Comment on the transfer channel correction to the heavy ion subbarrier fusion cross section

S. Y. Lee
Department of Physics, State University of New York at Stony Brook,
Stony Brook, New York 11794
(Received 7 October 1983)

We examine the penetrability (or fusion cross section) in the influence of a favorable Q value transfer channel. We found that the formula for the cross section cannot be expressed as that given by Broglia *et al.* Our result does support their idea that the transfer channel gives rise to a large enhancement to the sub-Coulomb heavy ion fusion cross section.

Recently, it has become clear that one-dimensional models cannot provide a satisfactory description for a heavy ion fusion cross section at subbarrier energies.[1] This has led to the belief[2] that the individual or the internal degrees of freedom of the colliding nuclei may play an important role in the fusion process. One of the interesting suggestions by Broglia *et al.*[3] is to explain the subbarrier fusion enhancement of the ^{58}Ni + ^{58}Ni and ^{58}Ni + ^{74}Ge systems by the coupling of nucleon transfer channels with favorable Q values. They are able to fit well the experimental fusion cross-section data by using the following formula:

$$\sigma(E)=[1-P(E)]\sigma_1(E)+P(E)\sigma_2(E+Q+\Delta E_c) \ , \qquad (1)$$

where $P(E)$ is the transfer probability and σ_1 and σ_2 are the fusion cross sections for the entrance channel and the transferred channel, respectively.

In this paper, we shall examine the validity of Eq. (1) in a quantum mechanical system. We start with a simple model of a two channel Hamiltonian:

$$H=\left(\frac{P^2}{2\mu}-\frac{1}{2}\mu\Omega^2x^2\right)(a_1^\dagger a_1+a_2^\dagger a_2)-Qa_2^\dagger a_2 + \frac{\lambda}{\sqrt{2}}\delta(x)(a_2^\dagger a_1+a_1^\dagger a_2) \ , \qquad (2)$$

where $a_1^\dagger$ and $a_2^\dagger$ are creation operators for channels 1 and 2, respectively. For $Q>0$, channel 2 has a favorable energy for barrier penetration. The coupling matrix between the channels 1 and 2 is simplified to be a δ function, which has the merit of giving us analytic solution.

Let us consider now the scattering experiment that a unit flux of channel 1 at energy E is incident from the left side of the barrier. The flux penetrated through the barrier can be obtained easily by solving the Schrödinger equation. The resulting wave functions are expressed in terms of parabolic cylinder functions;[4] i.e.,

$$\Psi(y)=c_1E(\epsilon_1,y)|1\rangle+c_2E(\epsilon_2,y)|2\rangle \ , \qquad (3)$$

for $x>0$, where $y=\sqrt{2\mu\Omega}x$, $\epsilon_1=E/\Omega$, and $\epsilon_2=-(E+Q)/\Omega$. The wave function $E(\epsilon,y)$ is the parabolic cylinder function of unit flux traveling to the right. The penetration coefficients c_1 and c_2 for channels 1 and 2, respectively, are obtained by solving the Schrödinger equation to be

$$c_1=[1-\nu^2f(\epsilon_1,\epsilon_2)f(\epsilon_2,\epsilon_1)]^{-1}t_1 \ , \qquad (4)$$

$$c_2=\nu f(\epsilon_1,\epsilon_2)[1-\nu^2f(\epsilon_1,\epsilon_2)f(\epsilon_2,\epsilon_1)]^{-1}t_1 = \nu f(\epsilon_2,\epsilon_1)[1-\nu^2f(\epsilon_1,\epsilon_2)f(\epsilon_2,\epsilon_1)]^{-1}t_2 \ , \qquad (5)$$

with $\nu=(\lambda/2)\sqrt{\mu/\Omega}$ and

$$f(a,b)=-\frac{1}{\sqrt{2}}\left(\frac{e^{\pi a}+i}{e^{\pi b}-i}\right)^{1/2}\left|\frac{\Gamma(\frac{1}{4}+\frac{1}{2}ia)\Gamma(\frac{1}{4}+\frac{1}{2}ib)}{\Gamma(\frac{3}{4}+\frac{1}{2}ia)\Gamma(\frac{3}{4}+\frac{1}{2}ib)}\right|^{1/2} . \qquad (6)$$

$t_1=(-i)/(1+e^{2\pi\epsilon_1})^{1/2}$ and $t_2=(-i)/(1+e^{2\pi\epsilon_2})^{1/2}$ are the penetration coefficients for channels 1 and 2 without coupling, respectively, at energy E.

