# Electromagnetic properties of <sup>230</sup>Th studied by Coulomb excitation

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The Coulomb excitation of <sup>230</sup>Th by 135 MeV <sup>32</sup>S ions was studied using the particle-gamma coincidence technique. Scattered projectiles were detected within an angular range of  $30^{\circ} \le \Theta \le 172^{\circ}$ , and the impact-parameter dependence of the gamma-ray intensities was measured. Excitation of the ground-state rotational band was observed up to the  $10^{+}$  state, while the  $\beta$ - and  $\gamma$ -vibrational bands were seen up to the  $4^{+}$  and  $8^{+}$  states, respectively. Levels belonging to the  $K^{\pi}=0^{-}$  and  $1^{-}$  octupole vibrational bands were also identified. From the scattering-angle dependent gamma-ray intensities E2 transition matrix elements within the ground band and interband matrix elements connecting levels of the  $\gamma$  band with the ground band are deduced. Gamma-ray branching ratios for transitions from the  $\gamma$ - and octupole-vibrational bands to ground-band levels are also determined. Both transition matrix elements and branching ratios are compared with nuclear-model predictions.

### I. INTRODUCTION

The deformed nucleus <sup>230</sup>Th is the lightest actinide nucleus having a half-life which is long enough to permit its use as a target nucleus for in-beam experiments. However, only a limited number of such experiments have been performed and, hence, <sup>230</sup>Th is less well studied than other nuclei in the actinide region. Previous work on <sup>230</sup>Th includes studies of the (d,d') reaction,<sup>1</sup> the  $\beta$  decay of <sup>230</sup>Pa (Ref. 2 and references cited therein), Coulomb excitation by  $\alpha$  particles,<sup>3-5</sup> the  $\alpha$  decay<sup>6,7</sup> of <sup>234</sup>U, and the (p,t) two-neutron pickup reaction<sup>8</sup> on <sup>232</sup>Th. From these studies, energies, spins, and parities of the lowest states in <sup>230</sup>Th, as well as a few electromagnetic transition rates, have been deduced.

Recently we showed<sup>9</sup> for <sup>232</sup>Th that Coulomb excitation by medium-heavy projectiles, such as <sup>32</sup>S ions, provides a sensitive method to obtain detailed information on the electromagnetic properties of both yrast states and levels belonging to side bands built upon vibration-like configurations up to spins I < 14%. To study the electromagnetic properties of <sup>230</sup>Th in more detail, we have again performed a Coulomb-excitation experiment using <sup>32</sup>S ions. The work reported here supplements the previous studies of <sup>230</sup>Th as new excited states are identified and electromagnetic matrix elements are determined. Moreover, gamma-ray branching ratios from the  $\gamma$ - and octupolevibrational bands into the ground band are found to be very sensitive to band-coupling phenomena and give insight into finer details of the wave functions describing these states. Selected aspects of this work were reported earlier.10

## **II. EXPERIMENTAL PROCEDURE**

The experiment was performed by exposing a highly enriched <sup>230</sup>Th target to a beam of <sup>32</sup>S ions obtained from the Emperor tandem accelerator at München. A beam energy of 135 MeV was chosen, which is considered low enough to make the influence of nuclear forces on the excitation process negligible. This energy corresponds to a minimum distance of 5.3 fm between the surfaces of the projectile and target nucleus in head-on collisions when a charge radius of  $R = 1.2A^{1/3}$  fm and a quadrupole deformation parameter<sup>4</sup>  $\beta_2 = 0.2$  are used to calculate the Coulomb energy.

The target was produced by electrodeposition of thorium enriched to 83.9% in <sup>230</sup>Th from an isopropanol solution onto a 0.94 mg/cm<sup>2</sup> Ti backing. The thickness of the deposit was  $\approx 0.8$  mg/cm<sup>2</sup> with inhomogeneities of the order of 20% across the target diameter of 5 mm. These specifications were obtained from measurements of the specific  $\alpha$  and  $\gamma$  activities, as well as the energy loss of  $\alpha$ particles emitted from a <sup>241</sup>Am source and passing through the target.

