Inelastic scattering of 160 MeV protons by ^{48}Ca

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Differential cross sections and analyzing powers were measured for the inelastic scattering of 160-MeV polarized protons over the angular range $6° \le \theta \le 49.5°$. The spectra covered excitation energies up to about 11 MeV in ^{48}Ca. Angular distributions and analyzing powers were determined for 28 separate peaks. The data are compared to distorted-wave impulse approximation calculations and spin assignments are suggested for many of the states. The resulting level diagram is compared to three different shell model calculations of the ^{48}Ca spectrum.

I. INTRODUCTION

Closed-shell nuclei present an opportunity to get a better understanding of the structure of nuclear wave functions by comparing experimental spectroscopic information with the results of theoretical calculations. A number of studies yielding spectroscopic information about levels in ^{48}Ca have been reported. Inelastic proton scattering has been studied at 12 MeV (Ref. 1) and from 25 to 40 MeV (Ref. 2) along with inelastic alpha particle scattering at 31 MeV (Ref. 3) and at 42 MeV (Ref. 4). Also available are a study of the ^{46}Ca(t,p) reaction,[5] measurements of ^{48}K β decay,[6] and some rather old electron-scattering data.[7] A summary of previous experimental work on ^{48}Ca is given in the Nuclear Data Sheets.[8]

Several calculations of energy levels in ^{48}Ca have been performed. Assuming a closed ^{40}Ca core but truncating the remainder of the space in different ways, both McGrory, Wildenthal, and Halbert[9] and Federman and Pittel[10] have calculated the spectrum of even-parity states. The spectrum of odd-parity states formed by particle-hole excitations has been calculated by Jaffrin and Ripka.[11]

Reported here are measurements of differential cross sections and analyzing powers for the inelastic scattering of 160-MeV polarized protons by ^{48}Ca over a region of excitation energy up to about 11 MeV and an angular range of from 6° to 49.5° with an energy resolution of 70—100 keV. The present study of ^{48}Ca is the first one done at an energy where the nucleon-nucleus cross section is close to a minimum; all other published studies have been done at lower energies where absorption greatly distorts the angular distributions at large momentum transfer. In the ensuing sections, first the experiment is briefly described and the results given. Next, the results of DWIA calculations are shown and l values are assigned for many of the excitations. Finally, comparisons are given between the assignments and the predictions of the various shell-model calculations. Tables with the differential cross sections and analyzing powers are available upon request.[12]

There has been a particular interest in the 10.22-MeV 1^+ state and the possibility that its excitation would show effects of mesonic degrees of freedom. The results for this state have been published separately.[13]

II. EXPERIMENT

A 10.4 mg/cm^2 target, enriched to $>97.3\%$ in ^{48}Ca, was bombarded by 159.8 MeV polarized protons from the Indiana University Cyclotron Facility. The beam polarization was about 75% and it was reversed about every 30 sec with the two alternate spectra stored separately. Outgoing protons were momentum analyzed in a quadrupole-dipole-dipole multipole magnetic spectrometer and detected in the focal plane by a helical-cathode proportional chamber. Particle identification was performed by use of ΔE signals obtained from two scintillation counters.[14] The energy resolution was about 70 keV at angles $<35°$ and about 100 keV at larger angles. At each angle the spectrometer setting covered the region from the first excited state up to a maximum excitation energy of about 11 MeV. Elastic scattering was measured in separate, shorter, runs. Data were taken from $\theta_{\rm lab}=6°$ to 49.5° in steps of 1.5°.

Known levels below 5 MeV were used to determine the energy calibration of the spectrometer. Extrapolating to the higher excitation energies yielded an excitation energy of 10.22 ± 0.04 MeV for the strong 1^+ state in this region,[13] in excellent agreement with the (10.212 ± 0.009) MeV found in a high resolution (p,p′) experiment at 44.4 MeV (Ref. 15) and the (10.227 ± 0.005) MeV observed in electron scattering.[16]