The total penetrability $|T|^2$, or the cross section σ, is given by $\sigma=|T|^2=|c_1|^2+|c_2|^2$,

$$\sigma=\frac{|t_1|^2}{|1-\nu^2f(\epsilon_1,\epsilon_2)f(\epsilon_2,\epsilon_1)|^2}+\left|\frac{\nu f(\epsilon_2,\epsilon_1)}{1-\nu^2f(\epsilon_1,\epsilon_2)f(\epsilon_2,\epsilon_1)}\right|^2|t_2|^2$$
$$=P_1(E)\sigma_1(E)+P_2(E)\sigma_2(E+Q) \ . \qquad (7)$$

Equation (7) differs from the ansatz of Eq. (1) by the fact that $P_1(E)+P_2(E)$ is not necessarily equal to 1. Using the first part of Eq. (5), we can also express the cross section as

$$\sigma=P_1(E)\sigma_1(E)+\zeta_2(E)\sigma_1(E)=\zeta(E)\sigma_1(E) \ , \qquad (8)$$

where the factor $\zeta_2(E)$ is the penetrability enhancement factor due to transfer channel coupling. $\zeta(E)$ is the total enhancement.

In Fig. 1, $P_1(E)$, $P_2(E)$, $\zeta_2(E)$, and $\zeta(E)$ are shown with $\mu=28300$ MeV, $\hbar\Omega=4$ MeV, $\lambda/\hbar c=0.015$, and $Q=4$ MeV appropriate for ^{58}Ni + ^{64}Ni entrance channel and ^{60}Ni + ^{62}Ni transfer channel. $\lambda/\hbar c=0.015$ is chosen so that the transfer probability is about 10% at and above the barrier energies in accord with the transfer probability of Ref. 3. We observe easily that indeed the favorable Q value transfer channel induces large enhancement to the barrier

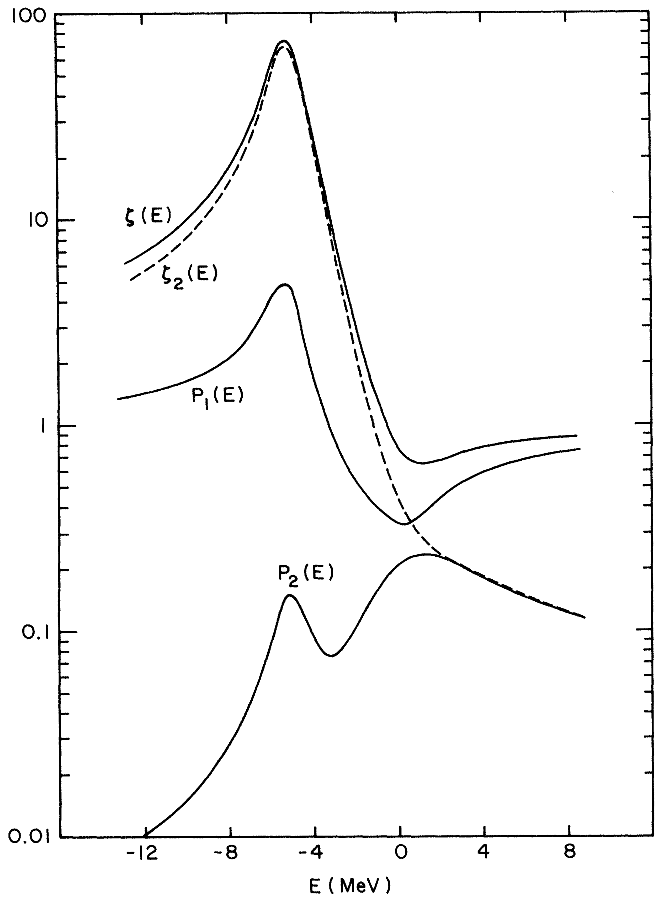

FIG. 1. $P_1(E)$, $P_2(E)$, $\zeta_2(E)$, and $\zeta(E)$ of Eqs. (9) and (10) are shown as a function of incident energy relative to the barrier height. In this calculation $\mu = 28300$ MeV, $\hbar\Omega = 4$ MeV, $\lambda/\hbar c = 0.015$, and $Q = 4$ MeV are used for the ^{58}Ni + ^{64}Ni entrance channel and ^{60}Ni + ^{62}Ni transfer channel. Note that $P_1(E)$ and $P_2(E)$ depend, respectively, on the energy and they do not agree with that of Eq. (1). $\lambda/\hbar c = 0.015$ is chosen so that the transfer probability is order of 10% at above barrier energies.