Gamma-ray spectra were measured in coincidence with the scattered <sup>32</sup>S projectiles as a function of the scattering angle. The experimental setup as shown in Fig. 1 included two Ge(Li) detectors positioned at  $(\Theta_{\gamma}, \Phi_{\gamma}) = (45^\circ, 180^\circ)$ and  $(112^\circ, 180^\circ)$  with respect to the beam direction. A large position-sensitive parallel-plate avalanche gas counter covering an angular range of  $30^\circ \le \Theta \le 150^\circ$  and an annular Si surface-barrier detector extending from 162° to 172° in the laboratory system were used to detect the scattered projectiles. The azimuthal range covered by the gas counter varied from  $-20^\circ \le \Phi \le 20^\circ$  to  $-28^\circ \le \Phi \le 28^\circ$ 



FIG. 1. Experimental setup used in the present Coulombexcitation study.

depending on the scattering angle. The cathode of this detector was divided into 12 segments, each having an angular width of  $\Delta \Theta = 10^{\circ}$ . These segments were connected by integrated 20-ns delays, thus forming a delay line which served to read out the position, i.e., scattering angle, information. To prevent recoiling <sup>230</sup>Th nuclei and <sup>32</sup>S ions scattered from the Ti backing from entering the particle detector and thus contributing to the background, absorber foils thick enough to stop these particles were placed in front of the gas counter. Sulphur ions scattered from <sup>230</sup>Th, however, suffered only a minor energy loss. For further details of the experimental setup and particulars of the position-sensitive gas counter, see Ref. 11.

For each event, the energy signal from either Ge(Li) detector, the scattering-angle information from the avalanche counter or the energy signal from the Si detector, and the time relations between the detectors were recorded on magnetic tape. From these data, particle-coincident gamma-ray spectra were accumulated for all scattering-angle intervals with random coincidences substracted. The energy and efficiency calibration of the gamma-ray spectra was evaluated using  $^{152}$ Eu and  $^{182}$ Ta sources.

A gamma spectrum measured with the Ge(Li) detector at  $\Theta_{\gamma} = 112^{\circ}$  is shown in Fig. 2. This spectrum contains gamma rays which are coincident with <sup>32</sup>S projectiles scattered into the angle interval  $50^{\circ} \le \Theta \le 150^{\circ}$  and  $162^{\circ} \le \Theta \le 172^{\circ}$ . Since the recoil velocity of the <sup>230</sup>Th nuclei varies from 0.007c to 0.02c depending on the scattering angle of the projectile, the individual particlecoincident gamma-ray spectra were corrected for their different Doppler shifts prior to summation. The gamma transitions seen in the spectrum of Fig. 2 are listed in Table I.

## **III. ANALYSIS OF THE DATA**

A partial level scheme of  $^{230}$ Th, which results from the gamma transitions listed in Table I in combination with data published previously,<sup>12</sup> is shown in Fig. 3. The ground-state rotational band was established by earlier work<sup>1</sup> up to the 6<sup>+</sup> (tentatively 8<sup>+</sup>) level, while the present experiment shows excitation of the ground band



FIG. 2. Gamma-ray spectrum resulting from the Coulomb excitation of  $^{230}$ Th by  $^{32}$ S ions measured in coincidence with the scattered projectiles (see the text). Lines labeled by c are contaminants due to transitions in  $^{232}$ Th.

up to the  $10^+$  state. The characteristic dependence of the gamma intensity on the projectile scattering angle served to identify the  $8^+$ - $6^+$  and  $10^+$ - $8^+$  transitions. Our interpretation is in excellent agreement with results of a very recent study of the  $^{232}$ Th( $\alpha, \alpha' 2n$ )<sup>230</sup>Th reaction,  $^{13}$  which also shows excitation of the ground band up to  $I^{\pi} = 10^+$ . Moreover, the Coulomb excitation induced by  $^{84}$ Kr and  $^{142}$ Nd ions was recently used  $^{14}$  to investigate yrast states

and levels of the  $K^{\pi}=0^{-}$  octupole band in <sup>230</sup>Th up to higher spins. In the present experiment strong excitation of the  $\beta$ - and  $\gamma$ -vibrational bands, as well as the  $K^{\pi}=0^{-}$ and  $1^{-}$  octupole bands, is observed in addition to the ground-band levels mentioned, and several new levels are assigned. These assignments are primarily based on rotational energy systematics [the I(I+1) law to second order] and the Ritz combination principle.

TABLE I. Gamma transitions in <sup>230</sup>Th.