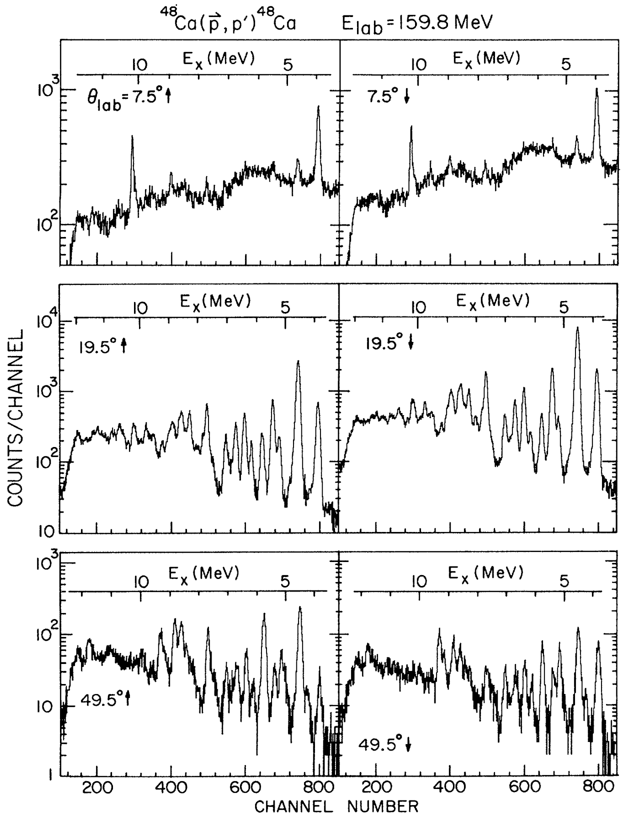

FIG. 1. Spectra for both polarization states (↑ or ↓) at (a) 7.5°, (b) 19.5°, and (c) 51°.

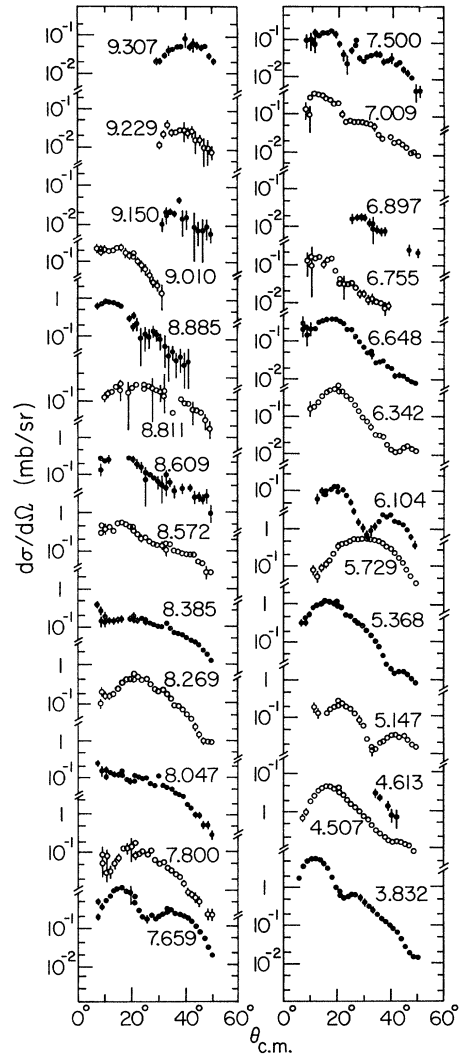

FIG. 2. Differential cross sections for the various peaks that were observed.

Spectra obtained at three angles are shown in Fig. 1. At 7.5° the spectrum is relatively simple, with only a few peaks discernible, reflecting the fact that only states reached by low angular momentum transfer will have large cross sections at small angles. Analyzing powers are small and positive. The background is high, due to slit scattering and the tail of the elastic peak. The presence of this continuum limited the overall count rate at small angles. At 19.5°, many more peaks are clearly visible. A striking effect at 19.5° is the large positive analyzing powers for virtually all of the peaks. At 49.5° the spectrum is still complex, although a detailed comparison shows that the states that dominate are not always the same as at 19.5°. For most, but not all, states the cross sections are much smaller at the larger angle. Analyzing powers are negative for most of the peaks and, on the average, smaller in magnitude than at 19.5°.

The greater the excitation energy, the more closely spaced the levels, and therefore the greater the chance that individual levels are not being resolved. The compilation[8] lists 10 levels between 6 and 7 MeV, 15 between 7 and 8 MeV, and 23 between 8 and 9 MeV. It therefore appears that with the present resolution above about 6.5 MeV there is a significant probability that a peak contains more than one state.

