

## Meson theory of nucleon-nucleon scattering up to 2 GeV

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We show that it is possible to construct a meson-exchange Hamiltonian for  $N$ ,  $\pi$ ,  $\Delta$ , and  $N^*$  (1470 MeV) to describe NN scattering up to 2 GeV. The model consists of: (a) vertex interactions  $\pi N \leftrightarrow \Delta$  or  $N^*$ , and  $\pi \Delta \leftrightarrow \Delta$  or  $N^*$  with which an isobar model is constructed to describe the  $P_{33}$  and  $P_{11}$   $\pi N$  scattering phase shifts up to 1 GeV; (b) the transition interactions from NN to  $N\Delta$ ,  $\Delta\Delta$ ,  $NN^*$ , and  $N^*N^*$  are determined from one-pion and one-rho exchange mechanisms; (c) the  $NN \rightarrow NN$  interaction is directly derived from the *Paris potential* by using a *momentum-dependent* procedure to subtract the contributions from intermediate states involving  $\Delta$  or  $N^*$ . The  $NN \rightarrow NN$  scattering equation is cast into the familiar coupled-channel form, but with a highly nonlocal isobar self-energy  $\Sigma(E, p)$  calculated from the vertex interactions  $\pi N \leftrightarrow \Delta$  or  $N^*$  in a dynamical three-body approach. Both the isospin  $T=1$  and  $T=0$  NN scattering phase shifts of Arndt *et al.* up to 1 GeV can be described to a very large extent by the model. The fits are, on the average, better than most of the previous NN calculations. The model also describes reasonably well both the magnitudes and signs of the NN total cross sections  $\sigma^{\text{tot}}$ ,  $\Delta\sigma_T^{\text{tot}}$ , and  $\Delta\sigma_L^{\text{tot}}$  up to 2 GeV, except the strong energy dependences in the region near 800 MeV. We discuss the origin of this problem in connection with future necessary improvements of the model and the questions about the dibaryon resonances. The model can be used for a unified approach to study the isobar-nucleus dynamics at both low and intermediate energies.

[ NUCLEAR REACTIONS Isobar model for  $\Delta$  and  $N^*$  excitation, meson-exchange theory of NN scattering from 0 to 2 GeV. ]

### I. INTRODUCTION

The meson theory of nucleon-nucleon (NN) interaction has long been developed<sup>1</sup> according to two essential assumptions: (a) the NN force at the long and intermediate ranges (roughly  $r \geq 1$  fm) can be described by the exchange of mesons; (b) the short range part can be treated phenomenologically by fitting the NN data. In the past decade, the development of the theory reached its peak with impressive results of two-pion-exchange calculations obtained by the Paris group<sup>2</sup> and others.<sup>3,4</sup> Many low energy nuclear properties have since been calculated from the Paris potential. An important question to ask is the following: Can this theory be extended to higher energies? In this paper, we extend the work of Ref. 5 to show that such a meson theory can be developed to describe NN scattering up to about 2 GeV (all collision energies referred to in this paper are defined in the laboratory frame).

In the higher energy regions  $E \lesssim 1$  or 2 GeV, the  $\Delta$  excitation and pion production are two essential ingredients of any theory. The existing studies of intermediate energy NN scattering (including  $\pi d$  scattering) can be roughly divided into two different approaches. The first one<sup>6-13</sup> is to emphasize the unitarity of the theory, which immediately leads to nontrivial numerical problems<sup>6</sup> of treating the  $\pi NN$  branch cut. Largely owing to the computational difficulties, most calculations along this line have been carried out by using either the low-rank separable interaction or an incomplete description of baryon-baryon in-

teractions. The simple isobar model of  $\Delta$  excitation is frequently introduced in these studies. The second approach is to construct coupled-channel models<sup>14-18</sup> with an appropriate prescription of the width of the *off-mass-shell*  $\Delta$ . These calculations are usually carried out by using the standard one-boson-exchange model,<sup>19-21</sup> without taking advantage of using the results of the nonperturbative  $2\pi$ -exchange calculation as achieved by the Paris potential. The success of either one of these two approaches in describing the NN and  $\pi d$  scattering is still very limited.<sup>22</sup> Major improvements are clearly needed for a detailed understanding of extensive NN and  $\pi d$  data from various meson facilities.

At the present time, it is reasonable to assume that the most realistic approach is to follow the unitary formulation<sup>6-13</sup> of the problem and use a meson theory of nuclear force which is closely related to the Paris potential. The purpose of this work is to continue the work of Ref. 5, to report the progress we have made in this direction.

In Ref. 5, we constructed a meson-exchange Hamiltonian for  $\pi$ ,  $N$ , and  $\Delta$ , which gives a satisfactory description of NN scattering phase shifts up to 1 GeV in the  $T=1$  channel, where  $T$  is total isospin. The model consists of a  $\pi N \leftrightarrow \Delta$  vertex interaction and three baryon-baryon interactions  $NN \rightarrow NN$ ,  $NN \leftrightarrow N\Delta$ , and  $NN \leftrightarrow \Delta\Delta$ . The success of the model is attributed to the following: (a) the  $\pi N \leftrightarrow \Delta$  effect on NN scattering is treated in a dynamical three-body approach<sup>6,8</sup>; (b) a nonlocal  $NN \rightarrow NN$  interaction of the model Hamiltonian is derived from the Paris potential by using a *momentum-dependent* procedure to

subtract contributions from the  $N\Delta$  and  $\Delta\Delta$  intermediate states; (c) the transition interactions  $NN\leftrightarrow N\Delta$ ,  $NN\leftrightarrow\Delta\Delta$  are determined by one-pion and one-rho exchange. In this paper, we report an extension of the model to also include the  $N^*$  (1470) resonance ( $S=I=\frac{1}{2}$ ). This extension is necessary for two reasons. First, since the threshold energies for exciting  $\Delta\Delta$  and  $NN^*$  are about the same,  $N^*$  and  $\Delta$  must be treated<sup>17</sup> on the same footing in order to realistically describe the inelasticities in the  $NN$   $T=0$  channels (note that the  $N\Delta$  state does not contribute to  $T=0$  channels). Second, to examine the problem of dibaryon resonances as suggested by pp polarization measurements,<sup>23</sup> it is necessary to have a careful description of channel coupling effects,<sup>15-17,24</sup> for which the  $N^*$  could be as important as the  $\Delta$  in the considered energy region. In addition, the extended model will allow us to describe  $NN$  scattering at higher energies  $\lesssim 2$  GeV, where  $2\pi$  production through  $N^*$  can also be investigated.

