{19}F(p,^{8}Be)^{12}C and ^{16}O(α,^{8}Be)^{12}C resonances report a weak 100-keV wide 4^+ state, but their resonant energy is over 110 keV too low. They also report a weak 200-keV wide 3^- level at 17.75 MeV.

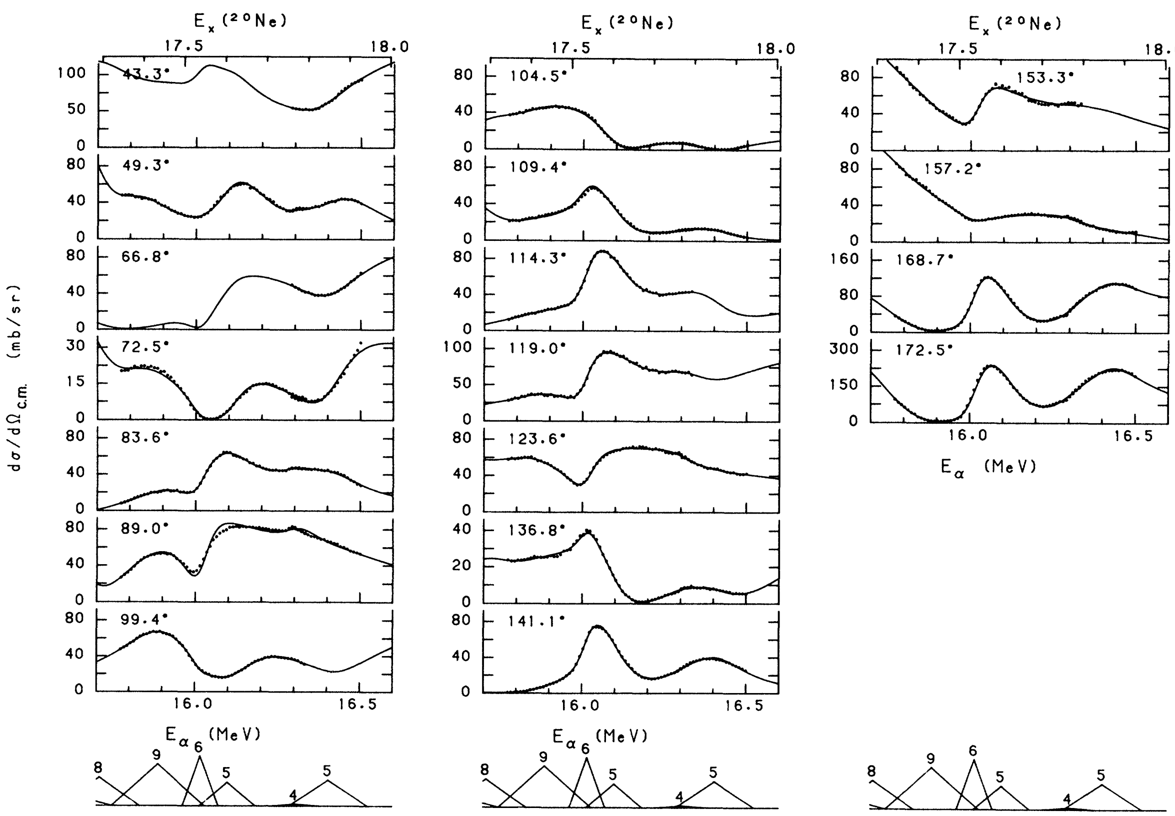

FIG. 5. The lower portion of region C fit to the elastic data ($\chi_\nu^2=2.49$) includes the six resonances shown as triangles below the E_α scale plus the tail of the broad 4^+ from region B. (Note the large overlap with region B.) The triangle height is again proportional to $(2J+1)\Gamma_{\alpha_0}/\Gamma$ and the base width is Γ. See Table I for parameters.

