Limitation on heavy ion fusion and nuclear level density at high excitation energies

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Over a wide energy range, the limitation on fusion is interpreted from the properties of the compound nuclear level density obtained by taking into account the finiteness of the nucleus. The essential point is to use the shell model single particle bound states in the calculation of the grand partition function. The critical angular momentum for fusion is estimated in connection with the critical temperature for fusion.

The fusion cross section observed as the yield of evaporation residues for example can be expressed in terms of the critical angular momentum $l_{cr}\hbar$ above which the contribution to fusion drops off. In light heavy-ion systems the experimental data on the fusion shows that l_{cr} deviates increasingly from l_{max} as the incident energy becomes larger, where l_{max} indicates the maximum partial wave leading to nuclear reactions.

Such a limitation on fusion has been interpreted by either the entrance channel limitation model[1-3] or by the compound nucleus (CN) limitation model.[4-6] In the latter, the theoretical prediction is based on the conventional level density formula, which has been established in the low energy region, but is not necessarily adequate in highly excited states and in high-spin states.[7] Furthermore, the evidence for incomplete fusion of projectiles reported recently[8-10] at high bombarding energies requires some modification to these limitation models.

In the present paper, the nuclear level density $\rho(E^*,J)$ applicable to high excitation energy E^* is calculated by taking into account the finiteness of the nucleus with an emphasis of dependence on spin J. To estimate l_{cr}, we use the thermodynamical temperature $T(E^*,J)$ obtained in the course of calculation of ρ. In accordance with the critical temperature $T_F(=1.47$ MeV) for fusion proposed in the recent work[11] by the São Paulo group, we determine l_{cr} for a given energy E^* from the condition $T(E^*,l_{cr})=T_F$. At high bombarding energies where incomplete momentum tansfer is appreciable, we assume that a loosely bound component of projectile is promptly emitted in the forward direction and we derive l_{cr} from the level density of the incompletely fused nucleus.

The most usual way to obtain the nuclear level density is the partition function method.[12,13] Usually the assumption of a constant single particle level (SPL) density is made to obtain the grand partition function in a convenient analytical form. Because of this assumption the applicability of such a formula is restricted to low excitation energies.[13] Among other methods proposed recently, we may refer to the combinatorial method[7,14] and the statistical spectroscopy one.[15] Although the former is exact and both methods give almost the same results in the low energy region, it is not yet known what level densities they could provide for the case of high excitation energies.

We adopt here the partition function method but solve the saddle point equation numerically by using shell model SPL's. Our calculation starts with the construction of the grand partition function: $Z_G \equiv \exp(\Omega) = \mathrm{Tr}[\exp(\alpha_1\hat{N}+\alpha_2\hat{Z}+\alpha_3\hat{M}-\beta\hat{E})]$, where α's and β are Lagrange multipliers, and $\hat{N}$, $\hat{Z}$, $\hat{M}$, and $\hat{E}$ are the operators corresponding to neutron number, proton number, z component of spin, and energy, respectively. Ω denotes the thermodynamical potential and is expressed as

$$\Omega = \sum_{j=1}^{2}\sum_{i}\ln[1+\exp(\alpha_j+\alpha_3 m_i^{(j)}-\beta\epsilon_i^{(j)})] \ .$$

Here, $\epsilon_i^{(1,2)}$ and $m_i^{(1,2)}$ are the energy and the z component of spin for neutron and proton of the ith SPL, respectively. Since we need to calculate the state density of long-lived excited states, the trace of the operator is to be taken over certain restricted nuclear states. We limit the available space for single particle motion to the bound states produced by the Woods-Saxon potential. The parameters involved in the potential are taken from the compilation work of Veje[16] for neutron, and for proton only the strength of the potential is deepened so as to fit the last proton binding energy. Thus to get Z_G in practice[17] the trace is taken over all the excited states which are expressed in terms of the particle-hole configuration composed of only these single particle bound states. The formal expression in the present calculation for the spin-dependent level density $\rho(E^*,J)$ is the same as that in the appendix of Ref. 18.