Differential cross sections for the various peaks are given in Fig. 2 and analyzing powers in Fig. 3.

III. ANALYSIS OF RESULTS

A calculation of the differential cross sections and analyzing powers for feeding a state requires, of course, a knowledge of both the ground-state and the excited-state wave functions. While a complete shell model calculation has not been done for ^{48}Ca, it can be taken that there is fairly good shell closure at both 20 and 28 nucleons, thus severely limiting the shell model configurations that need be considered. Furthermore, as can be seen in Fig. 4, the shapes of the angular distributions up to the position of the first maximum are largely determined by the angular momentum transfer, and are not very sensitive to the exact shell model wave functions. From the observed angular distributions it is therefore often possible to greatly limit, and perhaps uniquely determine, the spins of the various excited states.

Angular distributions were calculated in the distorted-wave impulse approximation (DWIA) for simple shell model wave functions with the code DW81.[17] The optical

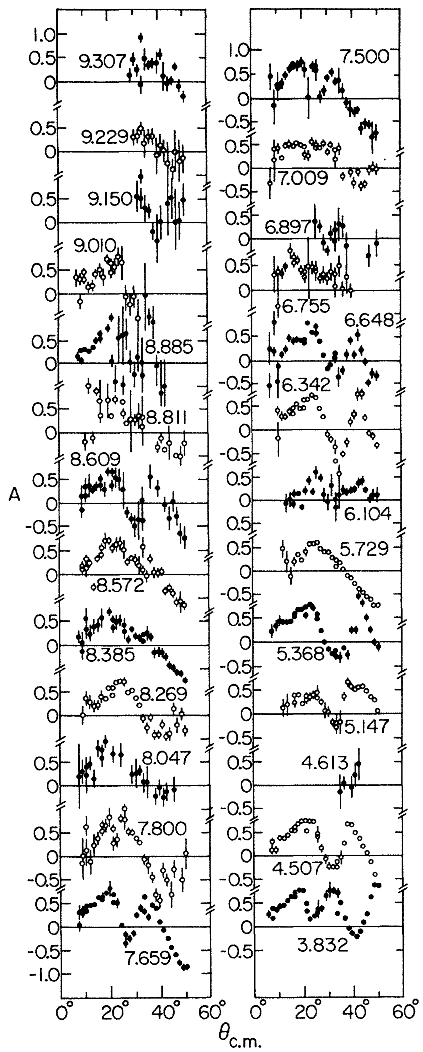

FIG. 3. Analyzing powers for the various peaks that were observed.

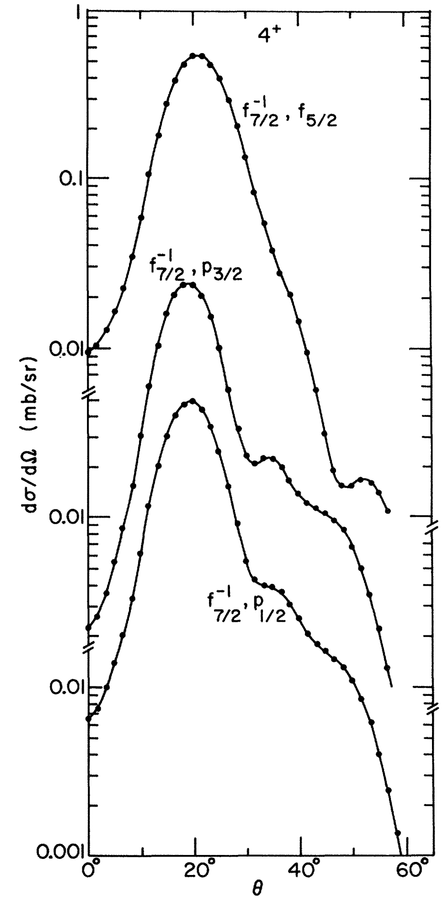

FIG. 4. DWIA calculations of angular distributions for exciting a 4^+ state assuming the ground state to be a closed ^{40}Ca core plus eight neutrons in the $f_{7/2}$ shell.

potential parameters were obtained using a search procedure applied to the elastic-scattering data obtained in this experiment, and are listed in Table I. The Love-Franey 140-MeV t matrix[18] was used for the effective interaction, and knockon exchange mechanisms were included exactly. Harmonic oscillator single-particle wave functions were used for bound states with an oscillator parameter $b=1.90$ fm, where the $l=0$ radial dependence is given by $\sim\exp(-\frac{1}{2}r^2/b^2)$. The value of b is an effective one and includes small center-of-mass corrections.[19] Small improvements in reproducing the angular dependence of positive-parity states could be obtained with the value $b=2.1$ fm. Since calculations were made only for the simplest pure configurations, center-of-mass renormalization corrections[19] were not included.