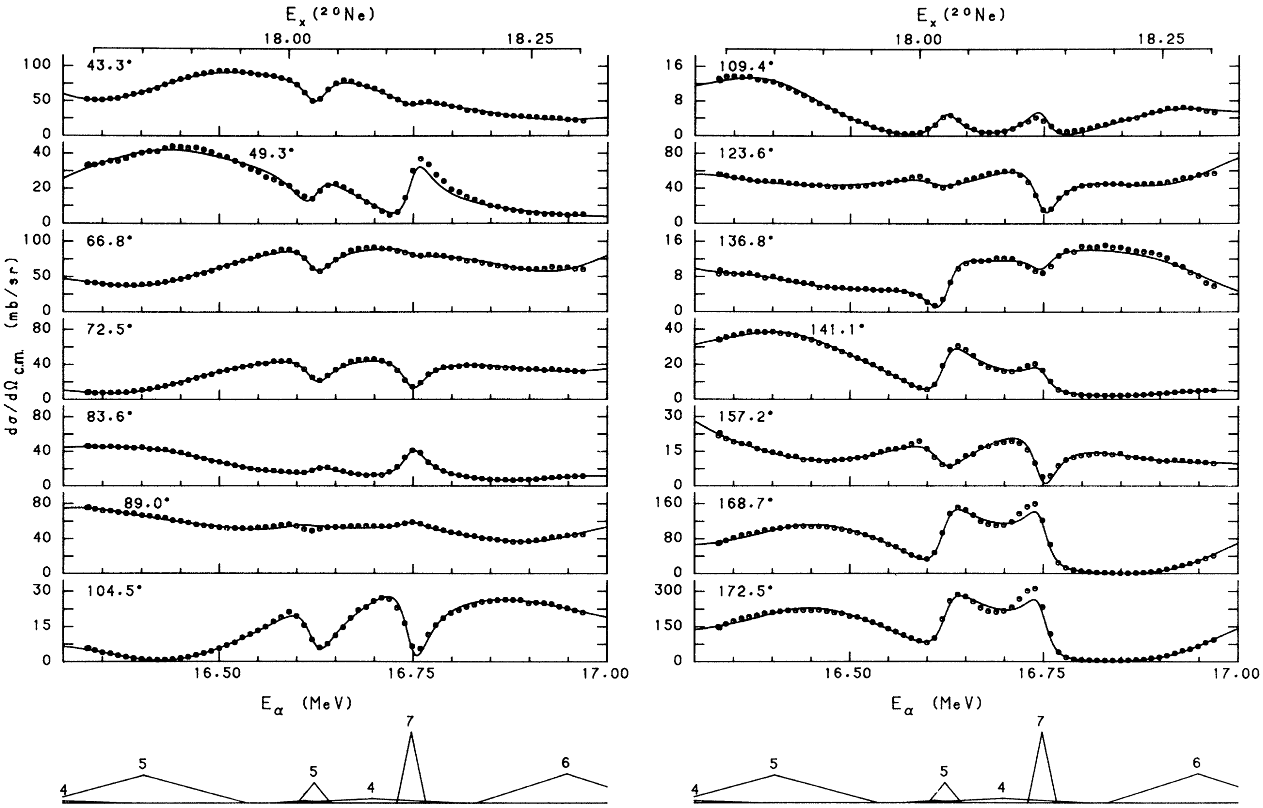

FIG. 6. The poorly fit ($\chi_\nu^2=6.26$) upper portion of region C includes the six resonances shown as triangles below the E_α scale plus the tail of a 7^- state at $E_x=18.427$ MeV. The triangle heights are proportional to $(2J+1)\Gamma_{\alpha_0}/\Gamma$ and the base width is Γ. See Table I for resonant parameters.

The last resonance in this lower region, another broad, relatively strong, 5^- level at 17.848 MeV, has never been reported before though Mehta *et al.*[1] suggest from visual inspection of their data a broad (unassigned spin) state at 17.77 MeV. This state is only $\Gamma/2$ from the edge of the fitting region, so its parameters may not be as reliable as for the other levels.

The fit of the upper portion of region C includes two levels from the lower region, three resonances of the upper region, and two resonances from region D whose resonant parameters were held fixed, but the χ_ν^2 is only 6.26.

The 4^+ and 5^- resonances at 17.765 and 17.848 MeV discussed above were included in this fit with their resonant parameters held fixed at the values from the lower portion of region C. Billen's energy steps were too coarse to detect the very narrow (<10 keV) 7^- state reported by Häusser *et al.* at 18.001 MeV.

The first resonance searched on by the program, a 5^- level at 18.022 MeV, was reported by Häusser *et al.* who left an ambiguity (2,5,6) in the spin assignment. Billen was able to fit this resonance, and removed the spin ambiguity. I obtain the same resonant parameters as Billen.

The 4^+ resonance at 18.080 MeV probably is the same level that Mehta *et al.* assigned (from visual inspection) a tentative spin of 6^+. Our widths agree very well. Garman[19] also reports a tentative 2^+ level at 18.09 MeV. Her assignment is based on inelastic alpha scattering to the 6.05-MeV level in ^{16}O, but she does not give any estimate of a width.

The narrow (29 ± 6 keV) 7^- level at 18.122 ± 0.004 MeV was observed also by Billen, who found nearly the same resonant parameters. Garman reports a 33 ± 8-keV wide, $J=$odd resonance at 18.130 ± 0.008 MeV in her ^{16}O$(\alpha,\alpha_2)^{16}$O* reaction. Hindi *et al.*[6] via ^{12}C(^{12}C,α)^{20}Ne(α)^{16}O claim a 7^- level at 18.153 ± 0.010 MeV with small ground state branch whereas by the same reaction Young *et al.*[5] find the 7^- state to be at 18.06 MeV with $\Gamma_{\alpha_0}/\Gamma=0.71\pm0.06$. So the presence of at least one 7^- state seems certain, but the poor fit around 18.12 MeV of the present data at $\theta=49.3°$ and at the back angles suggests also another level, probably odd parity since the fit is excellent at $\theta=89°$ where $P_{\rm odd}(\cos\theta)\approx0$.

D. Region D: $18.227\le E_x\le19.056$ MeV, Fig. 7, $\chi_\nu^2=4.19$

This region does not contain any of the resonances analyzed by Billen. Although the overall fit (Fig. 7) is not good, I identified seven resonances in this region. Probably more overlapping states are needed.

The first resonance is a 188 ± 27-keV wide 6^+ state at 18.283 ± 0.015 MeV. From visual inspection, Mehta *et al.*[1] reported a tentative 6^+ level at 18.32 MeV with a width of ~240 keV. There are two possible explanations for this discrepancy in widths. First, our 6^+ resonance is very close to the edge of the fitting region in both this fit