As can be seen in Fig. 1, it is confirmed that for exactly the same space of SPL of ^{56}Fe (31 orbits both for neutron and proton) used by Hillman and Grover,[14] our state density defined by $\omega(E^*)=\sum(2J+1)\rho(E^*,J)$ agrees with that of the combinatorial method[14] as well as the statistical spectroscopy one.[15] The spin-dependent level density of ^{56}Fe calculated by using the SPL's produced by the Woods-Saxon potential is shown in Fig. 2 and compared with the conventional one calculated with the parameters given by Gilbert and Cameron.[18] Our result deviates drastically from that obtained by the conventional level density formula, as excitation energy increases. This indicates an inadequacy of a simple extrapolation of the conventional Fermi-gas level density formula to higher energies.

We analyze the fusion reaction data[19,20] for the ^{20}Ne+^{26}Mg system. The spin dependence of the thermodynamical temperature T $(=\beta^{-1})$ is plotted in Fig. 3 for ^{46}Ti at $E^*=70$ MeV. $T\sim 0$ MeV at the highest spin means

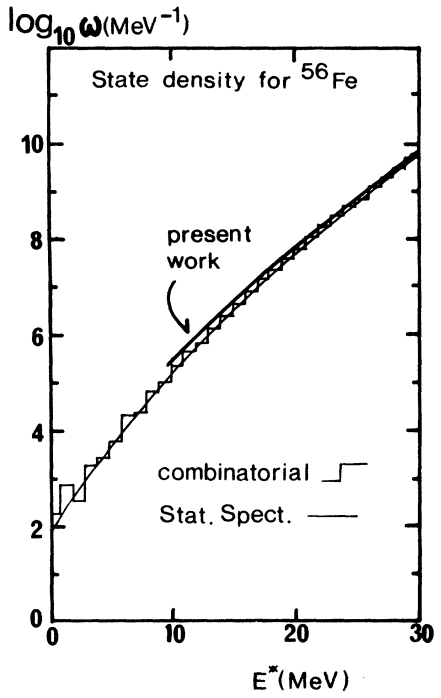

FIG. 1. State density of ^{56}Fe. The space of single particle level involves 31 orbits both for neutron and proton. The bold solid line shows our result. The step and the thin solid line indicate the result of the combinatorial method (Ref. 14) and the statistical spectroscopy one taken from Ref. 15.

that almost all of the excitation energy is exhausted by the rotational motion of nucleus as a whole ($E^* \sim E_{\rm rot}$). Corresponding to $T = 1.4$ meV which is the critical temperature for fusion proposed by Civitarese *et al.*,[11] we can determine $l_{\rm cr}$ as 31.6 from Fig. 3. The intrinsic excitation energy $E_{\rm int}(=E^*-E_{\rm rot})$ at this point is estimated to 14.0 MeV. This value is comparable to the minimum excitation energy for fusion ΔQ calculated from $0.27A$.[11] Here, the rotation energy $\hbar^2 l_{\rm cr}(l_{\rm cr}+1)/2I$ is evaluated with the moment of inertia $(401\times10^{-44}$ MeV sec): $I=\sum_i f_i(2/3)\int r^2|\psi_i|^2 dr^3$, where ψ_i and f_i are the wave function and the occupation probability of the ith level at $E^*=70$ MeV. It is also interesting that the level density corresponding to $T=1.47$ MeV is read as $\log_{10}\rho \sim 1$ MeV^{-1} from Fig. 3. We have determined $l_{\rm cr}$ up to $E^* \sim 90$ MeV in this way.

Since the decay width for neutron[21] Γ_n calculated from the present level density is about 6 MeV at $E^*(^{46}\text{Ti}) = 140$ MeV independently of spin and $\tau_n = \hbar/\Gamma_n$ becomes comparable to the time required to realize the thermal equilibration ($\sim 10^{-22}$ sec),[22] the compound nucleus ^{46}Ti can be formed with extremely small probability above this energy. Thus the incomplete fusion is considered to be dominant in Region III in Fig. 4. As the incomplete fusion reaction forms compund nuclei with a mass number less than 46, the prescription mentioned above is applied to determine $l_{\rm cr}$. For the incomplete fusion process we assume that the projectile breaks up into ^{16}O + α[8] and ^{19}Ne + n at early stage of the collision with the probability p_1 and p_2, respectively ($p_1+p_2=1$), and then ^{16}O or ^{19}Ne collides with almost the same velocity as the projectile to form the compound nucleus ^{42}Ca or ^{45}Ti. The velocities of the centroid for the ^{16}O + ^{26}Mg and the ^{19}Ne + ^{26}Mg system are calculated[8] as 0.876 and 0.971 times that for the initial ^{20}Ne + ^{26}Mg system

