Relativistic and nonrelativistic models of the nuclear optical potential

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Recently it has been shown that the standard nonrelativistic theory of the nuclear optical potential is unable to explain the spin-dependent observables for proton-nucleus scattering for projectile energies from about 400 MeV to 1 GeV if one uses the impulse approximation to calculate the potential. On the other hand, a relativistic impulse approximation leads to a (relativistic) optical potential for use in the Dirac equation; the relativistic model is able to provide good fits to the spin observables in the energy range noted above. We discuss the relationship between the nonrelativistic and relativistic models of the optical potential and show that the nonrelativistic potential may, to a first approximation, be identified with one of two terms in an expression for the relativistic optical potential. This identification is not precise since the density matrices of the target are different in the relativistic and nonrelativistic theories. The second term in our expression for the relativistic optical potential is associated with projectile propagation in negative-energy states. It may be seen that this term and the corrections arising from the use of a relativistic density matrix for the target are responsible for the success of the relativistic model.

I. INTRODUCTION

There have been several studies of nucleon-nucleus scattering which have shown the utility of a relativistic model of the nuclear optical potential.[1] Recently, a relativistic impulse approximation has been used to construct an optical potential which is then inserted in the Dirac equation.[2–5] If one compares the fits obtained for the polarization and spin-rotation parameter it is clear that the relativistic theory is able to account for the data, while the nonrelativistic analysis fails. This is most clearly seen when the impulse approximation is used to construct the optical potential for energies above 300 MeV. However, studies of the optical potential for projectile energies from 0 to 200 MeV, where medium corrections are quite important, have also shown that the relativistic approach is superior to the nonrelativistic analysis.[6]

We have several goals in this work. First we wish to show how a covariant model for the calculation of the relativistic optical potential may be formulated. We organize the model so that we may provide a satisfactory treatment of recoil effects and also isolate the specific features which are absent in the nonrelativistic models and which are responsible for the success of the relativistic formulation. In particular, we show that the relativistic optical potential may be organized into two terms. The first of these may be (approximately) identified with the nonrelativistic optical potential. (This identification is not exact since the density matrix of the target has a different structure in the relativistic and nonrelativistic theories. Also there are some minor propagator ambiguities which we comment upon at a later point.) The second term in the relativistic optical potential is associated with projectile propagation in negative-energy states.[7] Two effects are therefore responsible for the success of the relativistic description. These include the use of a relativistic density matrix of the target and the inclusion of the term describing projective propagation in negative-energy states.[8] (We note that it is possible for us to isolate the relativistic effects and relate them to specific diagrammatic elements since our organization of the relativistic calculation differs from that appearing in the literature.[1–5])

We now turn to a discussion of the significance of various aspects of our analysis. First let us consider the matter of a proper treatment of recoil. We have recently presented an anaysis of the general form of the relativistic optical potential[9] which is to be used in the Dirac equation. In that work we have shown that, in general, there are eight scalar invariants that determine the form of the potential. Three of these are nonzero in nuclear matter, that is, if one neglects "recoil effects." Almost all discussion of the relativistic optical potential has been limited to only two terms, often denoted as a Lorentz scalar, $U_S(r)$, and a Lorentz vector, $\gamma^0 U_V(r)$.[1–5] If we are to make contact between a theoretical analysis of the potential and the phenomenological studies we need to study the "recoil terms" referred to above since we must assess their size before comparing theoretical results with phenomenological values of the parameters of the relativistic optical potential. In light of these remarks one can appreciate the importance of a proper treatment of translational invariance.

We now turn to another motivation for this work. There has been some criticism of the use of the Dirac equation to describe nucleon propagation.[10] The argument rests upon the fact that the nucleon is not pointlike but is a composite particle with an underlying quark structure. Therefore propagation in negative energy states should be strongly suppressed, and the Dirac equation should not be used. One of our goals here has been to show that the *empirical evidence* strongly supports the use of the Dirac equation and that specific features of the relativistic description are responsible for its success; indeed, just those features whose use has been questioned. From our point of view, the success of the relativistic analysis in the study of nuclear structure physics[8] and in the study of nucleon-nucleus scattering[1-5] leads to the conclusion that a Dirac propagator theory is appropriate to describe the motion of the nucleon in spite of its composite nature. To our knowledge there has been little work on demonstrating that Feynman propagators may be used to describe the motion of strongly interacting composite particles. (There is little objection to using such a description for noninteracting particles.) These are interesting questions which deserve further study.