Energy <sup>a</sup>	Relative	in an	Energy <sup>a</sup>	Relative	
(keV)	intensity <sup>b</sup>	Assignment <sup>c</sup>	(keV)	intensity <sup>b</sup>	Assignment <sup>c</sup>
112.9	1838± 50	<sup>232</sup> Th	644.4	27± 15	$8^+_{\gamma} \rightarrow 8^+?$
120.9	$17901 \pm 125$	$4^+ \rightarrow 2^+$	651.1	47± 19	$3^{+}_{\gamma} \rightarrow 4^{+}$
151.4	$82\pm 24D$		667.8	$82\pm$ 18	<sup>232</sup> Th?
171.2	965± 34	<sup>232</sup> Th	675.7	82± 18	$2^+_B \rightarrow 2^+$
182.6	9267± 82	$6^+ \rightarrow 4^+$	681.4	265± 29	$6^+_{\gamma} \rightarrow 6^+$
187.9	$139\pm 23$		710.4	602± 34	$4^{+}_{\gamma} \rightarrow 4^{+}$
196.2	$141 \pm 22C$		719.6	64± 16	$4^{+}_{\beta} \rightarrow 2^{+}$
211.5	$131 \pm 19$	$9^- \rightarrow 7^-?$	727.9	$1889\pm 58D$	$2^{+}_{\gamma} \rightarrow 2^{+}$
223.8	$272\pm\ 25$	<sup>232</sup> Th	732.2	$252\pm 24D$	·
237.3	2163± 49	$8^+ \rightarrow 6^+$	734.9	$163 \pm 20$	<sup>232</sup> Th
240.9	$106 \pm 20C$		746.0	60± 15	$6^+_\beta \rightarrow 4^+??$
246.8	110± 15		753.9	$100 \pm 16$	5 <sup>-</sup> ′→6 <sup>+</sup>
251.9	$108 \pm 16$		761.6	50± 15	$8^+_{\beta} \rightarrow 6^+??$
257.8	222± 19	$7^- \rightarrow 8^+$	773.0	167± 19	$3^+_{\gamma} \rightarrow 2^+$
268.4	$185 \pm 19$	<sup>232</sup> Th	781.6	667± 34	$2^{+}_{\gamma} \rightarrow 0^{+}$ or
273.8	199± 21				$5^{+}_{\gamma} \rightarrow 4^{+}?$
285.9	$373 \pm 25$	$10^+ \rightarrow 8^+$	787.3	$160 \pm 18$	
303.4	$193 \pm 20$		791.4	87± 13	
313.6	110± 17		796.1	$111 \pm 11$	2 <sup>-</sup> ′→4 <sup>+</sup> ?
320.4	$101 \pm 18$	$12^{+} \rightarrow 10^{+}$	813.9	55± 11	
329.6	593± 31	$5^- \rightarrow 6^+$	824.3	91± 14	
335.5	$120\pm 18$		829.9	116± 15	$4^+_{\gamma} \rightarrow 2^+$
356.3	$129 \pm 19$		837.6	83± 13	3 <sup>-</sup> ′→4 <sup>+</sup> ?
397.7	407± 28	$3^- \rightarrow 4^+$	843.5	45± 10	
408.2	99± 17		875.8	51± 10	
420.5	$1628 \pm 22$	$5^{-\prime} \rightarrow 5^{-}$	896.0	80± 13	7−′→6+?
431.5	$168 \pm 21$		900.2	54± 11	$1^{-\prime} \rightarrow 2^{+?}$
454.9	979± 40	$1^- \rightarrow 2^+$	924.0	$67\pm12$	
460.0	$162\pm\ 22$	$2^{-\prime} \rightarrow 1^{-}$	929.5	$103 \pm 15$	
470.8	$218\pm 24$	9-→8+	934.6	$116 \pm 16$	5-'→4+
484.4	$143 \pm 21$	<sup>232</sup> Th	951.7	$93\pm 15D$	$1^{-'} \rightarrow 0^{+}$
495.6	539± 33	$7^- \rightarrow 6^+$	959.2	$191\pm 21D$	$3^{-\prime} \rightarrow 2^+$
508.7	$742\pm 38D$	$1^- \rightarrow 0^+$	966.8	73± 13	
513.0	$716\pm 37D$	$5^- \rightarrow 4^+$	976.2	77± 13	
518.3	485± 32	$3^- \rightarrow 2^+$	1015.6	33± 9	
554.9	$128 \pm 24$		1030.6	<b>43</b> ± 10	
581.8	437± 37	$0^+_B \rightarrow 2^+$	1080.3	73± 14	
597.3	47± 18	$5^{+}_{\nu} \rightarrow 6^{+}$ or	1170.4	70± 14	
		$4^{+}_{\beta} \rightarrow 4^{+}?$	1184.4	54± 12	
611.0	104± 22	<sup>232</sup> Th	1188.9	68± 14	
625.0	$112\pm 21$	$2^+_{B} \rightarrow 2^+$	1313.1	$35\pm 10$	
636.5	33± 17	۲ 	1323.0	88± 16	