Figure 4 shows calculations for scattering to a 4^+ state for three different neutron excitations, and it can be seen that the position of the first maximum is about the same in all three cases and, indeed, is very close to the observed maximum for the known 4^+ state at 6.342 MeV. Figure 5 shows the calculated angular distributions for the various spin states of a particular multiplet $(f_{7/2}^{-1},f_{5/2})$. The calculated angular distributions for a 2^+ state agree well with experiment. For a 3^- state, for both a $d_{3/2}^{-1}f_{7/2}$ and an $s_{1/2}^{-1}f_{7/2}$ configuration, the first maximum is predicted to be near to 18°, in good agreement with experiment. However, for a 5^- state for both a $d_{3/2}^{-1}f_{7/2}$ and $f_{7/2}^{-1}g_{9/2}$ configuration the calculated angular distribution peaks at about 26°, while experimentally the maximum for the state at 5.729 MeV that has been assigned[8] 5^- is at 32°.

The use of the analyzing powers in making spin assignments is less clear. Figure 6 shows calculated analyzing powers for exciting a 2^+ state of different configurations.

TABLE I. The optical model parameters used in the DWIA calculations. The form of the potential is the same as that used in Ref. 19.

$V=17.1$ MeV	$W=-15.0$ MeV
$V_0=1.28$ fm	$r'=1.25$ fm
$a=0.72$ fm	$a'=0.67$ fm
$V_{so}=-13.4$ MeV	$W_{so}=7.85$ MeV
$r_{so}=1.03$ fm	$r_{so}=1.03$ fm
$a_{so}=0.53$ fm	$a_{so}=0.53$ fm

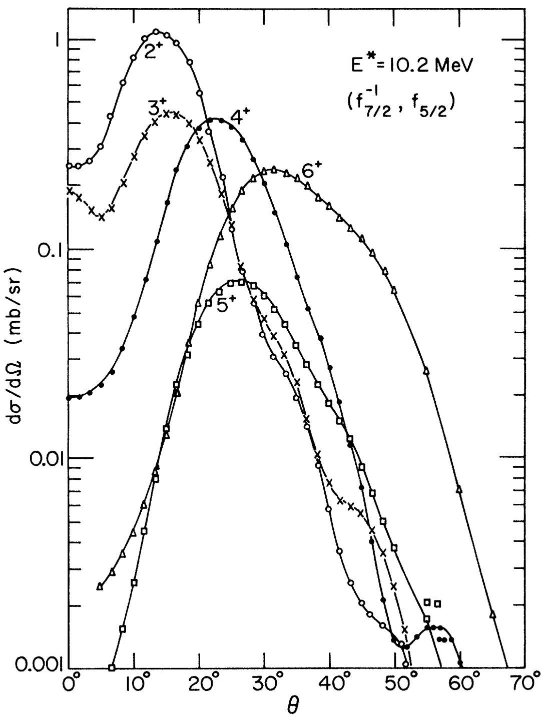

FIG. 5. Calculated angular distributions for the various states of an $(f_{7/2}^{-1},f_{5/2})$ multiplet.

It can be seen that the two calculated analyzing powers are nearly out of phase. On the other hand, it has been observed previously[19] that the analyzing powers in scalar-isoscalar transitions appear to exhibit a characteristic pattern of large variations, and thus might be useful in identifying natural-parity states. In the present work, a pattern of large positive lobes near 20° and 40° and a small negative lobe near 30° is discernible for several known natural parity states, namely those at 3.832 MeV (2^+), 4.507 MeV (3^-), 5.368 MeV (3^-), 6.342 MeV (4^+), 6.648 MeV (4^+), and 8.609 MeV (3^-).

For several of the states, it appears possible that by comparing the observed angular distributions to those from states of known spin and to calculations, significant limitations can be placed on the spin. Table II lists the "indicated spins" for such states. While the assignments are not definitive, it is considered unlikely that they are in error by more than one unit.