Since this work follows precisely the approach of Ref. 5, no detailed discussion of the model will be repeated here, except for new features owing to the presence of  $N^*$ . However, the formalism needed in practice to calculate the  $NN\rightarrow NN$  interaction of the model Hamiltonian and to treat the decay mechanism  $\Delta$  or  $N^*\leftrightarrow\pi N$  and  $\pi\pi N$  will be given explicitly.

In Sec. II, we present a simple isobar model for the excitations of  $\Delta$  and  $N^*$  in  $\pi N$  scattering. Compared with previous work,<sup>8,9</sup> the new feature of this model is to describe the  $\pi$  production from  $\pi N$  collision. This isobar model will then be used in Sec. III to describe pion production in  $NN$  collisions up to 2 GeV. In Sec. IV, we discuss two main results. First, we show that the model can describe to a large extent the  $NN$  phase shifts<sup>32</sup> up to  $\sim 1$  GeV in both the  $T=0$  and  $T=1$   $NN$  channels. Second, we discuss the calculations of the pp and np reaction cross sections and total cross sections  $\sigma^{\text{tot}}$ ,  $\Delta\sigma_T^{\text{tot}}$ , and  $\Delta\sigma_L^{\text{tot}}$  for various spin orientations defined in Ref. 23. The model gives, on the average, satisfactory descriptions of *both* the magnitudes and signs of these three observables in the entire energy region 0–2 GeV. However, the model does not describe very well the energy dependences of the total cross sections in the energy region from 600 to 1000 MeV. We discuss the origin of this problem in connection with future necessary improvements of the model and the questions about the dibaryon resonances. Section IV is devoted to summarizing our approach.

## II. ISOBAR MODEL FOR $\Delta$ AND $N^*$

To proceed, we follow earlier studies<sup>25</sup> of  $\pi N$  scattering to construct an isobar model for the  $\Delta$  and  $N^*$  excitations. The main feature<sup>26</sup> of  $N^*$  is its large decay width to the  $\pi\pi N$  state through the  $\pi\Delta$  state. It is therefore reasonable to extend the approach of Ref. 8 to assume that the  $\pi N$  scattering in  $P_{11}$  and  $P_{33}$  channels can be described by a model Hamiltonian (in the c.m. frame)

$$h = h_0 + h', \quad (1)$$

where  $h_0$  is the sum of relativistic free energy operators  $E_\pi(\vec{k})$ ,  $E_N(\vec{p})$ ,  $E_\Delta(\vec{p})$ , and  $E_{N^*}(\vec{p})$  for  $\pi$ ,  $N$ ,  $\Delta$ , and  $N^*$ , respectively. The interaction  $h'$  is the sum of vertex in-

teractions shown in Fig. 1(a)

$$h' = h_{01} + h_{02} + h_{31} + h_{32} + h_{10} + h_{20} + h_{13} + h_{23}, \quad (2)$$

where the subscripts 0, 1, 2, and 3 denote the  $\pi N$ ,  $\Delta$ ,  $N^*$ , and  $\pi\Delta$  states, respectively. Note that  $h_{ij}^\dagger = h_{ji}$  in Eq. (2), and hence  $h'$  is a Hermitian operator. The main feature of this model is to have pion production in  $\pi N$  scattering, as illustrated in Fig. 1(b). Our approach is to determine the vertex interaction  $h'$  by fitting the  $P_{11}$  and  $P_{33}$   $\pi N$  phase shifts up to  $\sim 1$  GeV laboratory energy. To achieve this, we need to solve the  $\pi N$  scattering equation defined by<sup>8</sup>

$$t(w) = h' + h' \frac{1}{w - h + i\epsilon} h'. \quad (3)$$

To account for pion production, we extend the method of Ref. 8 to solve Eq. (3) in the subspace

$$S = B \oplus \pi N \oplus \pi\Delta \oplus \pi\pi N,$$

where  $B$  is  $\Delta$  in the  $P_{33}$  channel and is  $N^*$  in the  $P_{11}$  channel. The major complication of the resulting scattering equation in  $S$  is owing to the coupling to the three-body  $\pi\pi N$  channel [Fig. 1(b)]. The pion can dress  $\Delta$  and  $N^*$  as illustrated in Fig. 2(a). In the intermediate  $\pi\Delta$  state [lower parts of Fig. 2(a)], the interaction  $\pi N\leftrightarrow\Delta$  can induce  $2\pi$  contributions as shown in Figs. 2(b) and (c). If we neglect the pion crossing mechanism [Fig. 2(c)] between any two intermediate  $\pi\Delta$  states, the partial-wave solutions of Eq. (3) in the subspace  $S$  take simple algebraic forms in the  $\pi N$  c.m. frame,

$$t_\alpha(q, q_0, w) = \frac{h_{0\alpha}(q) h_{0\alpha}^*(q_0)}{w - m_\alpha - \Sigma_\alpha(w)}, \quad \alpha = 1, 2, \quad (4)$$

where  $q = |\vec{q}|$  is the  $\pi N$  relative momentum,  $w$  is the collision energy, and  $m_1 = m_\Delta$  and  $m_2 = m_{N^*}$ , are, respec-