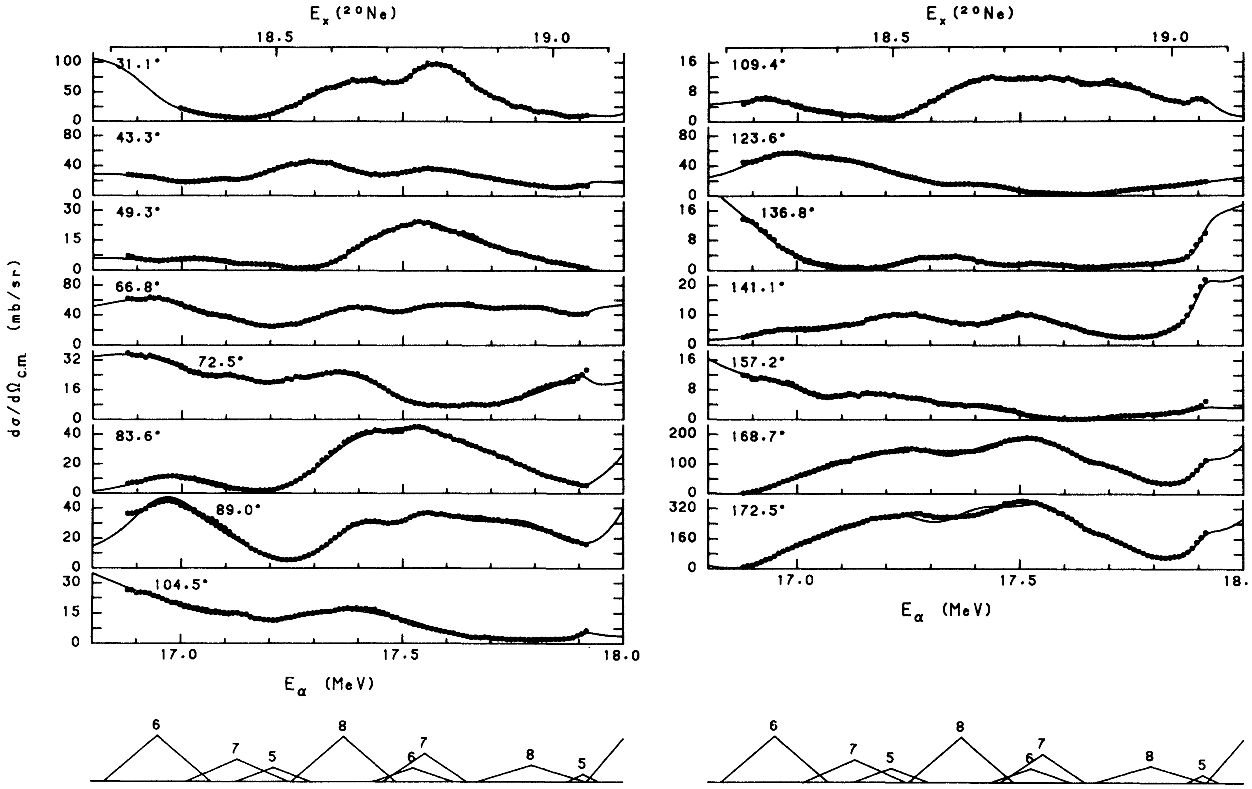

FIG. 7. The region D fit to the elastic data ($\chi_\nu^2=4.19$) employs the eight resonances shown as triangles below the E_α scale plus the tail of a 6^+ resonance from region E. The triangle height is proportional to $(2J+1)\Gamma_{\alpha_0}/\Gamma$ and the base width is Γ. See Table I for resonant parameters.

and in the fit to the upper portion of region C. In general, this situation seems to yield a too narrow width. Second, Mehta *et al.* did not resolve this level from the broad overlapping 7^- level at 18.427 MeV, thus they may have overestimated the width and mislocated the resonant energy.

The next resonance is a new 7^- assignment at 18.427 MeV. While Young *et al.*[5] report a level at 18.4 MeV via ^{12}C(^{12}C,α)^{20}Ne(α)^{16}O, they see only decay to the first and second doublets of ^{16}O and could not make a spin assignment. To be the same state as that of the present data requires Young *et al.* to have missed the $\sim 19\%$ ground state branch. Comparison with Hindi *et al.*'s data[6] indicates that the state seen by Young *et al.* is (except for a surprising E_x difference) more like the decay of the 8^+ state that Hindi *et al.* reported at 18.538 ± 0.007 MeV.

The new weak 132 ± 26-keV wide 5^- resonance at $E_x=18.491$ MeV shows visually at 43.3° and 141.1° where the nearby 7^- and 8^+ resonances have almost zero amplitude, i.e., $P_J(\cos\theta)\approx 0$.

The 8^+ resonance at 18.617 ± 0.018 MeV is another new assignment although Hindi *et al.*[6] report an 8^+ level nearby at $E_x=18.538\pm0.007$ MeV. However, the difference in energy and the Ref. 6 branching ratio of only 0.018 ± 0.009 to the ground state of ^{16}O make clear that these are different resonances.

The weak 6^+ level at 18.742 MeV with $\Gamma=141\pm46$ keV and $\Gamma_{\alpha_0}/\Gamma=0.17\pm0.04$ may be the same tentative 6^+ level suggested by Mehta *et al.*[1] at 18.69 ± 0.02 MeV.

A 7^- level is also needed at 18.764 MeV. Artemov *et al.*[10] report a 7^- level at 18.7 MeV via the ^{16}O(^{6}Li,d)^{20}Ne(α)^{16}O reaction, but give no further details. Young *et al.*,[5] via ^{12}C(^{12}C,α)^{20}Ne(α)^{16}O*(γ)^{16}O, see an unassigned state at 18.7 MeV having a small decay to ^{16}O$_{g.s.}$ compared to decays to ^{16}O*. At $E_x=18.957$ MeV is another new 8^+ level of nearly the same width as the lower one but only 60% as strong.

The last resonance in region D is a new, fairly narrow 5^- level at 19.048 MeV. While not very strong, its presence shows in the excitation functions at 72.5°, 104.5°, and 109.4° (which are near zeros of P_6). The wide 6^+ resonance at 19.161 MeV from region E was included with its resonant parameters held constant.

E. Region E: $19.105\le E_x\le 20.071$ MeV, Fig. 8, $\chi_\nu^2=2.22$

While the fit of Fig. 8 does not quite overlap region D, it does include the broad 8^+ level at 18.957 MeV (with the resonant parameters held fixed). The first resonance in this region is a 236 ± 38-keV wide, fairly strong, 6^+ level at 19.161 MeV. The parameters agree well with the visual estimate of Mehta *et al.*:[1] $E_x=19.17$ MeV, $\Gamma=200$ keV, tentative 6^+. This state probably also produces the inelastic resonance which both Billen and I report at $E_x\sim 19.1$ MeV; see region G (below) where I find a width of 184 ± 29 keV. Because there are no inelastic data below 19.06 MeV, our values for these parameters may not be