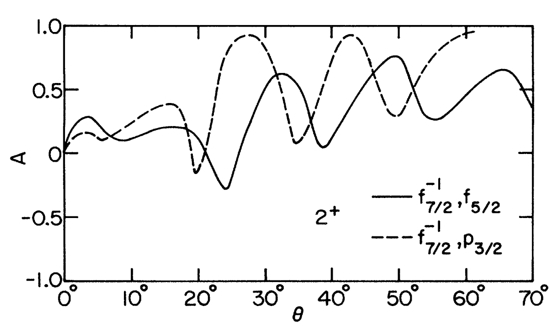

FIG. 6. DWIA calculation of the analyzing power for a 2^+ state for two different configurations.

TABLE II. States observed in ^{48}Ca and indicated spin assignments.

E_x	Indicated spin
0	0^{+a}
3.832	2^{+a}
4.507	3^{-a}
4.613	
5.147	4
5.368	3^{-a}
5.729	(5^-)
6.104	4
6.342	4^{+a}
6.648	4^{+a}
6.755	<3
6.897	$>\approx 3$
7.009	<3
7.500	3
7.800	4^{+a}
8.047	<2
8.269	4^{+a}
8.385	
8.572	3
8.609	3^{-a}
8.811	
8.885	2
9.010	<4
9.150	>4
9.229	$8^+,8^-$
9.307	$8^-,8^+$
10.22	1^+

[a]Previously assigned.

IV. DISCUSSION

As noted previously, the calculated angular distributions for the known 2^+ state at 3.832 MeV and the known 3^- state at 4.507 MeV agree well with experiment, at least as to the position of the first maximum. The state at 5.147 MeV had been assigned 5^- on the basis of (p,p′) studies[1,2] at 12–40 MeV. Angular correlation measurements[18] restrict the spin to 3, 4, or 5. The lifetime of the state has been reported as favoring a 5^- assignment,[20] but subsequent work showed that the lifetime measured was actually that of a lower state,[21] located at 4.504 MeV. Therefore, assignments of 3 or 4 are also consistent with the gamma-ray data. The angular distribution in the present work favors 4.

Considerable evidence[8] supports the 3^- assignment for the 5.368 MeV state and the observed angular distribution is in good agreement with that calculated for a 3^- state.

Although the nuclear data compilation[8] lists 5^- for the 5.729 MeV state, the assignment does not appear to be definitive. The 25–40 MeV (p,p′) data[2] favor $l=5$, but the lower energy data[1] favor $l=3$. An (α,α') study at 31 MeV reports[3] $l=5$, while one[4] at 42 MeV reports $l=2$. The first maximum in the angular distribution in the present work is at an angle some five degrees greater than that calculated for a 5^- state, which must be considered a large discrepancy in light of the good agreement found for other spin states. Furthermore, the analyzing powers do not fit the pattern, noted above, that appears to often be present in the excitation of natural parity states. However, it should also be noted that for the 5^- state in ^{28}Si,[22] the first maximum is considerably broader and centered at a larger angle than is the DWIA prediction.

The position of the first maximum favors 4 for the 6.104-MeV state, whose spin had not been previously assigned. Both the 6.342-MeV and the 6.648-MeV states have previously been tentatively assigned[8] 4^+, and the present angular distributions support these assignments. The angular distributions for the 7.659-MeV state support the previous[8] 3^- assignment. The compilation tentatively lists 4^+ for a state at 7.800 MeV and the present work indicates that there is, indeed, such a state at or near this energy. A similar statement holds for the 8.269-MeV state. Similarly, the tentative 3^- assignment[8] for the 8.609-MeV state, which also stems from lower energy (p,p′) work,[2] is in accord with the present angular distribution. On the other hand, the present study favors a low spin for the 8.885 MeV state, while the lower energy (p,p′) work[2] indicated 5^-. At this high an excitation energy, it is quite possible that individual levels were not resolved.