The plan of our work is as follows. In Sec. II we write the Dirac equation in momentum space in terms of a self-energy operator, $\Sigma(W)$. [The structure of $\Sigma(W)$ has been discussed in great detail in a previous publication.[9]] We then introduce a decomposition of the propagator into two parts, corresponding to propagation in positive and negative energy states. This allows us to construct an equation [Eq. (2.20)] which eliminates explicit reference to negative energy states *via* the introduction of an effective potential $U^{++}(W)$—see Eq. (2.19). In Sec. III we discuss the calculation of U^{++} and $\Sigma(W)$ making use of the impulse approximation. We also contrast the full relativistic calculation of these quantities with nonrelativistic approximations. Some discussion of the relativistic corrections is given in Sec. IV. Previous work concerning the optical potential for nuclear matter and for finite nuclei is recalled and estimates of the size of the relativistic correction terms are indicated. Finally, in Sec. V we provide a short summary.

II. THE RELATIVISTIC OPTICAL POTENTIAL

We consider a nucleon of momentum $\vec{k}$ incident on a nucleus of mass M_A and momentum $-\vec{k}$. The total energy is $W=E_N(\vec{k})+E_A(\vec{k})$, where $E_N(\vec{k})=[\vec{k}^2+m_N^2]^{1/2}$ and $E_A(\vec{k})=[\vec{k}^2+M_A^2]^{1/2}$. We have shown in Appendix A that we may write the following equation for the optical-model wave function:

$$[W-E_A(\vec{p})-\vec{\alpha}\cdot\vec{p}-\gamma^0 m_N]\langle\vec{p}\,|\,\psi^{(+)}_{\vec{k},s}\rangle=\gamma^0\int d\vec{p}\,'\langle\vec{p}\,|\,\Sigma(W)\,|\,\vec{p}\,'\rangle\langle\vec{p}\,'\,|\,\psi^{(+)}_{\vec{k},s}\rangle\ . \tag{2.1}$$

Here $\gamma^0\Sigma(W)$ plays the role of a generalized optical potential. We consider the integral equation corresponding to Eq. (2.1),

$$\langle\vec{p}\,|\,\psi^{(+)}_{\vec{k},s}\rangle=\delta(\vec{p}-\vec{k})u(\vec{k},s)+\frac{1}{W-E_A(\vec{p})-\vec{\alpha}\cdot\vec{p}-\gamma^0 m_N+i\epsilon}\gamma^0\int d\vec{p}\,'\langle\vec{p}\,|\,\Sigma(W)\,|\,\vec{p}\,'\rangle\langle\vec{p}\,'\,|\,\psi^{(+)}_{\vec{k},s}\rangle\ . \tag{2.2}$$

Further, with the definition (see Appendix A)

$$M(W)\,|\,\vec{k}\rangle u(\vec{k},s)=\Sigma(W)\,|\,\psi^{(+)}_{\vec{k},s}\rangle\ , \tag{2.3}$$

we have

$$\langle\vec{k}\,'\,|\,M(W)\,|\,\vec{k}\rangle=\langle\vec{k}\,'\,|\,\Sigma(W)\,|\,\vec{k}\rangle+\int d\vec{k}\,''\langle\vec{k}\,'\,|\,\Sigma(W)\,|\,\vec{k}\,''\rangle\frac{1}{W-E_A(\vec{k}\,'')-\vec{\alpha}\cdot\vec{k}\,''-\gamma^0 m_N+i\epsilon}\gamma^0\langle\vec{k}\,''\,|\,M(W)\,|\,\vec{k}\rangle\ . \tag{2.4}$$

It is useful to note that

$$\frac{1}{E-\vec{\alpha}\cdot\vec{k}-\gamma^0 m_N+i\epsilon}\gamma^0=\sum_s\frac{m_N}{E_N(\vec{k})}\left[\frac{u(\vec{k},s)\bar{u}(\vec{k},s)}{E-E_N(\vec{k})+i\epsilon}+\frac{w(\vec{k},s)\bar{w}(\vec{k},s)}{E+E_N(\vec{k})-i\epsilon}\right], \tag{2.5}$$

where $u(\vec{k},s)$ and $w(\vec{k},s)\equiv v(-\vec{k},-s)$ are spinor solutions of the Dirac equation without interaction.[11]