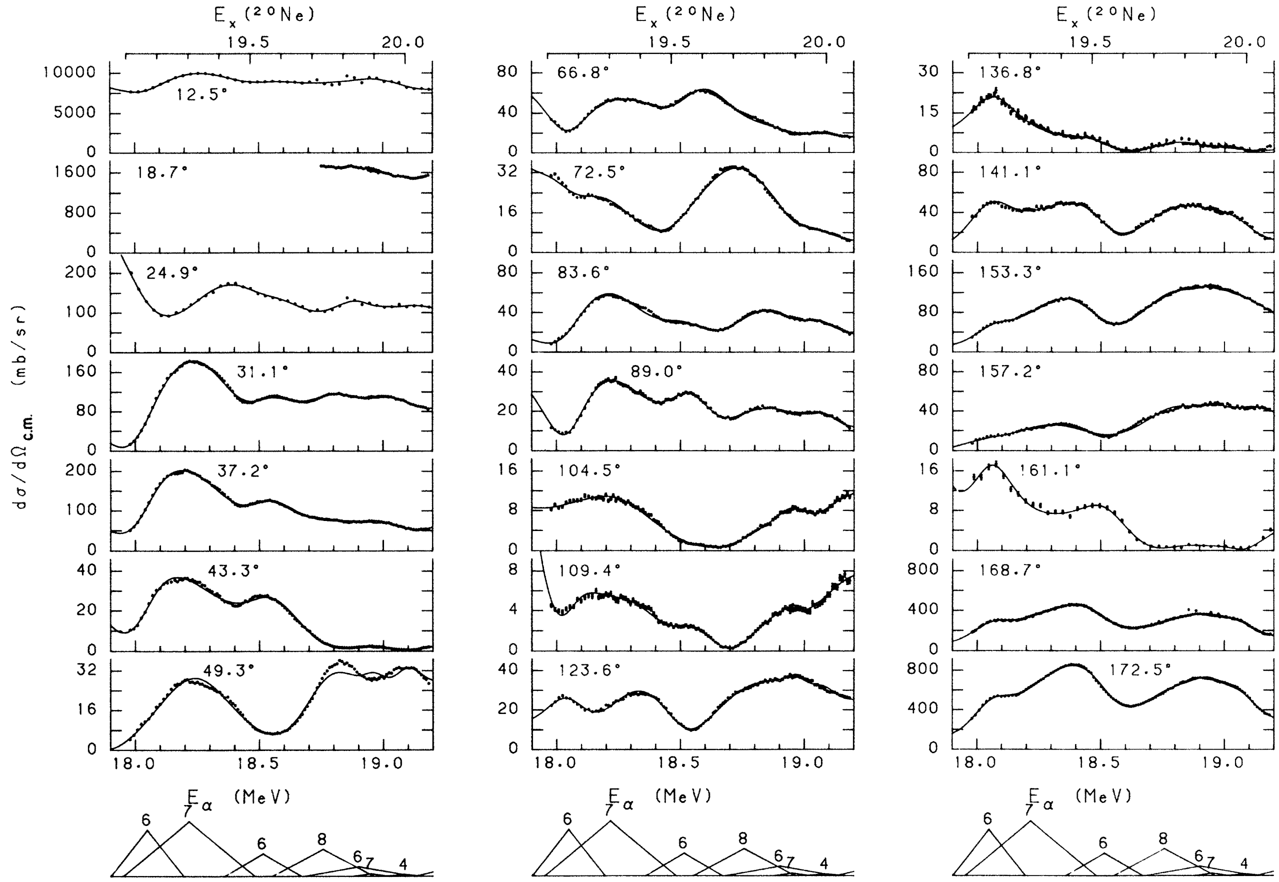

FIG. 8. The region E fit to the elastic data ($\chi_\nu^2=2.22$) employs the seven resonances shown as triangles below the E_α scale plus the tail of an 8^+ resonance from region D and a 6^+ state from region F. The triangle height is proportional to $(2J+1)\Gamma_{\alpha_0}/\Gamma$ and the base width is Γ. See Table I for resonant parameters. The fitting program could handle only 20 angles, therefore 18.7° was not fit.

very accurate (see Sec. II regarding the size of the fitting region).

The 7^- resonance at 19.295 MeV is the broadest level (426 ± 62 keV) that I see and probably corresponds to the 320-keV 7^- level reported via the ^{16}O(^{6}Li,d)^{20}Ne(α)^{16}O$_{\text{g.s.}}$ reaction by Artemov *et al.*[10] at 19.4 ± 0.1 MeV.

Another strong 6^+ level at 19.533 MeV is apparent at angles where the lower 7^- nearly vanishes (e.g., $\theta=43.3°$, 66.8°, and 89°). Mehta *et al.*[1] report from visual inspection a 6^+ resonance of about the same width but ~120 keV lower in energy.

The new broad (328 ± 56 keV) 8^+ level needed at 19.727 MeV is especially pronounced at $\theta=49.3°$ and 72.5°, i.e., near zeros of $P_6(\cos\theta)$—the spin of the two neighboring resonances.

I confirm the 6^+ level which Mehta *et al.*[1] visually estimated as at 19.85 MeV with $\Gamma\sim280$ keV. Artemov *et al.*[10] report a 320-keV wide 7^- level at 19.9 MeV but not as strong as the state at 19.4 MeV. Although such a state is not visually apparent in Billen's data, including it in my fit improved the χ_ν^2 by 15%. However, I find a much narrower (121 keV) and weaker ($\Gamma_{\alpha_0}/\Gamma=0.08$) level than Ref. 10.

The last level in region E, a 4^+ state at $E_x=19.988$ MeV, is most visible at 49.3°. However, its small strength and the relatively poor fit in this part of the energy range produce a large uncertainty (93 keV) in its width. Omitting either the 4^+ or the 7^- level results in a 10% increase in χ_ν^2.

F. Region F: $20.083\le E_x\le21.054$ MeV, Fig. 9, $\chi_\nu^2=2.69$

Except for the well-known narrow 9^- level at $E_x=20.683$ MeV, Billen[4] did not analyze any of these elastic scattering data. Besides the 9^- level, the fit of Fig. 9 used seven more levels plus the tails of the 6^+ and 4^+ states from region E (with their resonant parameters fixed).

The first level searched on, a 6^+ at $E_x=20.165$ MeV with $\Gamma=281\pm92$ keV and $\Gamma_{\alpha_0}/\Gamma=0.18$, is a new assignment. Bergman and Hobbie[2] from visual inspection of data at only eight angles had suggested a tentative 7^- state at 20.15 MeV of width 250 keV. The broad maximum in the data[4] near 20.15 MeV and at $\theta=66.8°$ and 89.0° [both near zeros of $P_7(\cos\theta)$] and the lack of structure at $\theta=49.3°$ and 104.5° [near zeros of $P_6(\cos\theta)$] are inconsistent with a 7^- assignment and support the present 6^+ assignment. Since the fitting region does not extend 2Γ below this weak resonance, the parameters are not well determined.