penetration [see $\zeta_2(E)$]. We however note that $P_1(E)$ is also enhanced at $E \lesssim -4$ MeV; i.e., $P_1(E) \neq 1 - P_2(E)$. $P_2(E)$ may become small at an energy below the barrier while $\zeta_2(E)$ is still large. Thus the coupled transfer channel is still important.

When $E >> 0$ above the barrier, we note that $f(\epsilon_1, \epsilon_2) \cong f(\epsilon_2, \epsilon_1) \cong -ig$, where

$$g = \frac{1}{\sqrt{2}} \left| \frac{\Gamma(\frac{1}{4} + \frac{1}{2} i\epsilon_1)\Gamma(\frac{1}{4} + \frac{1}{2} i\epsilon_2)}{\Gamma(\frac{3}{4} + \frac{1}{2} i\epsilon_1)\Gamma(\frac{3}{4} + \frac{1}{2} i\epsilon_2)} \right|^{1/2} . \tag{9}$$

Therefore if $\nu^2 g^2$ is small, Eq. (7) becomes

$$\sigma(E) \simeq (1 - 2\nu^2 g^2)\sigma_1(E) + \nu^2 g^2 \sigma_2(E + Q) , \tag{10}$$

where one can identify $P_2(E) = \nu^2 g^2$. Thus $P_1(E)$ is $1 - 2P_2(E)$ in contrast to $1 - P_2(E)$ of Eq. (1).

On the other hand, when $E << 0$ below the barrier, $f(\epsilon_1, \epsilon_2)$ and $f(\epsilon_2, \epsilon_1)$ are given approximately by

$$f(\epsilon_1, \epsilon_2) \simeq -e^{\pi Q/2\Omega} g ,$$

$$f(\epsilon_2, \epsilon_1) \simeq -e^{-\pi Q/2\Omega} g .$$

Thus

$$\sigma = \frac{1}{(1 - \nu^2 g^2)^2} \sigma_1(E) + \frac{\nu^2 g^2 e^{-\pi Q/\Omega}}{(1 - \nu^2 g^2)^2} \sigma_2(E + Q) . \tag{11}$$

Here the dependence of $P_2(E)$ on the Q value is explicitly expressed at energies below the barrier.

In conclusion, our simple model study does not yield the intuitive ansatz of Eq. (1) by Broglia *et al.* That is, the experimental fusion data may not be correctly analyzed in the model by assuming the ansatz of Eq. (1). However our model study does support their idea that the favorable Q value transfer channel can induce a large fusion cross section [see the enhancement factor $\zeta_2(E)$ of Fig. 1]. Thus a more consistent quantum mechanical analysis is urgently needed to understand the data of ^{58}Ni + ^{64}Ni and ^{58}Ni + ^{74}Ge.

[1]L. C. Vaz, J. M. Alexander, and G. R. Satchler, Phys. Rep. 69C, 373 (1981); L. C. Vaz, J. M. Alexander, M. Prakash, and S. Y. Lee, in *Proceedings of the International Conference on Nuclear Physics with Heavy Ions, Stony Brook, New York, April, 1983,* Nuclear Science Research Conference Series, Vol. VI (Harwood Academic, Amsterdam, 1984).

[2]H. Esbensen, Nucl. Phys. A352, 147 (1981); C. H. Dasso, S. Landowne, and A. Winther, *ibid.* A405, 381 (1983); R. Broglia, in Proceedings of the International Conference on Heavy Ion Physics and Nuclear Physics, Catania, Italy, 1983 (unpublished).

[3]R. A. Broglia, C. H. Dasso, S. Landowne, and A. Winther, Phys. Rev. C 27, 2433 (1983).

[4]M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1970).