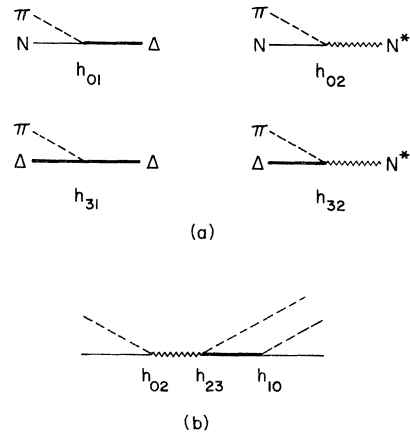


FIG. 1. (a) Vertex interactions  $h_{ij}$  in Eq. (2);  $i=0, 1, 2$ , and 3 denotes, respectively,  $\pi N$ ,  $\Delta$ ,  $N^*$ , and  $\pi\Delta$  states. (b) Pion production mechanism induced by vertex interactions  $h_{ij}$ .

tively, the *bare* masses of  $\Delta$  and  $N^*$ ; i.e.,  $\alpha=1$  and 2 represents, respectively, the  $\pi N P_{33}$  and  $P_{11}$  channels. The isobar self-energy  $\Sigma_\alpha$  has two components [Fig. 2(a)]

$$\Sigma_\alpha(w) = \Sigma_{\alpha,\pi}(w) + \Sigma_{\alpha,2\pi}(w), \quad \alpha=1,2, \quad (5)$$

which can be explicitly calculated from the vertex interactions

$$\Sigma_{\alpha,\pi}(w) = \int_0^\infty \frac{|h_{0\alpha}(q')|^2 q'^2 dq'}{w - E_\pi(q') - E_N(q') + i\epsilon}, \quad (6)$$

$$\Sigma_{\alpha,2\pi}(w) = \int_0^\infty \frac{|h_{\alpha 3}(q')|^2 q'^2 dq'}{w - E_\pi(q') - E_\Delta(q') - \Pi_\Delta(w, q')}, \quad (7)$$

with

$$\Pi_\Delta(w, q') = \int_0^\infty \frac{|h_{01}(q'')|^2 q''^2 dq''}{w - E_\pi(q') - \{[E_\pi(q'') + E_N(q'')]^2 + q''^2\}^{1/2} + i\epsilon}. \quad (8)$$

The isobar self-energies  $\Sigma_{\alpha,\pi}$  and  $\Sigma_{\alpha,2\pi}$  contain the contributions from one-pion and two-pion intermediate states as illustrated in Figs. 2(a) and (b). In our approximation,  $\Sigma_{\alpha,2\pi}$  only contains the  $\Delta$  self-energy  $\Pi_\Delta$  in the presence of a spectator pion [Fig. 2(b)]. If the pion crossing mechanism  $\Pi_\Delta^c$  [Fig. 2(c)] is included, the solution cannot be written in a simple algebraic form as given by Eqs. (4)–(8). When the  $\pi N$  phase shifts are fitted properly, the effects of the pion crossing mechanism and other neglected mechanisms, such as that owing to the pion rescattering,<sup>25</sup> are phenomenologically included in  $h'$ . The above simplified solution of  $\Sigma_\alpha$  will allow us to avoid unmanageable complications when the same solution of the model is needed in the calculations of NN scattering.

All vertex functions are parametrized as

$$h_{\alpha\beta}(q) = g_{\alpha\beta} \frac{q}{\mu} \frac{1}{\sqrt{2(M+\mu)}} \left[ \frac{\Lambda_{\alpha\beta}^2}{\Lambda_{\alpha\beta}^2 + q^2} \right]^2, \quad (9)$$

where  $m$  and  $\mu$  are, respectively, the masses of the nucleon and the pion. We adjust the parameters  $g_{\alpha\beta}$  and  $\Lambda_{\alpha\beta}$  and the bare masses  $m_\Delta$  and  $m_{N^*}$  to fit the  $P_{33}$  and  $P_{11}$   $\pi N$  phase shifts<sup>27</sup> up to 1 GeV kinetic energy. The results are shown in Fig. 3 and Table I. The large inelasticity  $\eta$  in  $P_{11}$  can be reasonably fitted, while the simple isobar model cannot describe the small but negative  $P_{11}$  phases at low energy. Other mechanisms must be considered to resolve this problem and get better fits to the data at high energy. We do not attempt to solve this nontrivial problem<sup>28</sup> here. Instead, we argue that the present model is sufficient to describe the main physics of pion production from NN collisions.

### III. NN SCATTERING EQUATION

We now extend the approach of Refs. 5 and 8 to include  $N^*$  in the study of NN scattering. Formal and somewhat

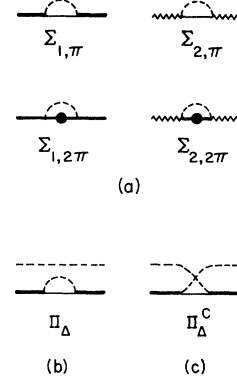


FIG. 2. (a) One-pion and two-pion contributions to the isobar self-energies defined by Eqs. (5)–(7). (b)  $\Delta$  self-energy calculated in the presence of a spectator pion [given by Eq. (8)]. (c) Pion crossing mechanism between two  $\pi\Delta$  states.

lengthy derivations of all equations are parallel to those given in Ref. 8, with some obvious extensions. Therefore, we will only focus the reader's attention here on the necessary formalism and numerical procedures by which our results are obtained.

