Analytic distorted wave approximation for kaon-nucleus interactions

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At 800 MeV/c momentum, with optical potentials defined from K-nucleon phase shifts, distortion effects in the K-nucleus wave function can be approximated by a simple form that may be useful in analyses of more complicated kaon induced nuclear reactions.

In recent years, with the development of precision measurements, pion induced nuclear reaction studies have taken their appropriate and complementary role in the determination of nuclear structure alongside studies of the equivalent electron and nucleon (proton) induced reactions. The electron interacts with nuclei via the electromagnetic interaction and thus is more sensitive to proton participation in nuclear transitions than to neutrons. Protons, on the other hand, because of the nature of the strong two-nucleon interaction are sensitive to both neutron and proton excitation properties. Polarized proton beams are also available so that very sensitive, spin dependent data can be obtained. The separate proton and neutron transition densities in nuclei are tested by pion reactions since the scattering of both π^+ and π^- can be measured, and at energies relevant to the 3-3 resonance. Kaon induced reactions offer the additional possibility of studying hypernuclei. Furthermore, as both K^+ and K^- beams can be produced, similar studies to π^+ and π^- reactions can be made, but now with a different sensitivity to proton and neutron participation. Of the two kaons, the K^+ meson has the attractive property of being a weakly interacting probe and, thus, simplifying approximations to facilitate evaluation of reaction amplitudes becomes reasonable. One such approximation is to replace the more common distorted waves [of distorted wave approximation (DWA) analyses] by analytic forms [and thus make the analytic distorted wave approximation (ADWA)]. Such has been found useful in analyses of electron scattering[1] (from atoms) as well as of high energy proton[2] and alpha particle[3] scattering from nuclei.

The prime test of any ADWA scheme, and the means by which parameters of the analytic functions are determined, is to fit elastic scattering data. The differential cross sections are defined by

$$\frac{d\sigma}{d\Omega}(\text{mb/sr}) = 10|f(\theta_{sc})|^2 \ , \tag{1}$$

where the scattering amplitude is usually taken as

$$f(\theta_{sc}) = -(\mu/2\pi\hbar^2)\int \exp(-i\vec{k}_f\cdot\vec{r})\,U(r)\,\psi_{k_i}^{(+)}(\vec{r})\,d\vec{r} \ , \tag{2}$$

with $U(r)$ being the optical model potential. The ADWA replaces the complete scattering wave function $\psi_{k_i}^{(+)}$ by an analytic form. As has been used in analyses of intermediate and higher energy proton reactions[2] we consider herein the ADWA representation

$$\psi_{k_i}^{(+)}(\vec{r}) = N\exp[i(\alpha+i\beta)\vec{k}_i\cdot\vec{r}\,] \ , \tag{3}$$

which preserves the simplicity of the plane wave. A reasonable approximation for optical model wave functions[4] is to choose unit normalization of the analytic form at the nuclear radius, whence

$$N = \exp(-\alpha k_i R) \tag{4}$$

gives a two parameter approximation for the scattering state to be used in evaluations of the scattering amplitudes. These now take the form of complex argument Fourier transforms of the optical potential as distinct from the standard Born approximation.

The optical potential can be generated using the Kisslinger approach[5] as used for pion scattering[6] and, recently, for kaon scattering.[7] This potential has the form

$$U(r) = -A(\hbar c)^2/2E[b_0k_i^2\rho(r) - b_1\vec{\nabla}\cdot\rho(r)\vec{\nabla}\,] \ , \tag{5}$$

in which $\rho(r)$ is the nuclear density profile and the complex parameters b_0, b_1 obtained from the relevant momentum kaon-nucleon phase shifts. Two such sets of phase shifts were considered for K^+-nucleon interactions in our calculations. They are the set specified by Martin[8] and the BGRT set.[9] The resultant optical potential parameters are given in Table I. Their use in calculations of elastic scattering give essentially the same results. For K^--nucleus scattering we have adopted the simpler, purely central, optical potential form of Dover and Walker[10] to account for necessary Fermi averaging in the 800 MeV/c region. With the potential form of Eq. (5) and using

$$\begin{aligned} \vec{K}_i &= (\alpha+i\beta)\vec{k}_i \ , \\ \vec{T} &= \vec{K}_i - \vec{k}_f = \vec{q} + \vec{K}_i - \vec{k}_i \ , \end{aligned} \tag{6}$$

TABLE I. Parameter values of the optical model potentials as determined from phase shift folding at $p_{\text{lab}} = 800$ MeV/c (units, fm^3).