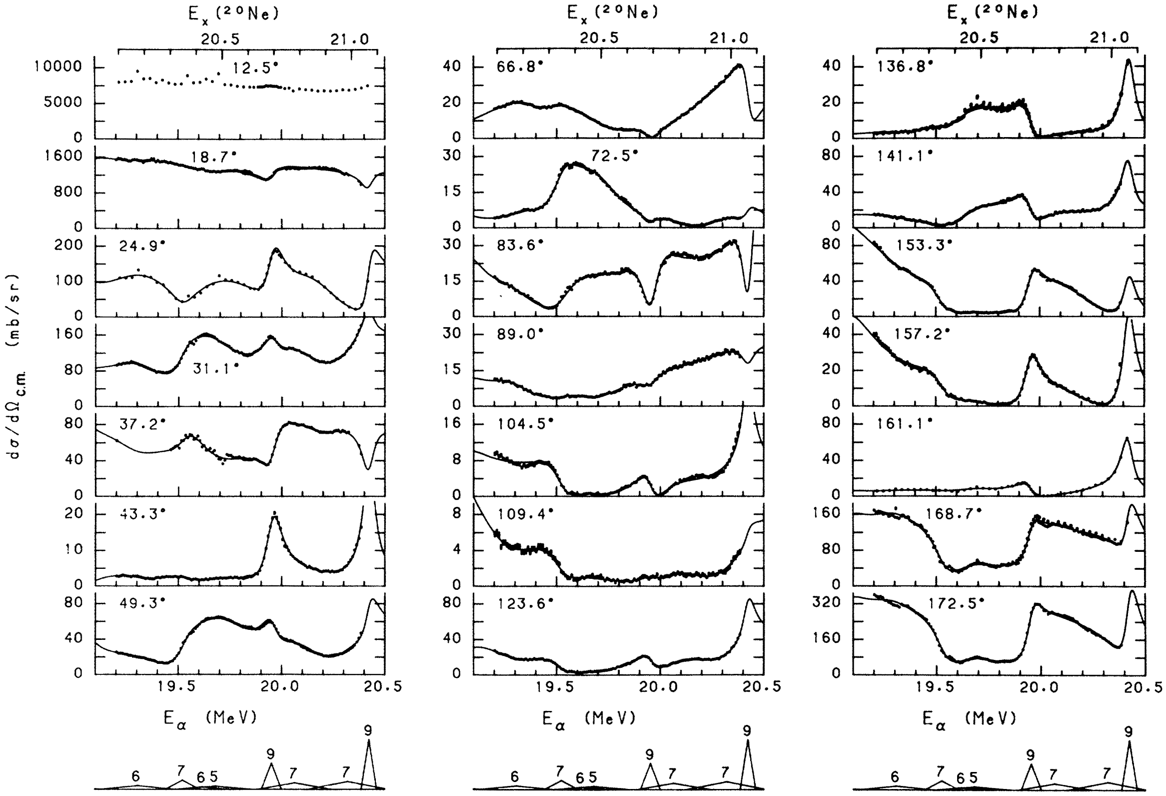

FIG. 9. The region F fit to the elastic data ($\chi_\nu^2=2.69$) employs the eight resonances shown as triangles below the E_α scale plus the tails of a 6^+ and a 4^+ resonance from region E. The triangle height is proportional to $(2J+1)\Gamma_{\alpha_0}/\Gamma$ and the base width equals Γ. See Table I for resonant parameters. The fitting program could handle only 20 angles so 12.5° was not fit.

The 7^- level I find at $E_x=20.341\pm0.013$ MeV has $\Gamma=133\pm33$ keV and a strength of 0.25. Bergman and Hobbie[2] performed a phase-shift analysis (not unique because of the many open channels) of their data near this energy and suggested two levels at 20.4 MeV: a 200-keV 7^- level and a 360-keV 6^+ level. They quote an error in E_x of "up to half the width" of the resonance. My analysis also gives a 6^+ level at 20.416 ± 0.031 MeV, but narrower (212 ± 84 keV) than they indicate.

The next level has $J^\pi=5^-$, is at 20.464 MeV, with 280 ± 69 keV and $\Gamma_{\alpha_0}/\Gamma=0.20\pm0.03$. Hindi *et al.*[6] have reported a tentative 8^+ assignment at $E_x=20.478\pm0.011$ MeV via the ^{12}C(^{12}C,α)^{20}Ne*(α)^{16}O reaction. They quote a branching ratio of 0.66 ± 0.26 and a width of 250 ± 30 keV. I have tried fits with only an 8^+ level and with an $8^+,5^-$ combination and get significantly worse fits compared to having only the 5^- level. An 8^+ level of the large (0.66) resonant strength quoted by Hindi *et al.*[6] would dominate our elastic cross section data. Therefore, the Ref. 6 assignment or branching ratio must be in error.

The most pronounced resonance in this region is a 9^- level at 20.683 MeV with a width (78 ± 11 keV) less than half that of any other level in this range. I agree with the parameters Billen[4] quotes except that I find a larger strength: 0.33 ± 0.03 compared to 0.25 ± 0.02. This state was first assigned 9^- by Bergman and Hobbie.[2] In 1979, via the ^{12}C(^{12}C,α)^{20}Ne*(α)^{16}O reaction, Young *et al.*[5] also found a 9^- assignment for an $\sim$100-keV wide state at $E_x=20.67$ MeV and quoted $\Gamma_{\alpha_0}/\Gamma=0.61\pm0.03$ but with a caveat that the g.s. coincidence peak was broad ($\sim$140 keV) and asymmetric; hence possibly two states were involved. In 1983, Hindi *et al.*,[6] using the same reaction, quote $E_x=20.704\pm0.011$ MeV, $\Gamma\sim120$ keV, but $\Gamma_{\alpha_0}/\Gamma\lesssim0.14$. Hindi *et al.* were unsuccessful in making a spin assignment and apparently believed they were dealing with a different state. However, it is puzzling why they did not also see the strong 9^- state reported by Young *et al.* at this energy. The $\Gamma_{\alpha_0}/\Gamma=0.33\pm0.03$ from the present analysis of Billen's data is certainly in strong disagreement with Hindi *et al.*'s limit of $\lesssim0.14$.

The present analysis also required a new 7^- level at 20.772 MeV with a width of 286 ± 66 keV and a strength of 0.21. This state probably also resonates in the α_1 and α_4 channels.