The model Hamiltonian for NN scattering is assumed to be

$$H = H_0 + h' + \sum_{i=1}^5 v^i, \quad (10)$$

where  $H_0$  is just the  $h_0$  of Eq. (1), the sum of free energy operators for  $\pi$ ,  $N$ ,  $\Delta$ , and  $N^*$ , and  $v^i$  for  $i=1, 2, 3, 4$ , and 5 are, respectively, the transition interactions from NN to NN,  $N\Delta$ ,  $\Delta\Delta$ ,  $NN^*$ , and  $N^*N^*$  [Fig. 4(a)]. We consider NN scattering in the subspace

$$C = BB \oplus BN\pi \oplus NN\pi\pi,$$

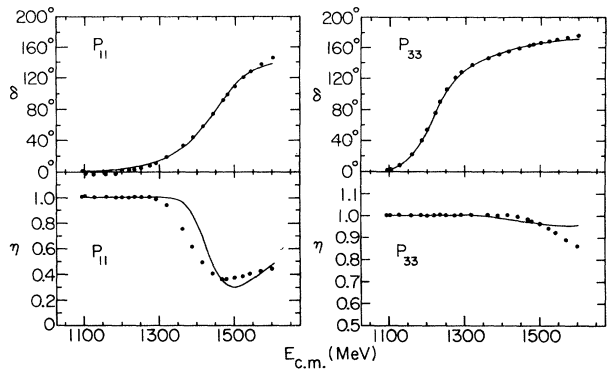


FIG. 3. The calculated  $\pi N$  scattering phase shifts are compared with the data of Ref. 27. ( $E_{c.m.}$  is the total  $\pi N$  energy in c.m. frame.)

TABLE I. The parameters of the isobar model for  $\Delta(P_{33})$  and  $N^*(P_{11})$  excitations. The parameters are defined in Eq. (9) and Fig. 1.

$h'_{\alpha\beta}$	$g_{\alpha\beta}$	$\Lambda_{\alpha\beta}$ (MeV/c)	
$\pi N \leftrightarrow \Delta$	0.98	358	$M_\Delta = 1300$ MeV
$\pi N \leftrightarrow N^*$	0.463	599	$M_{N^*} = 1575$ MeV
$\pi \Delta \leftrightarrow N^*$	2.013	251	
$\pi \Delta \leftrightarrow \Delta$	0.689	356.4	

where  $B$  is  $N$ ,  $\Delta$ , or  $N^*$ . The main feature of the model Hamiltonian Eq. (10) is to have pion production from  $NN$  collisions [Fig. 4(b)]. The interactions between isobar channels ( $i=2, 3, 4$ , and  $5$ ) within this model are automatically generated by the vertex interaction  $h'$ . The low order mechanisms are the pion contributions to the isobar self-energies in the presence of a spectator baryon [Figs. 5(a) and (b)], and the pion exchange between isobar channels [Fig. 5(c)]. Furthermore, two baryons can interact in the  $\pi$  plus two-baryon channels, such as shown in Fig. 5(d). A complete calculation including all of these mechanisms between isobar channels is simply beyond our present numerical capabilities in dealing with large complex matrix equations (except in the separable model). Instead, we follow Ref. 5 and other approaches<sup>14-18</sup> to keep only pion contributions to the isobar self-energies [Figs. 5(a) and (b)]. As suggested by the work of Betz and Lee,<sup>8</sup>

$$T_{l1}^\alpha(p', p, E) = V_{l1}^\alpha(p', p, E) + \sum_{l''} \int \frac{p''^2 dp'' V_{l''1}^\alpha(p', p'', E) T_{l''1}^\alpha(p'', p, E)}{E - 2E_N(p'') + i\epsilon}, \quad (11)$$

where  $l$  is the relative orbital angular momentum. The energy-dependent effective  $NN$  interaction  $V_{l1}^\alpha$  contains all contributions from the coupling of  $NN$  to inelastic channels involving pions. In deriving  $V_{l1}^\alpha$ , we limit the number of pions in any intermediate state to be less than 2 and keep only self-energy contributions from the pion to one of the baryons [such as Figs. 5(a) and (b)]. Then, it is straightforward to extend the procedures given in Sec. IV of Ref. 8 to obtain

$$V_{l1}^\alpha(p', p, E) = v_{l1}^{1\alpha}(p', p, E) + \sum_{i=2}^5 \sum_{l''s''} \int p''^2 dp'' \frac{v_{l''s''}^{i\alpha}(p', p'') v_{l''s''}^{i\alpha}(p'', p)}{E - [H_0(p'')]_i - \Sigma_i(w_i(E, p''))}, \quad (12)$$

where  $s''$  is the total spin of channels containing  $\Delta$  or  $N^*$ ;  $[H_0(p'')]_i$  is the free energy of the  $i$ th two-baryon state; e.g.,

$$[H_0(p'')]_2 = E_N(p'') + E_\Delta(p''),$$

etc.; and  $v_{l''s''}^{i\alpha}$  is the partial-wave matrix element of  $v^i$  of Eq. (10) in momentum space. All the pion contributions are contained in the isobar self-energy  $\Sigma_i(w_i(E, p''))$  which depends on the collision energy  $E$  and the intermediate relative momentum  $\vec{p}''$ . In practice, the baryons are treated nonrelativistically in calculating  $\Sigma_i(w_i(E, p''))$ . Then, for each  $E$  and  $p''$ , the isobar self-energies of each intermediate state  $i$  in Eq. (12) can be calculated from Eqs. (5)–(8) by substituting

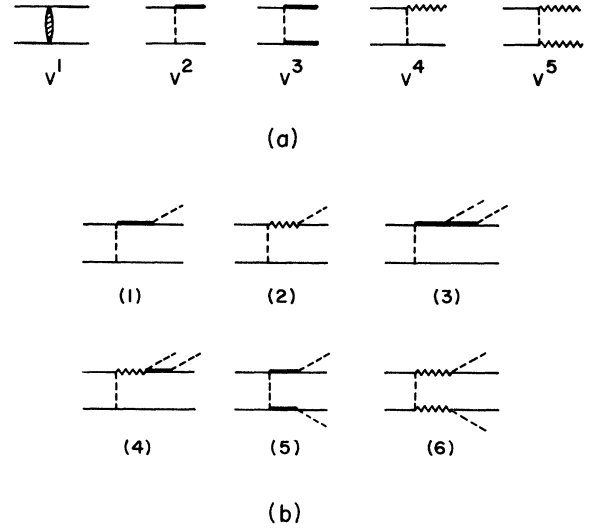


FIG. 4. (a) The baryon-baryon interactions  $v^i$  of the model Hamiltonian equation (10). (b) The mechanisms of one-pion and two-pion production generated by the model Hamiltonian equation (10).

this approximation is probably acceptable except in channels in which the isobar-nucleon state is in the relative  $l=0$  wave, such as  ${}^5S_2(N\Delta)$ .