With the definitions

$$\langle\vec{k}\,'\,|\,\tilde{T}(W)\,|\,\vec{k}\rangle=\left[\frac{m_N}{E_N(\vec{k}\,')}\right]^{1/2}\langle\vec{k}\,'\,|\,M(W)\,|\,\vec{k}\rangle\left[\frac{m_N}{E_N(\vec{k})}\right]^{1/2}, \tag{2.6}$$

$$\langle\vec{k}\,'\,|\,\tilde{V}(W)\,|\,\vec{k}\rangle=\left[\frac{m_N}{E_N(\vec{k}\,')}\right]^{1/2}\langle\vec{k}\,'\,|\,\Sigma(W)\,|\,\vec{k}\rangle\left[\frac{m_N}{E_N(\vec{k})}\right]^{1/2}, \tag{2.7}$$

we can write Eq. (1.4) as

$$\langle \vec{k}' | \tilde{T}(W) | \vec{k} \rangle = \langle \vec{k}' | \tilde{V}(W) | \vec{k} \rangle + \int d\vec{k}'' \langle \vec{k}' | \tilde{V}(W) | \vec{k}'' \rangle \left[\frac{u(\vec{k}'',s)\bar{u}(\vec{k}'',s)}{\mathscr{E}_{\vec{k}''} - E_N(\vec{k}'') + i\epsilon} + \frac{w(\vec{k}'',s)\bar{w}(\vec{k}'',s)}{\mathscr{E}_{\vec{k}''} + E_N(\vec{k}'') - i\epsilon} \right] \langle \vec{k}'' | \tilde{T}(W) | \vec{k} \rangle , \tag{2.8}$$

with $\mathscr{E}_{\vec{k}''} = W - E_A(\vec{k}'')$.

The following definitions are useful for further work:

$$g_0^{(+)}(\vec{k} \mid W) = g_+(\vec{k} \mid W) + g_-(\vec{k} \mid W) , \tag{2.9}$$

$$g_+(\vec{k} \mid W) = \sum_s \frac{u(\vec{k},s)\bar{u}(\vec{k},s)}{\mathscr{E}_{\vec{k}} - E_N(\vec{k}) + i\epsilon} , \tag{2.10}$$

$$g_-(\vec{k} \mid W) = \sum_s \frac{w(\vec{k},s)\bar{w}(\vec{k},s)}{\mathscr{E}_{\vec{k}} + E_N(\vec{k}) - i\epsilon} , \tag{2.11}$$

$$\langle \vec{k}',s' | T^{++}(W) | \vec{k},s \rangle = \bar{u}(\vec{k}',s') \langle \vec{k}' | \tilde{T}(W) | \vec{k} \rangle u(\vec{k},s) , \tag{2.12}$$

$$\langle \vec{k}',s' | T^{+-}(W) | \vec{k},s \rangle = \bar{u}(\vec{k}',s') \langle \vec{k}' | \tilde{T}(W) | \vec{k} \rangle w(\vec{k},s) , \tag{2.13}$$

$$\langle \vec{k}',s' | T^{--}(W) | \vec{k},s \rangle = \bar{w}(\vec{k}',s') \langle \vec{k}' | \tilde{T}(W) | \vec{k} \rangle w(\vec{k},s) , \tag{2.14}$$

etc.

Thus Eq. (2.8) may be written as

$$\tilde{T} = \tilde{V} + V g_0^{(+)} \tilde{T} , \tag{2.15}$$

where T, V, and $g_0^{(+)}$ are 4×4 Dirac matrices. Equation (2.15) is equivalent to the following two equations:

$$\tilde{T} = \tilde{U} + \tilde{U} g_+ \tilde{T} , \tag{2.16}$$

$$\tilde{U} = \tilde{V} + \tilde{V}(g_0^{(+)} - g_+)\tilde{U} = \tilde{V} + \tilde{V} g_- \tilde{U} . \tag{2.17}$$

From these equations one may obtain (see Appendix B)

$$T^{++} = U^{++} + U^{++} g_+ T^{++} , \tag{2.18}$$

where

$$U^{++} = V^{++} + V^{+-} \left[\frac{1}{(g_-)^{-1} - V^{--}} \right] V^{-+} . \tag{2.19}$$

Equation (2.18) may be written as

$$\langle \vec{k}',s' | T^{++}(W) | \vec{k},s \rangle = \langle \vec{k}',s' | U^{++}(W) | \vec{k},s \rangle + \sum_{s''} \int d\vec{k}'' \frac{\langle \vec{k}',s' | U^{++}(W) | \vec{k}'',s'' \rangle \langle \vec{k}'',s'' | T^{++}(W) | \vec{k},s \rangle}{\mathscr{E}_{\vec{k}''} - E_N(\vec{k}'') + i\epsilon} . \tag{2.20}$$