<sup>a</sup>The energy errors are typically 0.5 keV for stronger lines and up to 1 keV for weaker transitions.

<sup>b</sup>Extracted from the gamma-ray spectrum coincident with projectiles scattered into the angle intervals  $50^{\circ} \le \Theta \le 150^{\circ}$  and  $162^{\circ} \le \Theta \le 172^{\circ}$  (see Fig. 2). The error given includes the statistical error and the uncertainty associated with the background subtraction.

<sup>c</sup>D=doublet, C=complex structure. Odd-parity levels marked by a prime belong to the  $K^{\pi} = 1^{-}$  octupole band. No prime indicates  $K^{\pi} = 0^{-}$ .



FIG. 3. Partial level scheme of <sup>230</sup>Th as based on the gamma transitions observed in the present experiment.

The determination of electromagnetic transition rates in <sup>230</sup>Th has been a major objective of this work. For this purpose, calculations of excitation probabilities and gamma-decay intensities were performed within the framework of the semiclassical theory of Coulomb excitation<sup>15</sup> using a sophisticated version<sup>16</sup> of the Winther-de Boer computer code.<sup>17</sup> These calculations served to determine electromagnetic transition matrix elements. In addition, the particle-gamma angular correlation was obtained, which is needed to relate the measured gamma intensities to branching ratios.

The internal conversion coefficients used in these calculations were taken from the tables of Hager and Seltzer.<sup>18</sup> Finite solid-angle corrections for the Ge(Li) detectors were evaluated on the basis of the work of Camp and Van Lehn.<sup>19</sup> Corrections to the Rutherford trajectory of the projectile due to dipole polarization, atomic screening, vacuum polarization, and relativistic effects, as derived in Ref. 20, were found to be negligible.

For the present experiment, the Sommerfeld parameter<sup>15</sup> is  $\eta = 110$ , and the semiclassical theory is believed to provide a reasonable description as long as only a few steps are involved in the excitation. For multistep processes, quantal corrections are expected to become nonnegligible, however. Thus far, exact quantum mechanical calculations, which take into account all relevant states, are extremely time consuming and have not been performed. The influence of quantal corrections on the Coulomb excitation of <sup>236</sup>U by <sup>40</sup>Ar ions was estimated (see, e.g., Ref. 21) to range from approximately zero for the  $4^+$  state to -7% for the  $10^+$  level of the ground-state band. Since these values are of the same order of magnitude as the uncertainties of the measured gamma strengths, quantal corrections were not applied in the analysis.

Our calculations of the gamma intensities included corrections for the magnetic hyperfine interaction, which causes the recoiling target nuclei to precess, thus leading to a more isotropic particle-gamma angular correlation. This alignment attenuation is accounted for by coefficients  $G_k$  which enter the angular distribution function given by Winther and de Boer.<sup>17</sup> We have adopted the Abragam-Pound<sup>22</sup> formalism, which is based on statistical perturbations, to parametrize the deorientation effect. In this model, the time-integrated attenuation coefficients are given by

$$G_k(N) = [1 + \lambda_k(N)\tau(N)]^{-1}, \qquad (1)$$

where  $\tau(N)$  denotes the lifetime of the nuclear state N. The quantities  $\lambda_k(N)$  are proportional to the g factor of the state N, which has been assumed to be constant within the ground band,<sup>23</sup> and the magnetic field. The attenuation coefficients  $G_k(N)$  were determined from the scattering-angle dependence of the gamma-intensity ratios  $I(\Theta;\Theta_{\gamma}=45^{\circ})/I(\Theta;\Theta_{\gamma}=112^{\circ})$ , which is influenced by the deorientation effect. For the 6<sup>+</sup> level of the ground band a value of  $G_2(6^+)=0.56\pm0.05$  was obtained. Correction factors  $G_k(N)$  for all other relevant states were calculated from this value by using Eq. (1) and the relation<sup>22</sup>  $\lambda_2(N)/\lambda_4(N)=3/10$ , which is valid for pure dipole interaction. A study of the deorientation effect in other actinide nuclei will be the subject of a forthcoming publication.<sup>24</sup>