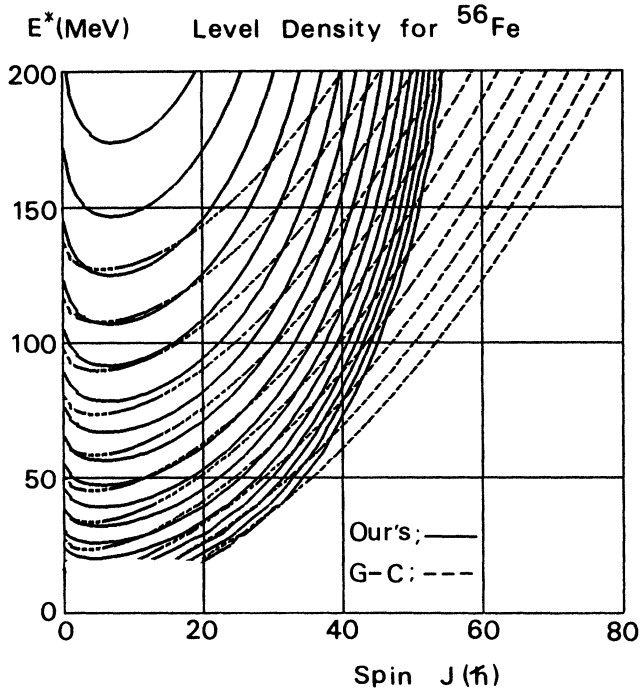

FIG. 2. Contour lines of the level density $\rho(E^*,J)$ for ^{56}Fe. The solid lines ($\log_{10}\rho = 0, 1, 2, \ldots$) show our result in the restricted space of single particle levels. The dashed lines ($\log_{10}\rho = 0, 2, 4, \ldots$) present the conventional level density (Ref. 18).

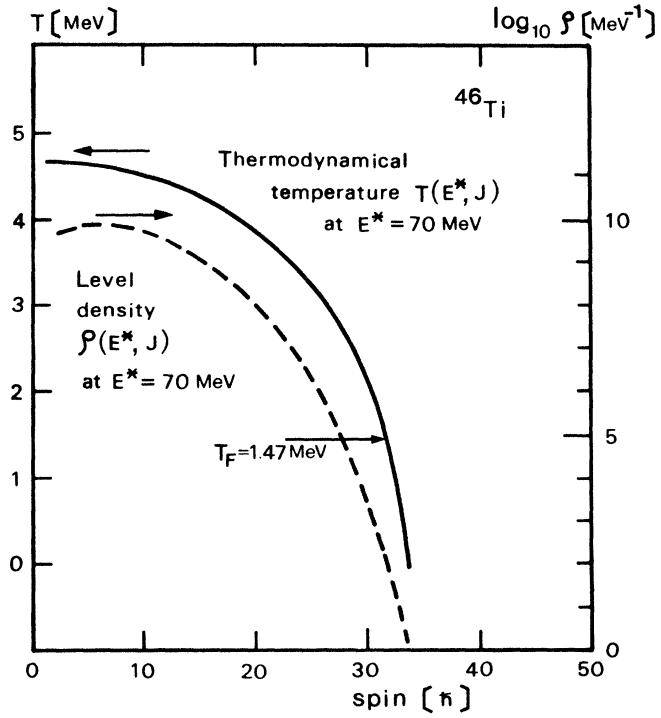

FIG. 3. Level density $\rho(E^*,J)$ of ^{46}Ti at $E^*=70$ MeV (dashed line) and thermodynamical temperature $T(E^*,J)$ (solid line).