Another broad 7^- level occurs at $E_x=20.977\pm0.033$ MeV and corresponds to that reported by Bergman and Hobbie[2] at 21.05 MeV. They estimated a width of 200 keV from a nonunique phase shift analysis of their elastic scattering data. When I attempted to combine these two 7^- resonances (at 20.772 and 20.977 MeV) into one broad resonance the χ_ν^2 of the fit almost doubled, and so these two 7^- levels are not just an overparametrization of a single broad state.

The parameters of the final level in this region are very tentative because the resonant energy is just beyond the high energy end of the data. Bergman and Hobbie list a

9^- resonance at $E_x = 21.10$ MeV of width 80 keV. I included a level with these parameters in a fit of this region, and the fitting procedure moved the resonance to 21.059 MeV and reduced the width to 60 keV. As noted earlier, if a resonance is not well inside the fitting region, the resonant parameters are likely to be inaccurate, and the errors underestimated. Hindi *et al.*[6] list a level with width 140 keV at 21.05 ± 0.02 MeV. It is clearly visible in their $^{12}C(^{12}C,\alpha)^{20}Ne$ spectrum, but not in any of the α-decay channels so it may be the same state. Sanders *et al.*[7] via $^{16}O(^{12}C,^{8}Be)^{20}Ne$ report a 9^- resonance with a ground state branching ratio of 0.65 ± 0.15 and a width of 100 ± 50 keV at 21.08 ± 0.03 MeV. These agree well with my tentative results.

G. Region G: $19.063 \leq E_x \leq 20.035$ MeV (inelastic channel), Fig. 10, $\chi_\nu^2 = 1.04$

The Fig. 10 fit employs seven resonances, all of them reported earlier by Billen.[4] (The seventh, from region H, is the 6^+ state at 20.024 MeV and had its parameters held fixed.) However, as described in Sec. II, the small (compared to the widths of the levels) energy range in Billen's fit probably led to considerable imprecision in the resonant parameters he quoted. Table II compares the parameters determined by Billen and this work.

Initial fits to the region $19.26 \leq E_x \leq 19.74$ MeV were made both with and without the weak 7^- level at 19.57 MeV. The larger statistical errors in the α_1 cross sections compared to the α_0 cross sections make obtaining a fit with a good χ_ν^2 much easier. However, it also is more difficult to decide which of two fits with "good" χ_ν^2 is the better. The fit with the 7^- is slightly better than the fit without it, but not overwhelmingly so. However, the data in Fig. 10 at $\theta = 76.6°$ [near a zero of $P_6(\cos\theta)$] certainly suggest a state at 19.57 MeV. Based on these two results, I decided to include the weak 7^- level in the remaining fits to this region and to footnote it as not needed for an acceptable χ_ν^2. This approach modifies a preliminary report which omitted the 7^- level (see note h of Table 20.21 in Ref. 20). The three levels (19.28, 19.44, and 19.57 MeV) were fit as a group by Billen, with the 7^- just outside the fitting region. Thus his resonant parameters for the 7^- probably are not accurate.

The first 6^+ level at $E_x = 19.122$ MeV is at the low energy end of the α_1 data, and so both in Ref. 4 and the present case the resonant parameters are not well determined and the error analysis underestimates the errors. The present level parameters are consistent with the parameters of the 6^+ level seen in the elastic channel at 19.161 MeV (see discussion in region E). Assuming that it is the same 6^+ state in both channels, one has $\Gamma_{\alpha_0} = 90$

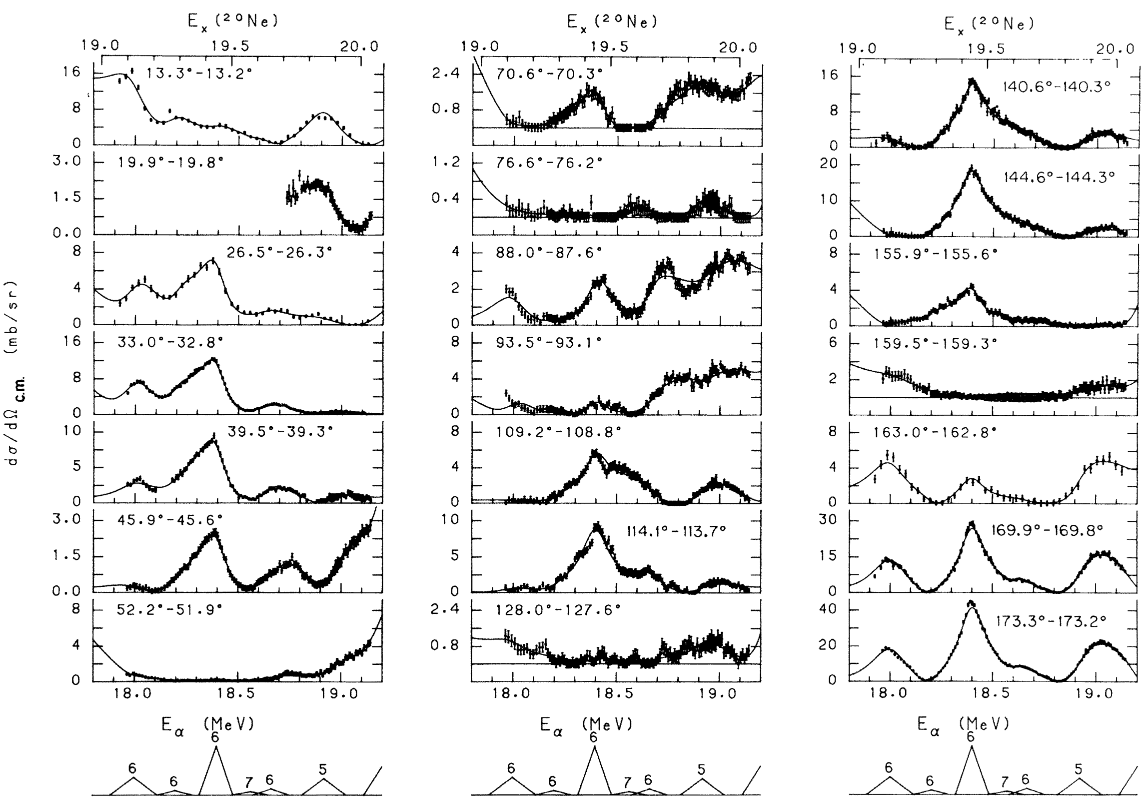

FIG. 10. The region G fit to the *inelastic* data ($\chi_\nu^2 = 1.04$) employs the six resonances shown as triangles below the E_α scale plus the tail of a 7^- resonance from region H. The triangle height is proportional to $(2J+1)(\Gamma_{\alpha_0}\Gamma_{\alpha_1})^{1/2}/\Gamma$ and the base width equals Γ. See Table I for resonant parameters. The fitting program could handle only 20 angles; therefore 19° was not fit. Note also that the ordinate zero level is offset at several angles.