	K^+ Martin	K^+ BGRT	K^- Dover-Walker
Re b_0	−0.26	−0.26	0.51
Im b_0	0.20	0.17	0.87
Re b_1	0.077	0.1	···
Im b_1	0.14	0.19	···

from which

$$\mathrm{Re}(T) = \{[(\alpha-1)k_i]^2 + 2q(\alpha-1) \times k_i \cos[(\pi-\theta_{sc})/2] + q^2\}^{1/2} ,$$

$$\mathrm{Im}(T) = \beta k_i , \tag{7}$$

the elastic scattering amplitude is simply

$$f(\theta_{sc}) = A\mu N/E\{(b_0 k_i^2 + b_1 K_i^2)I_1 + 2b_1 K_i \cos(\eta) I_2\} , \tag{8}$$

where

$$I_1 = \int_0^\infty j_0(Tr)\rho(r)r^2 dr , \tag{9}$$

$$I_2 = \int_0^\infty j_1(Tr)[0.006\exp(-0.354r^2) - 0.354\rho(r)]r^3 dr , \tag{10}$$

and

$$\eta = (\pi-\theta_{sc})/2 - \sin^{-1}\{(\alpha-1)k_i \sin\{(\pi-\theta_{sc})/2\}/\mathrm{Re}(T)\} . \tag{11}$$

Two parameters (α,β) thus specify the elastic scattering cross sections and, by using the values (1.0, −0.052) and (1.205, −0.055) for K^+ and K^- scattering, respectively, the 800 MeV/c data[7] can be quite well fit as is shown in Fig. 1. The fits to data are very similar to those obtained from the complete optical model potential (OMP) calculations.[7] The OMP results, however, obtain a better trend in the very small angle K^+ scattering region. Somewhat surprisingly the ADWA estimates are better fits to the large scattering angle data. But is is known[7] that the optical potential is sensitive to the elementary t matrix. Nevertheless, the optical potentials (for K^+ particularly) are not very strong and the associated wave functions can be reasonably represented by modified plane waves. In fact, the K^+ case is very close to the plane wave limit (1.0, 0.0) and the elastic scattering data is well fit over the entire scattering angle range. The result is not quite so good for the K^- case at the larger scattering angles but even so the ADWA scheme would seem to be a useful, simple prescription for more complicated kaon induced reactions at least at 800 MeV/c. Thereby more detailed nuclear structure may be used in analyses if such were precluded in full scale distorted wave analyses.

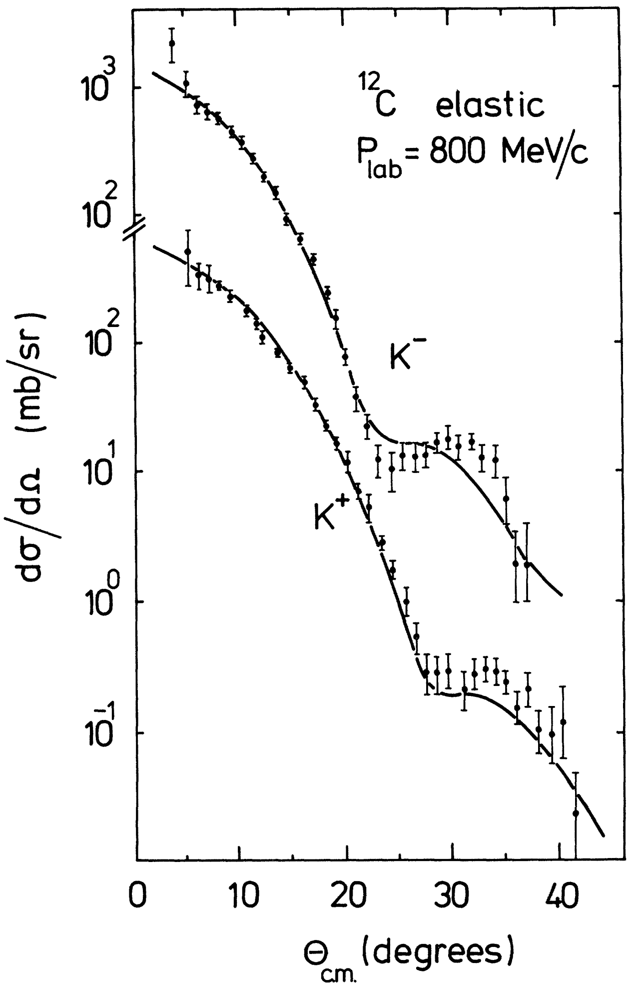

FIG. 1. Elastic scattering cross sections for $K^{\pm}$ scattering at $p_{lab} = 800$ MeV/c compared with ADWA calculations.

At the very least the ADWA approach can be a valued guide to the best distorted wave study.

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