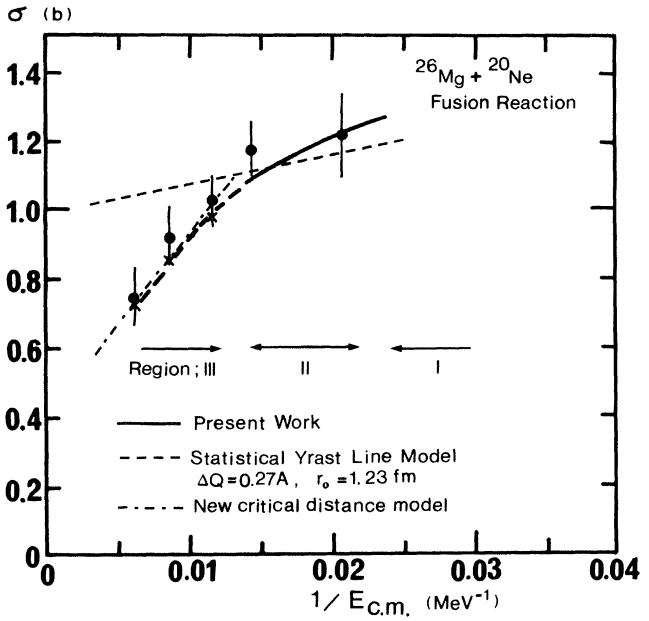

FIG. 4. Fusion excitation function for the $^{26}Mg+^{20}Ne$ reaction (Ref. 19 and 20). The solid curve and the crosses show the cross section calculated in the present work and the crosses are smoothly connected by the bold dashed curve. The prediction for fusion by the statistical yrast line model (Ref. 4) is drawn by the dashed line, where $\Delta Q = 0.27A$ and the moment of inertia is calculated by using the range parameter $r_0 = 1.23$ fm.

neglecting the separation energy which arises less than $\sim 10\%$ modification. If we determine $p_1 = 0.12, 0.33, 0.54$ corresponding to the laboratory energies 150, 200, and 291 MeV, respectively, the measured mean velocity[8] of the evaporation residue at these energies can be reproduced. The fusion cross section at these energies is then expressed as

$$\pi k^{-2}[p_1 l_1(l_1+1)+p_2 l_2(l_2+1)] ,$$

where l_1 and l_2 are the angular momenta which are transformed[23] from the limiting angular momenta estimated by the present level density of ^{42}Ca and ^{45}Ti, respectively.

The fusion cross section thus obtained is compared with the experimental data in Fig. 4. The characteristic energy dependence of the fusion cross section is well explained over a wide energy range. In "Region III," however, the statistical yrast line model overestimates the cross section and hence another mechanism is being called for. On the contrary the present calculation with the realistic level density gives a strong decrease in agreement with the experiment as E_{cm}^{-1} approaches zero. The failure of the statistical yrast line model in "Region III" could be ascribed to their use of the simple Fermi-gas level density formula with the rigid rotator for yrast levels, which is not valid in this energy region.

There is a maximum angular momentum of the system in the present calculation of the nuclear level density, which is responsible to the rapid drop off of the fusion cross section at high energies. The limitation of the angular momentum that the system can accommodate originates from the fact that available SPL's are limited in finite nuclei. The present origin of the limitation is thus different from that of the new critical distance model[24] and from that of the liquid drop limitation.[25]

It is concluded that the limitation to fusion at high incident energies can be interpreted by the present level density calculated realistically by taking into account the finiteness of CN. The essential point in the present work is the use of the restricted space of the SPL's in the calculation of the grand partition function. From the nature of the present level density, we have guessed that the formation of the CN ^{46}Ti will be almost impossible above $E_{lab} = 220$ MeV. It is a challenging problem to investigate theoretically the assumption of projectile breakup leading to incomplete fusion at high bombarding energies and to confirm the limit of the complete fusion. It is also interesting to investigate their applicability to heavier systems, in which the spherical Woods-Saxon potential should be replaced by the deformed one. Finally it is worthy of noting that the nuclear partition function at high temperatures which is necessary in the astrophysical field[26] can easily be obtained by the use of experimental data on complete fusion.

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