TABLE II. Comparison of parameters for inelastic resonances. The errors are determined by varying the parameter (holding the others fixed) until the χ^2 of the fit within 2Γ of E_x doubles. The E_α and E_x errors include the absolute energy uncertainty.

Present work					Ref. 4		
E_α (MeV±keV)	E_x (MeV±keV)	$\Gamma_{c.m.}$ (keV)	RS[a] (%)	J	E_x[b] (MeV±keV)	$\Gamma_{c.m.}$[b] (keV)	RS[a,b] (%)
17.999±23	19.122±18	184±29	23±2	6	19.113±10	149±18	42±1
18.198±17	19.281±13	137±32	12±2	6	19.322± 9	123±10	27±1
18.397±11	19.440± 9	131±13	38±2	6	19.437±10	102± 7	47±1
18.563±25[c]	19.573±20[c]	142±49[c]	9±2[c]	7[c]	19.577±11	50± 8	20±3
18.662±23	19.652±18	139±35	14±2	6	19.648±10	89± 8	33±1
18.918±11	19.856± 9	172±24	26±2	5	19.914±12	203±19	38±2
19.227±28	20.103±22	187±33	29±3	7	20.130±17	156±21	30±2
19.464±19	20.293±15	252±36	28±3	7	20.317±12	203±19	34±2
19.651±32	20.442±25	366±54	32±3	6	20.433±16	346±32	44±2
20.025±34	20.741±27	198±59	13±2	7	20.782±11	122±13	40±2
20.260±32	20.929±25	302±61	21±2	7	20.920±12	181±22	34±2

[a]RS (resonant strength)$=(\Gamma_{\alpha_0}\Gamma_{\alpha_1})^{1/2}/\Gamma$.
[b]The analysis by Ref. 4 included a linear energy dependence of the nonresonant phase for each level and sometimes did not include a fitting region large compared to Γ. As a consequence (see my discussion Sec. II) the Ref. 4 errors may be seriously underestimated and the resulting width tends to be too small. Because width and strength parameters seem correlated, the too small width often resulted in a too large strength parameter.
[c]An acceptable χ_ν^2 occurs without this weak state, but including it gives an 8.5% improvement in χ_ν^2.

keV, $\Gamma_{\alpha_1}=26$ keV, and $\theta_{\alpha_1}^2=5\pm1\%$.

The last resonance in this region is the 5^- at 19.856 MeV. Billen's parameters for this level come from a fit of this level (at 19.91 MeV), two 7^- levels at 20.13 and 20.32 MeV, and a 6^+ level at 20.43 MeV in a fitting region from 19.8 to 20.6 MeV, but his fitting region starts only $\Gamma/2$ below his resonant energy, while my fitting region ends $\sim\Gamma$ above my resonant energy.