The states at 9.229 and 9.307 MeV come up at large angles, indicating that they have high spin. An 8^- state will result from a "stretched" $(f_{7/2}^{-1}g_{9/2})$ configuration, and from the excitation energy of these states in[23] ^{54}Fe and[24] ^{58}Ni, it can be expected that an 8^- state will lie at about 9 MeV excitation energy in ^{48}Ca. It is therefore tempting to ascribe one of these states as being of this configuration. Since an 8^- state can be made in only one way, if the ground state is assumed to be pure $(f_{7/2})$,[8] it is unlikely that both of these states are of this assignment. Figure 7 shows calculated angular distributions for a 7^- and an 8^- state. Comparing the distributions for the 9.229 and 9.307 MeV states with these calculations, it appears likely that these states have $J=7$, 8, or 9. With the shell model orbitals that are available, there is no simple way to make $9^{\pm}$ or 7^+ states. An 8^+ state of the configuration $(f_{7/2})^6_6(p_{3/2})^2_2$ can be expected at about 9 MeV. A simple calculation of the spacings for an $f_{7/2}^{-1}g_{9/2}$ multiplet showed that the 7^- state would be expected to be at least 800 keV below the 8^- state.[25] Furthermore, the cross section for exciting an 8^- state is expected to be about 6 times that for exciting a 7^- state (Fig. 7), and no such strong excitation is seen at about 10 MeV. The cross section at the maximum for the 9.307-MeV state is about $\frac{1}{3}$ that calculated for an 8^- state, while the cross section for the 9.229-MeV state is about $\frac{1}{8}$ of the calculated value. Since a quenching to about 30% of the calculated value might be expected for the $f_{7/2}\rightarrow g_{9/2}$ magnetic transition, the cross sections favor the 9.307-MeV state as being the 8^- state. An 8^- state at 9.276 MeV, whose excitation carries 28% of the single particle strength, has been observed in an electron scattering experiment.[26] Figure 8 shows the calculated analyzing powers, and the 9.229-MeV state better fits the calculation for an 8^- state; neither state gives a good fit to the calculated analyzing powers of a 7^- state.

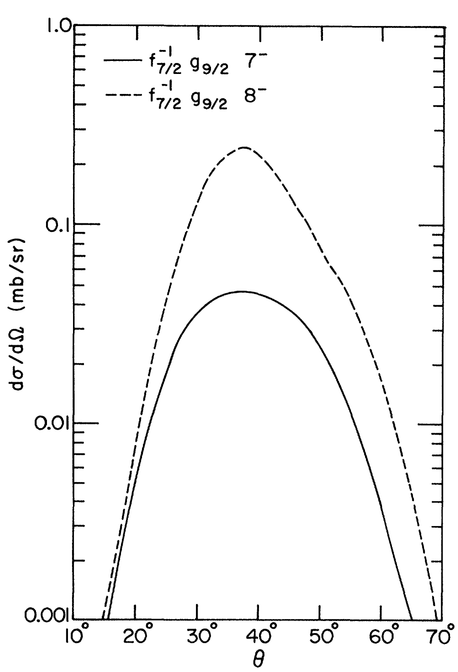

FIG. 7. Calculated angular distributions for the 7^- and 8^- members of the $(f_{7/2}^{-1},g_{9/2})$ octet.

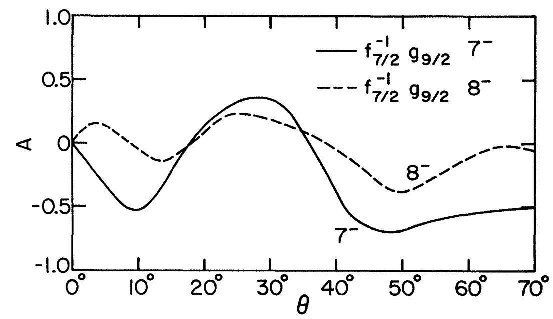

FIG. 8. Calculated analyzing powers for the 7^- and 8^- members of the $(f_{7/2}^{-1},g_{9/2})$ octet.

V. COMPARISON WITH SHELL MODEL CALCULATIONS AND CONCLUSIONS

As noted in the Introduction, while there have been calculations of the energy level spectrum of ^{48}Ca, none have used a large enough shell model space so that anything like a complete description of the low-lying levels could be expected. Table II lists 28 levels and several more are known from other experiments. While it cannot be expected that there is much to be gained from a level by level comparison between theory and experiment, something might be learned by comparing some of the features of the various calculations with the observations.