Equation (2.20) is not of the standard Lippmann-Schwinger form because of the structure of the propagator. This equation can be put into a standard form (see Appendix C) or may be used as it stands to fit nucleon-nucleus scattering data. We may remark, however, that if we consider massive targets such that $|\vec{k}| \ll M_A$ and values of $|\vec{k}''| \ll m_N$, we have

$$\mathscr{E}_{\vec{k}''} - E_N(\vec{k}'') = E_A(\vec{k}) + E_N(\vec{k}) - E_A(\vec{k}'') - E_N(\vec{k}'') \tag{2.21}$$

$$\simeq \frac{\vec{k}^2}{2\mu_{NA}} - \frac{\vec{k}''^2}{2\mu_{NA}} , \tag{2.22}$$

where

$$\frac{1}{\mu_{NA}} = \frac{1}{M_A} + \frac{1}{m_N} \tag{2.23}$$

is the usual effective mass parameter.

In Eq. (2.20) there is no explicit reference to negative-energy states of the projectile. The effects arising from such negative-energy states have been isolated in the second term of Eq. (2.19). In the next section we discuss the calculation of U^{++} making use of the impulse approximation.

III. THE IMPULSE APPROXIMATION

We may write Eq. (2.19) as

$$\begin{aligned}\langle \vec{k}',s'|U^{++}(W)|\vec{k},s\rangle &= \langle \vec{k}',s'|V^{++}(W)|\vec{k},s\rangle + \sum_{s'',s'}\sum_{\alpha\beta}\int\int d\vec{k}''d\vec{k}'''\langle \vec{k}',s'|V^{+-}(W)|\vec{k}'',s''\rangle \bar{w}_\alpha(\vec{k}'',s'') \\ &\quad \times \langle \vec{k}''|[\mathscr{E}_{\vec{k}''}-E_N(\vec{k}'')-V^{--}(W)]^{-1}_{\alpha\beta}|\vec{k}'''\rangle w_\beta(\vec{k}''',s''') \\ &\quad \times \langle \vec{k}''',s'''|V^{-+}(W)|\vec{k},s\rangle \ , \end{aligned} \tag{3.1}$$

where α and β denote spinor indices.

We turn to a consideration of the first term,

$$\langle \vec{k}',s'|V^{++}(W)|\vec{k},s\rangle = \left[\frac{m_N}{E_N(\vec{k}')}\right]^{1/2} \bar{u}(\vec{k}',s')\langle \vec{k}'|\Sigma(W)|\vec{k}\rangle u(\vec{k},s)\left[\frac{m_N}{E_N(\vec{k})}\right]^{1/2} . \tag{3.2}$$

The self-energy, $\Sigma(W)$, may be calculated in the impulse approximation,

$$\langle \vec{k}'|\Sigma(W)|\vec{k}\rangle = \int d\vec{p}\langle \vec{k}',\vec{p}+\vec{k}-\vec{k}'|t|\vec{k},\vec{p}\rangle\rho(\vec{p};\vec{p}+\vec{k}-\vec{k}') \ , \tag{3.3}$$

where ρ is the (relativistic) density matrix of the target and t is a (medium-modified) nucleon-nucleon scattering amplitude which is a 4×4 Dirac matrix with respect to *both* the projectile and target nucleon coordinates.[4,5] If we make a simple (relativistic) shell-model description of the target we would have

$$\rho_{\alpha\beta}(\vec{p};\vec{p}') = \sum_{i=1}^{N}\phi_\alpha^{(i)}(\vec{p})\bar{\phi}_\beta^{(i)}(\vec{p}') \ . \tag{3.4}$$

Here the $\phi^{(i)}$ denote the target wave functions and the α and β are spinor indices. Except for the differences between the density matrices of the relativistic and nonrelativistic theories, it is clear that the approximation given in Eq. (3.3) can be identified with the "$t\rho$" approximation of the nonrelativistic scattering theories.[12] More precisely, if we make the further approximation that the density matrix only has matrix elements in the space spanned by the positive-energy spinor states, we would have

$$\begin{aligned}\langle \vec{k}',s'|\Sigma^{++}(W)|\vec{k},s\rangle &= \sum_{s'''s''}\int d\vec{p}\,\bar{u}(\vec{k}',s')\bar{u}(\vec{p}+\vec{k}-\vec{k}',s'')\langle \vec{k}',\vec{p}+\vec{k}-\vec{k}'|t|\vec{k},\vec{p}\rangle \\ &\quad \times u(\vec{k},s)u(\vec{p},s''')\rho_{s'''s''}(\vec{p};\vec{p}+\vec{k}-\vec{k}') \end{aligned} \tag{3.5}$$

$$= \int d\vec{p}\langle \vec{k},s';\vec{p}+\vec{k}-\vec{k}',s''|t|\vec{k},s;\vec{p},s'''\rangle\rho_{s''',s''}(\vec{p};\vec{p}+\vec{k}-\vec{k}') \ , \tag{3.6}$$

which may readily be identified with the form of the nonrelativistic "$t\rho$" approximation.[12] If one has a satisfactory model for the (relativistic) density matrix of the target,[5] one can assess the differences of a calculation of Σ^{++} using Eqs. (3.3) and (3.4) or Eqs. (3.5) and (3.6). Some calculations of these relativistic combinations have been made in the case of nuclear matter.[7,8] These results are noted in the next section.