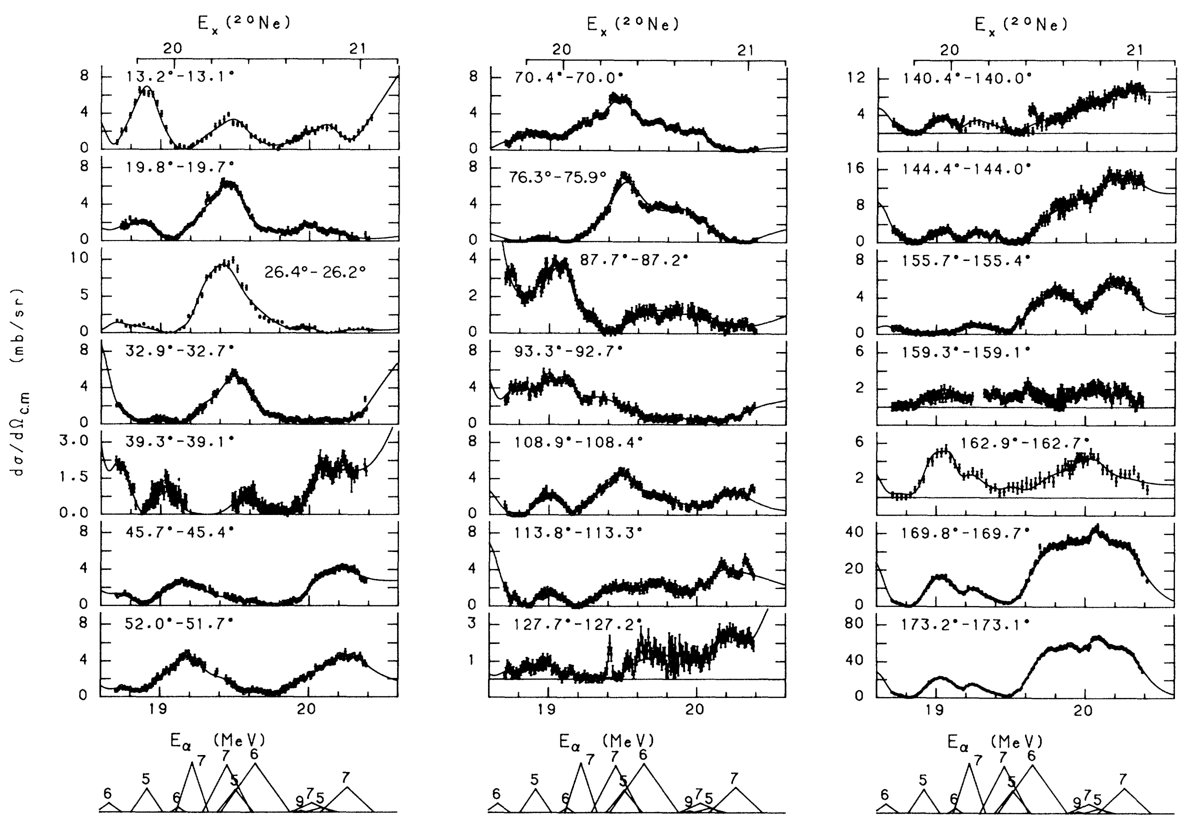

FIG. 11. The region H fit to the *inelastic* data ($\chi_\nu^2=1.23$) employs the 11 resonances shown as triangles below the E_α scale. The triangle height is proportional to $(2J+1)(\Gamma_{\alpha_0}\Gamma_{\alpha_1})^{1/2}/\Gamma$ and the base width equals Γ. See Table I for resonant parameters. The fitting program could handle only 20 angles so 159° was not fit. Note that the ordinate zero level is offset at several angles.

H. Region H: $19.684 \le E_x \le 21.054$ MeV (inelastic channel), Fig. 11, $\chi_\nu^2 = 1.23$

This region contains nine resonances, five of them previously reported by Billen. Table II compares the parameters determined by Billen and this work. The fit in Fig. 11 includes two resonances with parameters fixed from region G.

The first level searched on is the weak 6^+ at 20.024 MeV. It was not observed by Billen, but poor fits from 19.9 to 20.1 MeV at $\theta = 87.7°$, 108.9°, 113.8°, 144.4°, and 169.8° and a nearly zero cross section at 76.3° indicated that a 6^+ state was needed. Adding this 6^+ reduced the χ_ν^2 by 15%.

The two 7^- resonances agree within errors with the values Billen determined. Another new level is a 5^- at 20.338 MeV with $\Gamma = 189 \pm 40$ keV and a strength of 0.26 ± 0.03. However, the 6^+ level at 20.442 MeV was also seen by Billen.

Although not apparent in the inelastic data, the strong 9^- seen in the elastic channel at 20.683 MeV was included in the inelastic fits. Letting all the resonant parameters vary made little change in either the E_x or the Γ found from the elastic channel analysis, but the strength needed was < 0.04. In subsequent fits the Γ and E_x were held fixed at the elastic channel values. The resultant strength of 0.04 ± 0.02 corresponds to $\Gamma_{\alpha_1} \sim 0.4$ keV. Based on the small decrease in χ^2, such a weak level normally would not be included in the final fit, but it gives a useful limit for the branch to ^{16}O (6.05) and is consistent with the Young *et al.*[5] limit of $\Gamma_{\alpha_1}/\Gamma < 0.02$ observed via the $^{12}C(^{12}C,\alpha)^{20}Ne^*(\alpha)^{16}O^*$. They also quote a Γ_{α_2}/Γ of ≥ 0.22, and Billen's α_2 and α_3 data do show pronounced structure at this energy. Hindi *et al.*[6] apparently also see this level by the same $^{12}C(^{12}C,\alpha)^{20}Ne$ reaction at 20.704 ± 0.011 MeV but with a larger width (120 keV vs 78 keV) and a Γ_{α_0}/Γ in strong disagreement with present (α,α_0) data; see discussion of region F.

Another new state is the 5^- level at 20.797 MeV. When fitting a somewhat smaller region (20.23—21.03 MeV) with four resonances (7^- at 20.293 MeV, 6^+ at 20.442 MeV, 7^- at 20.741 MeV, and 7^- at 20.929 MeV), adding the 5^- level reduced the χ_ν^2 by 40%. It improved the fit most at $\theta = 70.0°$, 144.0°, 169.7°, and 173.1°.

The final state is the broad 7^- at 20.929 MeV. Since the level is near the upper limit of the available data, the parameters found both by Billen[4] and in the present analysis may have larger uncertainties than indicated. Quite likely this level is the same as that in the elastic channel at 20.977 MeV (see discussion of region F). If it is, then one has $\Gamma_{\alpha_0} = 72$ keV, $\Gamma_{\alpha_1} = 57$ keV, and $\theta_{\alpha_1}^2 = 13.5 \pm 3.5\%$. While Hindi *et al.*[6] via $^{12}C(^{12}C,\alpha)^{20}Ne$ report a level at $E_x = 20.89 \pm 0.03$ MeV, they give insufficient information to tell whether it could correspond.

IV. CONCLUSIONS

I have deduced resonant parameters for 56 levels in ^{20}Ne formed by elastic and inelastic scattering of alpha particles from ^{16}O. Of the 56 levels, 41 show as resonances in the elastic channel, 11 as resonances in the first inelastic channel [$E_x(^{16}O) = 6.05$ MeV], and 4 as resonances in both the elastic and inelastic channels. The analysis confirms several resonances reported by other investigators who use a variety of reactions to form states in ^{20}Ne, but 16 of the levels have not been previously reported.

The present analysis generally gave larger widths and smaller resonant strengths than Billen reported.[4] The difference appears to arise from a change in the parametrization for the nonresonant phases (see Sec. II). The change removed the linear energy dependence of the nonresonant phase of each resonance, but instead let the background term's phase vary linearly with energy over the fitting region.

The levels deduced by both Billen[4] and myself are in good agreement except that the above change makes my levels broader and weaker than the values cited by Billen. The present parametrization, and therefore the present resonant parameters, should be more realistic than those reported by Billen.

This analysis also shows that for accurate parameters one must include energy regions which are large compared to the widths of the resonances. Since the level density in ^{20}Ne is high and the levels broad at these energies, the cross section at any energy is influenced by the tails of several resonances. Thus many levels must be included (even some outside the fitting region). Satisfactory fits usually required the inclusion of the nearest (sometimes two) broad resonance on each side of the fitting region.

As discussed in Sec. II, to calculate parameter uncertainties, a subset of the data points which were within $E_{res} \pm 2\Gamma$ was used. Since the present fitting regions were much larger than Billen[4] used, the error analysis generally included more data points affected by each resonance; thus both more realistic parameters and more realistic uncertainties should result. As with the errors quoted by Billen,[4] the present error analysis neglects correlations between resonant parameters.

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