McGrory, Wildenthal, and Halbert[9] calculated only even-parity levels in ^{48}Ca. Their paper shows the results of a variety of calculations, but the only one for which the first excited state was not at much too low an energy made use of a modified Kuo-Brown interaction. For this calculation, only f and p orbits were allowed with the restriction that at least six particles be in the $f_{7/2}$ or $p_{3/2}$

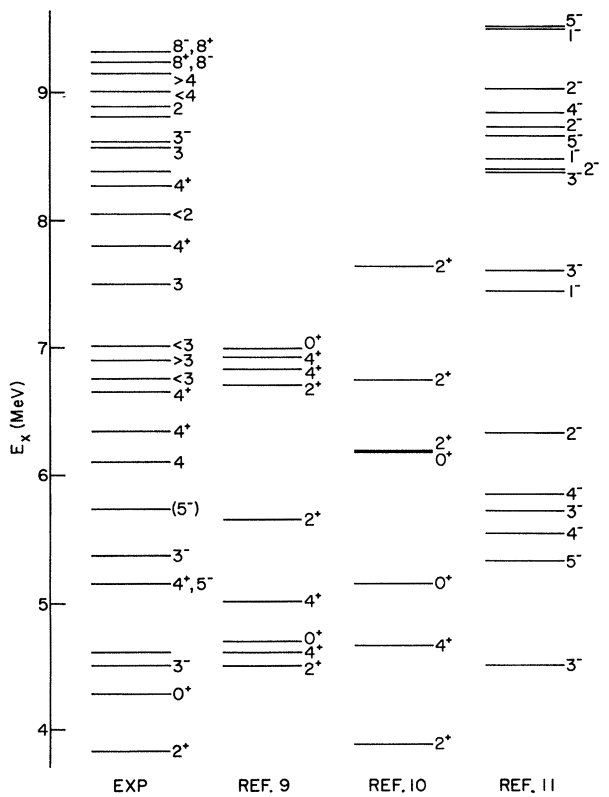

FIG. 9. Levels observed in the present experiment (plus the 0^+ state at 4.284 MeV) and the results of various shell model calculations.

orbits. Results are shown in Fig. 9. Inelastic proton scattering is not likely to excite 0^+ states, and it is therefore not surprising that none are observed in the present work. The position of the lowest predicted 0^+ state is in good agreement with that of the known 0^+ state at 4.284 MeV. A state at 5.461 MeV has been assigned 0^+ from (t,p) measurements,[5] while the next predicted 0^+ state is at about 6.9 MeV. In addition to the first excited state, two other 2^+ states below 7 MeV are predicted. Two low-spin states at around 7 MeV are listed in Table II and others could have been missed. The calculation predicts four 4^+ states below 7 MeV, which is about the number observed. No even-parity states with $J \geq 5$ are predicted and, indeed no states have been observed below 7 MeV that are likely to fall into this category.

Federman and Pittel[10] have also calculated the even parity states. Their space consisted of an inert ^{40}Ca core plus the $f_{7/2}$ and $p_{3/2}$ orbitals, with no more than two neutrons allowed in the $p_{3/2}$ orbit. Effective interactions were found by fitting to known excitations. Figure 9 also shows their results. Qualitatively, their results for 0^+ and 2^+ states are similar to those of McGrory, Wildenthal, and Halbert.[9] Only the lowest 4^+ state is predicted, however.

Particle-hole excitations forming odd-parity states have been calculated by Jeffrin and Ripka.[11] These authors took the ground state of ^{48}Ca to be a closed proton *s-d* shell and a closed neutron $f_{7/2}$ shell. Calculations were made by using various interactions and approximations, and Fig. 9 shows one of their spectra (CAL. 2 force, random phase approximation). The calculation correctly predicts the position of the first 3^- state. The prediction of five states near 6 MeV appears to be consistent with experiment, with the 5^- state that at 5.729 MeV, the 3^- state that at 5.368 MeV, and the state at 6.104 MeV being one of the two predicted 4^- states. The 7.659 MeV 3^- state seems to be correctly predicted, while in the 8—10 MeV region the situation is too complicated to permit a detailed comparison between theory and experiment. The calculation apparently did not consider 8^- states even though it allowed excitations to the $g_{9/2}$ orbital.

In each calculation the shell-model space was severely truncated and therefore none of them could be expected to reproduce the observed spectrum. For both the odd- and even-parity states there are significant areas of agreement between calculation and experiment, thus indicating that calculations of this type can be fruitful for ^{48}Ca. Perhaps with the greater amount of data now available, more elaborate calculations will be undertaken.