IV. RELATIVISTIC CORRECTIONS TO THE NONRELATIVISTIC MODEL

The formalism presented here allows one to isolate the relativistic correction terms and assess their magnitude. To some degree this has been done for finite nuclei in Ref. 7, making use of estimates obtained from the study of nuclear matter. In Ref. 7 we limited our considerations to low-energy projectiles. The range of projectile energies from 0 to 200 MeV has been considered in Refs. 6 and 8.

We consider the analog of U^{++} in nuclear matter. This quantity may be represented diagrammatically as in Fig. 1. Here the heavy lines represent the (self-consistent) spinor wave function of a nucleon in nuclear matter, $f(\vec{p},s)$. This wave function may be written in terms of the spinor solutions of the free Dirac equation, $u(\vec{p},s)$ and $w(\vec{p},s)$,[6,8]

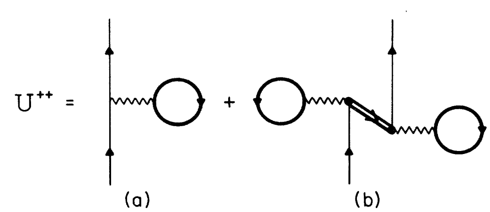

FIG. 1. The quantity U^{++} in nuclear matter. Here the wavy line is a reaction matrix and the heavy circles denote particles in occupied orbits which are described by (self-consistent) relativistic wave functions. The heavy double line represents particle propagation in a negative-energy state. (Only direct terms are shown for simplicity.) Part (a) corresponds to a *relativistic* "$t\rho$" approximation, while part (b) describes the correction due to particle propagation in negative-energy states (Refs. 7 and 8).

$$f(\vec{p},s)=a(\vec{p})u(\vec{p},s)+b(\vec{p})\sum_{s'}\langle s'|\vec{\sigma}\cdot\hat{p}|s\rangle w(\vec{p},s') , \tag{4.1}$$

$$=\frac{1}{[1+\alpha^2(\vec{p})]^{1/2}}\left\{u(\vec{p},s)+\alpha(\vec{p})\sum_{s'}\langle s'|\vec{\sigma}\cdot\hat{p}|s\rangle w(\vec{p},s')\right\} . \tag{4.2}$$

This expansion allows for a further diagrammatic representation of the process shown in Fig. 1(a), which is presented in Fig. 2. If one sets $\alpha(\vec{p})=0$, only Fig. 2(a) is nonzero. The process shown in Fig. 2(a) may be associated with the nonrelativistic "$t\rho$" approximation. [Note that in Fig. 1(b) we indicate the term analogous to the second term in Eq. (2.19).]

It should be clear that the terms indicated in Figs. 1(b), 2(b), 2(c), and 2(d) represent corrections to the nonrelativistic theory. [The relativistic correction arising from Fig. 2(d) is estimated to be quite small.] First, we may consider these correction terms as they affect the real part of U^{++}. One may refer to Fig. 10 of Ref. 6, where it is seen that these relativistic corrections are repulsive and contribute about +20 MeV to the real part of U^{++} (at nuclear matter densities) for projectile energies in the range of 0—200 MeV. It can be seen that this additional repulsion leads to agreement between theory and experiment for the magnitude of the real part of the central part of the nuclear optical potential.[6,8] These terms also give rise to a strong density dependence of the optical potential which corresponds to the density dependence found in empirical studies.[8] Second, the relativistic correction terms enhance the spin-orbit part of the nuclear optical potential. This enhancement is of the order of 30 percent,[5,6] and is clearly a very important effect. The importance of these various relativistic corrections for optical model studies is clearly demonstrated in Refs. 2—5. The importance of relativistic effects for understanding other aspects of nuclear structure physics is discussed in Refs. 6 and 8.