With the above simplifications, the  $NN \rightarrow NN$  scattering equation in each partial-wave eigenchannel  $\alpha = JST$  can be cast, in the c.m. frame into

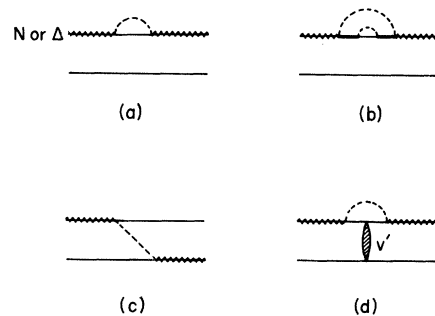


FIG. 5. (a) and (b) are, respectively, the one-pion and two-pion contributions to the isobar self-energies in the presence of a spectator baryon. (c) One-pion-exchange  $N\Delta \leftrightarrow \Delta N$  and  $NN^* \leftrightarrow N^*N$  interactions induced by the vertex interactions  $h'$ . (d)  $NN$  interaction in the  $\pi NN$  intermediate state.

$$w \rightarrow w_i(E, p'') = E - \tilde{m}_i - \frac{p''^2}{2\tilde{m}_i} - \frac{p''^2}{2[\tilde{m}_i + E_\pi(q')]} ,$$

$$i = 2, 3, 4, 5 , \quad (13)$$

where  $\tilde{m}_2 = \tilde{m}_4 = m$ ;  $\tilde{m}_3 = m_\Delta$ ;  $\tilde{m}_5 = m_{N^*}$ ;  $\bar{m} = m$  and  $m_\Delta$ , respectively, for calculating  $\Sigma_{\alpha, \pi}$  and  $\Sigma_{\alpha, 2\pi}$ ; and  $E_\pi(q')$  is the pion energy evaluated in the  $\pi N$  or  $\pi\Delta$  c.m. frame in which the integrands of Eqs. (6)–(8) are defined. In doing this calculation, the nucleon is also treated nonrelativistically in Eqs. (6)–(8). Clearly, our procedures of calculating  $\Sigma_i$  are very different from those in Refs. 14–18.

As emphasized in Ref. 5, the energy and momentum dependences of  $\Sigma_i(w_i(E, p''))$  are the consequences of treating pion production inelastic cuts. They have important dynamical effects on NN scattering. Similar forms of  $\Sigma_i$  are also seen in any approach<sup>6</sup> which rigorously accounts for the  $\pi NN$  cut in NN scattering. Including the  $\pi\pi NN$  in  $\Sigma_i$  [through Eqs. (7) and (8)] is a new feature of this work. At energies below the pion production threshold  $E \leq 280$  MeV in the laboratory frame,  $\Sigma_i$  is real and leads to pure elastic scattering from solving Eq. (11). The model will produce NN inelasticity when  $\Sigma_i$  becomes complex at higher energies. This “off-shell width” effect of the isobar has been found in Ref. 5 to give a satisfactory description of  $T=1$  NN scattering phase shifts up to 1 GeV. Including  $N^*$  here, we can now examine also the inelasticities in  $T=0$  channels and NN scattering at higher energies up to  $\sim 2$  GeV.

Our remaining tasks are to define the matrix elements of the  $NN \rightarrow NN$  interaction  $v_{i_1 i_2}^{\alpha} (p', p)$  and transition interactions  $v_{i_1 i_2}^{\alpha} (p', p)$  for  $i = 2, 3, 4$ , and 5. All transition interactions have a one-pion-exchange evaluated by taking appropriate static limits of Feynman amplitudes. A form factor

$$(\Lambda^2 - \mu^2)/(q^2 + \Lambda^2)$$

is introduced in each meson-baryon-baryon vertex to regularize the interaction at a short distance. Following Ref. 21, we also include the one-rho exchange in the transitions to  $N\Delta$  and  $\Delta\Delta$  states. The resulting potentials in  $r$  space have been explicitly given by Niephaus *et al.*<sup>21</sup> for transitions to  $N\Delta$  and  $\Delta\Delta$ , and by Lomon<sup>17</sup> for  $NN^*$  and  $N^*N^*$ . To limit the number of parameters, we also take their coupling constants determined from the decay widths of  $\Delta$  and  $N^*$ . In this way, the cutoff parameter  $\Lambda$  of the form factors is the only parameter of the transition interactions. The matrix elements  $v_{i_1 i_2}^{\alpha} (p', p'')$  can be straightforwardly expressed<sup>29</sup> in terms of combinations of Legendre functions of the second kind  $Q_l(z)$  and can be calculated very accurately.

The last step of our calculation is to define the  $NN \rightarrow NN$  interaction  $v^1$ . Following the procedure introduced in Ref. 5,  $v^1$  is defined by subtracting an *energy-independent* contribution of the intermediate  $N\Delta$ ,  $\Delta\Delta$ ,  $NN^*$ , and  $N^*N^*$  from the Paris potential. In our model, we assume that  $v^1$  is defined by the following matrix elements:

$$v_{i_1 i_2}^{\alpha} (p', p) = [V_{i_1 i_2}^{\alpha} (p', p)]_{\text{Paris}} - [\text{second term of Eq. (12)}]_{E=E_s} , \quad (14)$$

where  $E_s$  is chosen to be well below the pion production threshold so that the second term of Eq. (14) is real as required by the hermiticity of  $v^1$ . This construction is consistent with the Paris potential which calculates the  $2\pi$ -exchange mechanism in a nonperturbative approach based on dispersion relations. Equation (14) is simply a procedure to avoid double counting and to extract from the best meson theory a realistic  $NN \rightarrow NN$  interaction at intermediate and long ranges, when  $\pi$ ,  $\Delta$ , and  $N^*$  degrees of freedom are treated explicitly. The partial-wave matrix elements of the Paris potential can also be expressed in terms of Legendre functions of the second kind and can be calculated very accurately.