Little use has been made of the extensive analyzing power data that have been obtained in this experiment. The reason for this is, as noted above, the fact that the analyzing powers are sensitive to the actual configurations, unlike the angular distributions, which are primarily sensitive to the spin. Should extensive shell model calculations be performed for ^{48}Ca, the measured analyzing powers can be expected to be useful in determining their validity.

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[1]A. Tellez, R. Ballini, J. Delaunay, and J. P. Fouan, Nucl. Phys. A127, 438 (1969).
[2]C. R. Gruhn, T. Y. T. Kuo, C. J. Maggiore, and B. M. Preedom, Phys. Rev. C 6, 944 (1972).
[3]E. P. Lippincott and A. M. Bernstein, Phys. Rev. 163, 1170 (1967).
[4]R. J. Peterson, Phys. Rev. 140, B1479 (1965).
[5]J. H. Bjerrgaard, O. Hansen, O. Nathan, R. Chapman, S. Hinds, and R. Middleton, Nucl. Phys. A103, 33 (1967).
[6]L. G. Multhauf, K. G. Tirsell, S. Raman, and J. B. McGrory, Phys. Lett. 57B, 44 (1975).
[7]R. A. Eisenstein, D. W. Madsen, H. Theissen, L. S. Cardman, and C. K. Bockelman, Phys. Rev. 188, 1815 (1969).
[8]J. R. Beene, Nucl. Data Sheets 23, 1 (1978).
[9]J. B. McGrory, B. H. Wildenthal, and E. C. Halbert, Phys. Rev. C 2, 186 (1970).
[10]P. Federman and S. Pittel, Nucl. Phys. A155, 161 (1970).
[11]A. Jaffrin and G. Ripka, Nucl. Phys. A119, 529 (1968).

[12]See AIP document No. PAPS PRVCA-29-1703-55 for 55 pages of cross sections and analyzing powers. Order by PAPS number and journal reference from American Institute of Physics, Physics Auxiliary Publication Service, 335 E. 45th Street, New York, N.Y. 10017. The price is $1.50 for microfiche, or $8.75 for photocopies. Airmail additional. Make checks payable to the American Institute of Physics.

[13]K. E. Rehm, P. Kienle, D. W. Miller, R. E. Segel, and J. R. Comfort, Phys. Lett. 114B, 15 (1982).

[14]V. C. Officer, X. S. Henderson, and I. D. Svable, Bull. Am. Phys. Soc. 20, 1169 (1975).

[15]G. P. A. Berg, W. Hürlimann, I. Katayama, S. A. Martin, J. Meissburger, J. Römer, B. Styczen, F. Osterfeld, G. Gaul, R. Santo, and G. Sonderman, Phys. Rev. C 25, 2100 (1982).

[16]W. Steffen, H. D. Gräf, W. Gross, D. Meuer, A. Richter, E. Spamer, O. Titze, and W. Knüpfer, Phys. Lett. 95B, 23 (1980).

[17]Program DWBA70 by R. Schaeffer and J. Raynal (unpublished); extended code DW81 by J. R. Comfort (unpublished).

[18]W. G. Love and M. A. Franey, Phys. Rev. C 24, 1073 (1981).

[19]J. R. Comfort, G. L. Moake, C. C. Foster, P. Schwandt, and W. G. Love, Phys. Rev. C 26, 1800 (1982).

[20]J. W. Tape, R. Hensler, N. Benczer-Koller, and J. R. MacDonald, Nucl. Phys. A195, 57 (1972).

[21]G. Lo Bianco, N. Molho, P. A. Mando, and G. Poggi, Phys. Rev. C 15, 2245 (1977).

[22]S. Yen, R. J. Sobie, T. E. Drake, A. D. Bacher, G. T. Emery, W. P. Jones, D. W. Miller, C. Olmer, P. Schwandt, W. G. Love, and F. Petrovich, Phys. Lett. 105B, 421 (1981).

[23]R. A. Lindgren, J. B. Flanz, R. S. Hicks, B. Parker, G. A. Peterson, R. D. Lawson, W. Teeters, C. F. Williamson, S. Kowalski, and X. K. Maruyama, Phys. Rev. Lett. 46, 706 (1981).

[24]H. D. Wohlfahrt, O. Schwentker, G. Fricke, H. G. Andresen, and E. B. Shera, Phys. Rev. C 22, 264 (1980).

[25]R. D. Lawson, private communication.

[26]J. Wise, private communication.