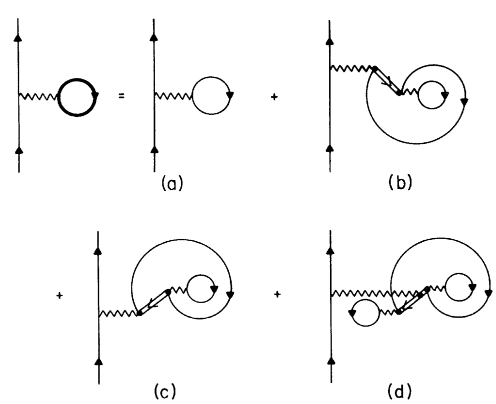

FIG. 2. Diagrammatic representation resulting from the expansion of the density matrix associated with the relativistic "$t\rho$" approximation—see Fig. 1(a). Here a thin line denotes a particle in a positive energy state, $u(\vec{p},s)$. A light double line denotes a particle in a negative-energy state, $w(\vec{p},s)$. The $u(\vec{p},s)$ and $w(\vec{p},s)$ are solutions of the free Dirac equation.

V. SUMMARY AND CONCLUSIONS

The formalism presented here has various advantages. First, it is rather general and may be used for the nonlocal potentials which would arise if one were to calculate the optical potential using a general many-body theory.[7] Second, our formalism isolates the relativistic correction in such a manner as to identify it with projectile propagation in negative-energy states and with effects arising from the use of a relativistic density matrix for the target.

The physical interpretation is less clear in the usual analysis which is presented for the local optical models which have been studied extensively,[1] For example, if one writes

$$\Sigma(r)=A(r)+\gamma^0 B(r) ,$$

one can write a Schrödinger equation whose solution determines the upper two components of the relativistic (spinor) solution of the Dirac equation. The effective central potential then has the form

$$U_{\text{cent}}(r)=A(r)+\frac{E}{m_N}B(r)+\frac{A^2(r)-B^2(r)}{2m_N}+\cdots . \tag{5.1}$$

[In Eq. (5.1) we have not written some complicated derivative terms which appear in the effective potential.[1]] While it is clear that the use of the Dirac equation to describe nucleon-nucleon scattering leads to the quadratic terms in Eq. (5.1), the physical interpretation of these terms is not easily expressed. On the other hand, our approach involves the expansion of wave functions and operators in terms of positive- and negative-energy solutions of the Dirac equation without interactions. In our case the physical interpretation is readily made and, if desired, a diagrammatic analysis may be developed such as that given in Refs. 7 and 8 and in Figs. 1 and 2 of this work.

As a final remark, we again state that the success of the relativistic theory in the description of nuclear structure and scattering provide support for the use of a relativistic propagator description of nucleon motion.

APPENDIX A

In this appendix we develop some relativistic equations which may be used in the description of nucleon-nucleon scattering. (These ideas have already been applied in the construction of a relativistic scattering theory for the description of pion-nucleus scattering.[13]) We use a propagator formalism to describe the motion of the target and projectile. Since the massive target is placed on mass shell throughout, the use of a relativistic formalism for the target plays only a very minor role. One advantage of these techniques, however, lies in our ability to include target-recoil effects. In addition, the formalism is covariant so that the transformation properties of the various quantities which appear in the analysis are manifest.

We consider a Bethe-Salpeter equation describing nucleon-nucleus scattering:

$$M=K+KG_F M . \tag{A1}$$

Here G_F is a propagator for the projectile and the ground state of the nucleus. Therefore all reference to the excited states of the target is implicit in the quantity K, which plays the role of a relativistic optical potential. We can write G_F as

$$G_F(p,P)=\frac{i}{(2\pi)}\frac{1}{\gamma\cdot p-m_{\rm N}+i\epsilon}\frac{1}{P^2-M_A^2+i\epsilon} , \tag{A2}$$

where p is the four-momentum of the nucleon and P is the four-momentum of the target of mass M_A. We also write

$$P=\left\{\frac{W}{2}-k^0,-\vec{\rm k}\right\}$$

and

$$p=\left\{\frac{W}{2}+k^0,\vec{\rm k}\right\}$$

in the center-of-mass frame.