We solve the integral equation (11) using a standard matrix method<sup>29,30</sup> in momentum space. All *nonlocal* effects contained in Eq. (12) can therefore be treated exactly. It has been shown in several previous studies<sup>30,31</sup> that the relativistic kinematic effects in the propagator of Eq. (11) can give, quantitatively depending on the potential, effects of about a few percent on the NN scattering phase shifts even at low energies. To reproduce the low energy Paris phase shifts at  $E = E_s$  [because of Eq. (14)], the results shown in Figs. 6–9 are obtained by using nonrelativistic kinematics in solving Eq. (11). We also have used the same numerical method to carry out relativistic calculations. The qualities of the fits to the data are comparable to those shown in Figs. 6–9. The largest differences are in the  $s$  wave phase shifts, which are about  $2^\circ$  lower than the nonrelativistic Paris result at  $E = E_s = 10$  MeV, and are only about  $5^\circ$  lower even at 1 GeV. Those differences can be reduced if the parameters of the Paris potential are also readjusted in our search of the values of  $\Lambda$  and  $E_s$ . This kind of many-parameter-searching is a much harder task, and we leave this problem to future study, which will also include other necessary improvements of the model, as discussed in Sec. IV.

#### IV. RESULTS AND DISCUSSION

The model Hamiltonian defined in previous sections only has two free parameters:  $E_s$  for the subtraction in Eq. (14) and the cutoff  $\Lambda$  of transition potentials  $v^{i>2}$ . No attempt was made in this work to try other form factors and hence, for simplicity, the same  $\Lambda$  is used for all meson-baryon-baryon vertices. It was shown in Ref. 5 that by choosing  $E_s = 10$  MeV and  $\Lambda = 650$  MeV/c, the Arndt<sup>32</sup> phase shifts in the  $NN$   $T=1$  channels can be satisfactorily described. For completeness, we also show these results in Fig. 6 (note that the new NN scattering phase parametrizations are given in Ref. 32). By including the  $N^*$  and using the same parameters, the model now predicts the  $T=0$  phase shifts. In particular, we can investigate the isoscalar inelasticities which are not well determined owing to the lack of sufficient np scattering data at higher energies. The results are compared with the Arndt phase shifts in Fig. 7. Clearly, the fits to the  $T=0$  data are also acceptable. It is important to note that both the model and data show very small inelasticities  $\rho$  in all partial waves. But the model predicts nonzero  $\rho$  in  $l \geq 2$  partial waves, while some of them are set to zero in the analysis of Arndt *et al.*<sup>32</sup> Our predictions can only be

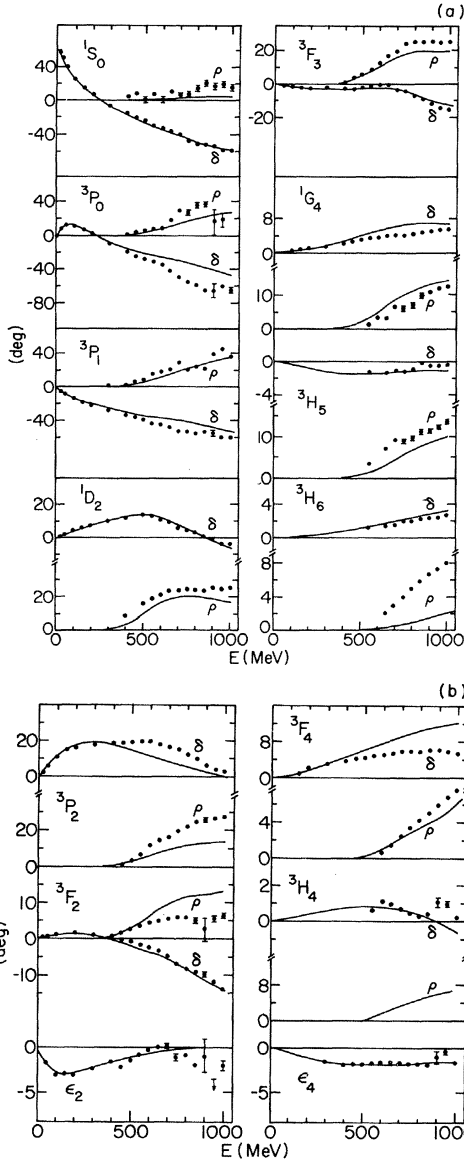


FIG. 6. The calculated  $T=1$  NN scattering phase shifts are compared with the energy independent analysis of Arndt *et al.* (Ref. 32).

verified from more precise np scattering measurements, which are currently being carried out in several meson facilities.