It is useful to reduce Eq. (A1) to a three-dimensional equation. This can be accomplished by writing

$$M=\Sigma+\Sigma\tilde{g}_0^{(+)}M , \tag{A3}$$

$$\Sigma=K+K(G_F-\tilde{g}_0^{(+)})\Sigma , \tag{A4}$$

where $\tilde{g}_0^{(+)}$ is a propagator with the same right-hand cut as G_F. We choose a form for $\tilde{g}_0^{(+)}$ which places the massive nucleus on its mass shell,

$$\begin{aligned}\tilde{g}_0^{(+)}(k\mid W)&=\frac{1}{\gamma\cdot p-m_{\rm N}+i\epsilon}\left[\frac{1}{2E_A(\vec{\rm k})}\right]\delta[P^0-E_A(\vec{\rm k})]\\&=\frac{1}{\gamma^0\left[\frac{W}{2}+k^0\right]-\vec{\gamma}\cdot\vec{\rm k}-m_{\rm N}+i\epsilon}\left[\frac{1}{2E_A(\vec{\rm k})}\right] .\\&\quad\times\delta\left[\frac{W}{2}-k^0-E_A(\vec{\rm k})\right] .\end{aligned} \tag{A5}$$

Thus Eq. (A3) may be written as

$$\begin{aligned}\langle\vec{\rm k}'\mid M(W)\mid\vec{\rm k}\rangle&=\langle\vec{\rm k}'\mid\Sigma(W)\mid\vec{\rm k}\rangle\\&\quad+\int d\vec{\rm k}''\langle\vec{\rm k}'\mid\Sigma(W)\mid\vec{\rm k}''\rangle\frac{1}{\gamma^0\left[\frac{W}{2}+k''^0\right]-\vec{\gamma}\cdot\vec{\rm k}''-m_{\rm N}+i\epsilon}\langle\vec{\rm k}''\mid M(W)\mid\vec{\rm k}\rangle .\end{aligned} \tag{A6}$$

Here

$$\langle\vec{\rm k}'\mid M(W)\mid\vec{\rm k}\rangle=\frac{1}{\sqrt{2E_A(\vec{\rm k}')}}\left\langle\vec{\rm k}',k'^0=\frac{W}{2}-E_A(\vec{\rm k}')\mid M\mid\vec{\rm k},k^0=\frac{W}{2}-E_A(\vec{\rm k})\right\rangle\frac{1}{\sqrt{2E_A(\vec{\rm k})}} , \tag{A7}$$

$$\langle\vec{\rm k}'\mid\Sigma(W)\mid\vec{\rm k}\rangle=\frac{1}{\sqrt{2E_A(\vec{\rm k}')}}\left\langle\vec{\rm k}',k'^0=\frac{W}{2}-E_A(\vec{\rm k}')\mid\Sigma\mid\vec{\rm k},k^0=\frac{W}{2}-E_A(\vec{\rm k})\right\rangle\frac{1}{\sqrt{2E_A(\vec{\rm k})}} . \tag{A8}$$

Let us define a wave function $\psi^{(+)}_{\vec{\rm k},s}$ such that

$$M(W)\mid\vec{\rm k}\rangle u(\vec{\rm k},s)=\Sigma(W)\mid\psi^{(+)}_{\vec{\rm k},s}\rangle . \tag{A9}$$

We find

$$\langle\vec{\rm k}'\mid\psi^{(+)}_{\vec{\rm k},s}\rangle=\delta(\vec{\rm k}'-\vec{\rm k})u(\vec{\rm k},s)+\frac{1}{\gamma^0[W-E_A(\vec{\rm k}')]-\vec{\gamma}\cdot\vec{\rm k}'-m_{\rm N}+i\epsilon}\int d\vec{\rm k}''\langle\vec{\rm k}'\mid\Sigma(W)\mid\vec{\rm k}''\rangle\langle\vec{\rm k}''\mid\psi^{(+)}_{\vec{\rm k},s}\rangle \tag{A10}$$

or

$$\{\gamma^0[W-E_A(\vec{\rm k}')]-\vec{\gamma}\cdot\vec{\rm k}'-m_{\rm N}\}\langle\vec{\rm k}'\mid\psi^{(+)}_{\vec{\rm k},s}\rangle=\int d\vec{\rm k}''\langle\vec{\rm k}'\mid\Sigma(W)\mid\vec{\rm k}''\rangle\langle\vec{\rm k}''\mid\psi^{(+)}_{\vec{\rm k},s}\rangle . \tag{A11}$$

Equation (A11) may be written

$$\{W-E_A(\vec{\rm k}')-\vec{\alpha}\cdot\vec{\rm k}'-\gamma^0 m_{\rm N}\}\langle\vec{\rm k}'\mid\psi^{(+)}_{\vec{\rm k},s}\rangle=\gamma^0\int d\vec{\rm k}''\langle\vec{\rm k}'\mid\Sigma(W)\mid\vec{\rm k}''\rangle\langle\vec{\rm k}''\mid\psi^{(+)}_{\vec{\rm k},s}\rangle . \tag{A12}$$

APPENDIX B

We may use Eq. (2.17) and write

$$U^{++}=V^{++}+V^{+-}g_-U^{-+}\ , \quad \text{(B1)}$$

$$U^{-+}=V^{-+}+V^{--}g_-U^{-+}\ . \quad \text{(B2)}$$

From the last equation we have

$$(g_-U^{-+})=g_-V^{-+}+g_-V^{--}(g_-U^{-+})\ , \quad \text{(B3)}$$

from which we obtain

$$g_-U^{-+}=\frac{1}{(g_-)^{-1}-V^{--}}V^{-+}\ . \quad \text{(B4)}$$

Thus

$$U^{++}=V^{++}+V^{+-}\frac{1}{(g_-)^{-1}-V^{--}}V^{-+}\ . \quad \text{(B5)}$$

From Eq. (2.16) we then have

$$T^{++}=U^{++}+U^{++}g_+T^{++}\ , \quad \text{(B6)}$$

with U^{++} given by Eq. (B5).

APPENDIX C

Here we consider Eq. (2.20) and note that the equation is equivalent to the following equation:

$$\langle \vec{k}',s'\,|\,t_{LS}(W)\,|\,\vec{k},s\rangle=\langle \vec{k}',s'\,|\,V_{LS}(W)\,|\,\vec{k},s\rangle+\sum_{s''}\int d\vec{k}''\frac{\langle \vec{k}',s'\,|\,V_{LS}(W)\,|\,\vec{k}'',s''\rangle\langle \vec{k}'',s''\,|\,t_{LS}(W)\,|\,\vec{k},s\rangle}{\dfrac{\vec{k}_W^{\,2}}{2\mu_{NA}}-\dfrac{\vec{k}''^2}{2\mu_{NA}}+i\epsilon}\ , \quad \text{(C1)}$$

where

$$\mu_{NA}=\frac{m_N M_A}{m_N+M_A}\ , \quad \text{(C2)}$$

and $\vec{k}_W$ is defined *via*

$$W=[\vec{k}_W^{\,2}+M_A^2]^{1/2}+[\vec{k}_W^{\,2}+m_N^2]^{1/2}\ . \quad \text{(C3)}$$

To see this one writes

$$W-E_A(\vec{k}'')-E_N(\vec{k}'')\equiv W-W'' =\frac{(\vec{k}_W^{\,2}-\vec{k}''^2)}{2\mu_{NA}}R^{-2}(W,\vec{k}'')\ , \quad \text{(C4)}$$

where

$$R^{-2}(W,\vec{k}'')=\frac{8\mu_{NA}}{(W+W'')\left[1-\dfrac{(M_A^2-m_N^2)^2}{W^2W''^2}\right]}\ . \quad \text{(C5)}$$

Note that if $W''=W$, $\vec{k}''\rightarrow\vec{k}_W$ and

$$R^{-2}(W,\vec{k}'')\rightarrow\frac{\mu_{NA}}{E_N(\vec{k}_W)E_A(\vec{k}_W)/W}\ . \quad \text{(C6)}$$

With the definition

$$\langle \vec{k}',s'\,|\,V_{LS}(W)\,|\,\vec{k},s\rangle =R(W,\vec{k}')\langle \vec{k}',s'\,|\,U^{++}(W)\,|\,\vec{k},s\rangle R(W,\vec{k})\ , \quad \text{(C7)}$$

$$\langle \vec{k}',s'\,|\,t_{LS}(W)\,|\,\vec{k},s\rangle =R(W,\vec{k}')\langle \vec{k}',s'\,|\,T^{++}(W)\,|\,\vec{k},s\rangle R(W,\vec{k})\ , \quad \text{(C8)}$$

one then obtains Eq. (C1). We see that in the nonrelativistic limit where $|\vec{k}_W|\ll m_N$, $R(W,\vec{k}_W)\rightarrow 1$. In that case, $V_{LS}(W)=U^{++}(W)$. In the relativistic limit $|\vec{k}_W|\geq m_N$, the potential to be used in the Lippmann-Schwinger equation differs from that to be used in Eq. (2.20) by the kinematic factors noted above. These factors are only needed if one insists upon using the Lippmann-Schwinger equation in a kinematic domain in which another question would be more appropriate, Eq. (2.20), for example.

We close this appendix by noting that our experience with relativistic models of pion-nucleus scattering[13] has indicated that the ambiguities that arise from the possibility of choosing various propagators in the reduction of the Bethe-Salpeter equation to a three-dimensional form do not appear to be numerically significant. Only quite small changes in the observables are noted if the same optical potential is inserted into three-dimensional equations, such as Eqs. (2.20) and (C1), which have different propagators.

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