A direct test of our model is to compare our predictions of NN cross sections to the experimental data. In view of the great interest in dibaryon resonances, we compare in Figs. 8 and 9 the data<sup>33</sup> with the calculated reaction cross section  $\sigma^R$ , and total cross sections  $\sigma^{\text{tot}}$ ,  $\Delta\sigma_L^{\text{tot}}$ , and  $\Delta\sigma_T^{\text{tot}}$  corresponding to various spin orientations in the incident beam and target nucleons (as defined in Ref. 23). First, we see that the model gives good descriptions of the pp reaction cross section  $\sigma^R$ . The three-body model of Kloeet and Silbar<sup>9</sup> is also very successful in describing these data, which contain the information of pion production. In

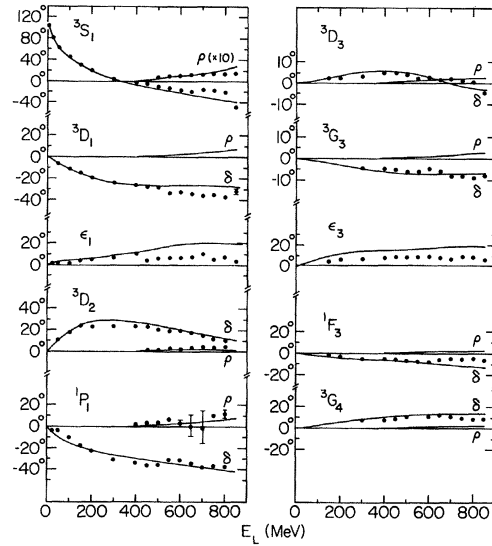


FIG. 7. The calculated  $T=0$  NN scattering phase shifts are compared with the energy independent analysis of Arndt *et al.* (Ref. 32).

fact, any model with vertex interactions properly fitted to the  $\pi N$  scattering could achieve the same success, if the two- and three-body unitarities are retained. Our Hamiltonian formulation of the problem of course satisfies these requirements, and hence the agreement in  $\sigma^R$  is not unexpected. The main new feature of our model is to also have  $2\pi$  production through  $N^*$ . By comparing the calcula-

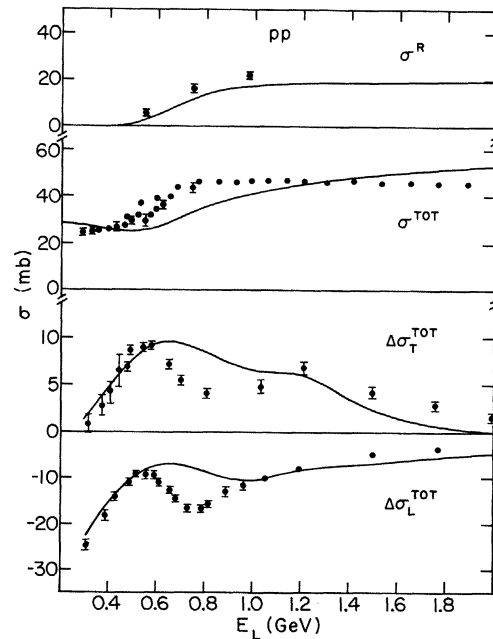


FIG. 8. The calculated pp reaction cross section  $\sigma^R$ , total cross sections  $\sigma^{\text{tot}}$ ,  $\Delta\sigma_T^{\text{tot}} = [\sigma^{\text{tot}}(\uparrow\uparrow) - \sigma^{\text{tot}}(\uparrow\downarrow)]$ , and  $\Delta\sigma_L^{\text{tot}} = [\sigma^{\text{tot}}(\vec{\rightarrow}) - \sigma^{\text{tot}}(\vec{\leftarrow})]$  are compared with the data (Ref. 33).

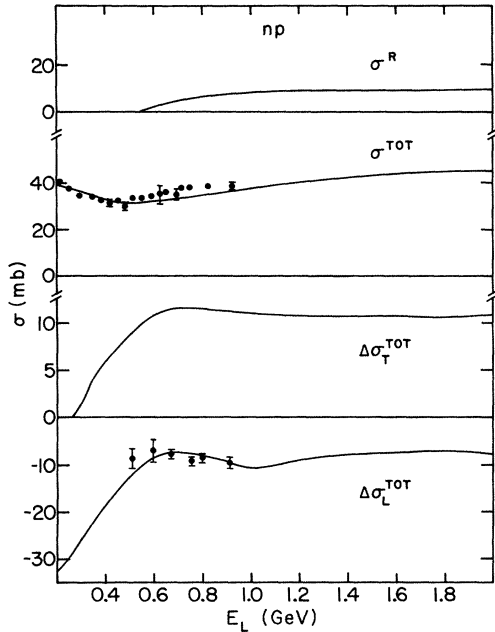


FIG. 9. Same as Fig. 8, except for the np scattering.

tions with and without  $N^*$ , we find that the  $2\pi$  production to  $\sigma^R$  is less than 1 mb, even at 2 GeV. Compared with Ref. 9, the main achievement of the present theory is to give an overall correct description of both the magnitudes and signs of the considered total cross sections in the entire energy region from 0–2 GeV. In particular, the model also does a remarkable job in the higher energy region 1–2 GeV.

A major difference between our model and others<sup>7–18</sup> is to use Eq. (14) to define the  $NN \rightarrow NN$  interaction of the

model Hamiltonian. The results shown in Figs. 6–9 indicate that the use of a nonperturbative  $2\pi$  exchange, as given by the Paris potential, is probably essential for a successful description of NN scattering at medium and higher energies.

The model, however, does not give sufficient energy dependences of all pp total cross sections in the region from 0.6 to 1 GeV. The calculated total cross section  $\sigma^{\text{tot}}$  in this region is only about 80% of the data. To see the origin of this problem, we show in Table II the contributions from each partial wave. First, note that both  $\Delta\sigma_L^{\text{tot}}$  and  $\Delta\sigma_T^{\text{tot}}$  involve cancellations between contributions of different signs. An overall agreement both in the magnitudes and signs for these two data is a nontrivial test of the dynamical content of the model. The main feature of Table II is that none of the contributions from each partial wave at  $E \geq 0.6$  GeV show any strong variation. That is why the calculations fail to reproduce the pronounced minima of  $\Delta\sigma_L^{\text{tot}}$  and  $\Delta\sigma_T^{\text{tot}}$  near 0.8 GeV. It is clear that if one or two partial waves had stronger energy dependences, the shapes of  $\Delta\sigma_L^{\text{tot}}$  and  $\Delta\sigma_T^{\text{tot}}$  could be well described. Within our model, we have investigated this possibility by examining the sensitivity of the calculation to the only free parameter of the model, the cutoff  $\Lambda$  of the form factor of transition interactions. (As discussed in Ref. 5, the value of another parameter  $E_s = 10$  MeV is pretty much fixed by getting good fits to the phase shifts at low energy.) We found that by changing  $\Lambda$  from 650 to 1000 MeV/c, we can get the correct energy dependence of  $\sigma^{\text{tot}}$  up to 1 GeV. But the resulting  $\Delta\sigma_L^{\text{tot}}$  has wrong signs at  $\sim 0.7$  GeV. The calculated phase shifts are also in severe disagreement with the Arndt phase shifts, indicating poor descriptions of all NN scattering observables. Furthermore, the calculations with  $\Lambda = 1000$  MeV/c do not come close to yielding any pronounced minima in  $\Delta\sigma_L^{\text{tot}}$  and  $\Delta\sigma_T^{\text{tot}}$  near 800 MeV.

TABLE II. The energy dependence of the contribution from each partial wave to the pp total cross sections  $\Delta\sigma_L^{\text{tot}}$  and  $\Delta\sigma_T^{\text{tot}}$  in the regions  $400 \text{ MeV} \leq E_L \leq 1000 \text{ MeV}$ .

$E_L$ (MeV)	$^1S_0$	$^3P_0$	$^3P_1$	$^1D_2$	$^3F_3$	$^3P_2 + ^3F_2$	$^1G_4$	$^3F_4 + ^3H_4$	Sum <sup>a</sup>
$\Delta\sigma_L^{\text{tot}}$ (mb)									
400	2.3	1.5	-20.4	9.2	-0.84	-5.3	0.72	-2.0	-15.1
500	3.1	2.5	-20.6	9.8	-1.3	-2.6	1.4	-2.5	-10.5
600	3.8	3.3	-20.4	10.5	-3.3	-0.96	2.4	-2.7	-7.8
700	4.3	4.0	-20.4	9.9	-6.4	0.04	4.1	-2.9	-7.5
800	4.6	4.7	-20.5	8.5	-9.1	0.6	6.0	-2.9	-7.9
900	5.0	5.3	-20.6	7.0	-10.8	1.1	7.4	-2.8	-8.3
1000	5.1	5.8	-20.5	5.9	-11.5	1.8	7.6	-2.5	-8.4
$\Delta\sigma_T^{\text{tot}}$ (mb)									
400	2.3	-1.5		9.2		-5.5	0.72	-0.41	4.7
500	3.1	-2.5		9.8		-3.5	1.4	-0.8	7.3
600	3.8	-3.3		10.5		-2.7	2.4	-1.4	9.2
700	4.3	-4.0		9.9		-2.9	4.1	-2.0	9.5
800	4.6	-4.7		8.5		-3.3	6.0	-2.7	8.5
900	5.0	-5.3		7.0		-3.8	7.4	-3.4	6.9
1000	5.1	-5.7		5.9		-4.4	8.0	-3.9	5.1

<sup>a</sup>Note that the sum also includes small contributions from higher partial waves up to  $l = 7$ .

One possible way to resolve the problem is to introduce dibaryon resonances in some of the partial waves. However, we feel that it is premature to proceed immediately in this direction. The meson-exchange Hamiltonian presented in this paper seems to contain most of the correct NN dynamics, since all of the predictions come very close to the data. The first important next step is to carefully investigate the energy dependence of other meson-exchange mechanisms which are omitted in this calculation. The most important one is the effect owing to NN interactions in the  $\pi$ NN three-body intermediate state [Figs. 5(c) and (d)]. In Ref. 8, it was shown that this effect can have  $\sim 20\%$  effect on  $^1D_2$  phase shifts and is *very energy dependent*. The input  $P_{11}$  isobar model should also be improved to also describe the negative  $\pi$ N phase shifts at low energies. In particular, we must have a careful treatment of the nucleon-pole term,<sup>7,10,34</sup> which can sensitively affect some of the NN and  $\pi d$  polarization observables. Finally, we should explore a better description of the meson-baryon-baryon vertex interaction. Chiral (cloudy) quark-bag model calculations<sup>35</sup> of the form factors for N,  $\Delta$ , and  $N^*$  could be useful in this regard.

## V. CONCLUSION

In conclusion, we have shown that it is possible to construct a meson-exchange Hamiltonian for  $\pi$ , N,  $\Delta$ , and  $N^*$  for NN scattering up to 2 GeV. The model gives reasonable descriptions of the Arndt phase shifts up to 1

GeV in both the  $T=0$  and  $T=1$  channels. The calculated total cross sections  $\sigma^{\text{tot}}$ ,  $\Delta\sigma_L^{\text{tot}}$ , and  $\Delta\sigma_T^{\text{tot}}$  agree to a large extent with the data in both the magnitudes and the signs. This is the first time, to our knowledge, that a meson-exchange Hamiltonian model has achieved such a quantitative agreement with the data in the *entire* energy region 0–2 GeV. The present calculation gives a sound starting point for future refinements. Among them, a large scale three-body calculation<sup>6</sup> could be needed to investigate the energy dependence of the effect owing to NN interactions in the  $\pi$ NN channel. Until this effect is carefully studied, it is premature to extract information on dibaryon resonances, if they exist, from the data. Our model also gives definite predictions of np scattering. Precise np polarization measurements at higher energies ( $\geq 0.6$  GeV) are needed to have a complete test of our model. Finally, as demonstrated in Ref. 36, the present model Hamiltonian can be used to carry out many-body calculations. Therefore, we have a realistic starting point of developing a unified approach to study the isobar-nucleus dynamics, which clearly plays an important role in resolving many nuclear problems,<sup>37</sup> both at low and intermediate energies.

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