By employing the details just described, Coulombexcitation and gamma-decay calculations were performed to compute gamma intensities as a function of the impact parameter. To deduce electromagnetic transition matrix elements from the measured gamma-ray intensities, the procedure applied in a previous study<sup>25</sup> of yrast bands in actinide nuclei, and described there in detail, was used.

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The method is based on the fact that a gamma-ray intensity ratio, in a limited scattering-angle interval, depends strongly on one transition matrix element only and may be used to determine this particular matrix element. In the work of Ref. 25 successive gamma transitions within the yrast sequence were used to calculate the intensity ratios. For the present study, which also includes an analysis of interband transitions, the method was generalized by introducing intensity ratios

$$r(\Theta; M, N, M', N') = I(\Theta; M \to N) / I(\Theta; M' \to N') . \quad (2)$$

In most cases, this ratio depends not only on transition matrix elements connecting states M with N and M' with N' with intensities  $I(\Theta; M \rightarrow N)$  and  $I(\Theta; M' \rightarrow N')$ , respectively, but also on a large number of matrix elements between other states M'' and N''. By selecting a suitable transition  $M' \rightarrow N'$ , a ratio r may be found that depends on just a few or even only one matrix element. To find such a normalization, a "sensitivity matrix" defined as

$$S(\Theta, M, N, M', N', M'', N'') = \frac{\Delta r(\Theta; M, N, M', N')}{\Delta \langle N'' | | M(E\lambda) | | M'' \rangle}$$
(3)

was determined from the Coulomb-excitation calculations. The elements of this seven-dimensional matrix are expansion coefficients in a Taylor series of the intensity ratio in terms of matrix elements  $\langle N''||M(E\lambda)||M''\rangle$  and, hence, provide a measure to what extent these elements influence the ratio r. For transitions within the ground-state band and selected interband transitions from the  $\gamma$ -vibrational band to ground-band levels, it was possible to find suitable intensity ratios to evaluate matrix elements.

## A. The ground-state band

Transition matrix elements  $\langle (I-2)_g | | M(E2) | | I_g \rangle$  within the ground band were deduced from intensity ratios

$$r(\Theta; I_g) = I[\Theta; I_g \to (I-2)_g] / I[\Theta; (I-2)_g \to (I-4)_g] .$$
(4)

At forward scattering angles, this ratio depends essentially on the  $\langle (I-2)_g | | M(E2) | | I_g \rangle$  matrix element, whereas the influence of other matrix elements becomes more important with increasing scattering angles. Hence, the latter angular region is less well suited for a determination of matrix elements. At small scattering angles, on the other hand, the level under consideration may be only weakly excited, with large statistical uncertainties associated with its gamma decay. By multiplying the r values with weight factors, which depend on both the statistical error and the strength of the correlation, the intermediate angular range was given the greatest significance for the determination of matrix elements. Transition matrix elements were obtained by an iteration procedure in which a set of matrix elements was varied until the experimental intensity ratios were reproduced. The "start" values were calculated using the rigid-rotor relations, with the  $\langle 2^+ || M(E2) || 0^+ \rangle$  and  $\langle 4^+ || M(E4) || 0^+ \rangle$  matrix elements taken from precise Coulomb-excitation studies with  $\alpha$  projectiles.<sup>4</sup> For further details of the computational method the reader is referred to Ref. 25.

The E2 diagonal matrix elements were found to affect the intensity ratios only slightly and, hence, were kept constant in the calculations, while the E4 matrix elements did have some influence. The correlations, however, were too complex to allow an unambiguous evaluation of E4 matrix elements. Higher-spin states are increasingly sensitive to the phase of the E4 matrix elements and allow a determination of the sign of the  $\langle 4^+ || M(E4) || 0^+ \rangle$  element in the frame of the rigid-rotor model. The positive sign was found to give better agreement with the particlegamma angular correlation. This is in accordance with the positive E4 matrix element established for <sup>232</sup>Th, <sup>236</sup>U, and <sup>238</sup>U by Guidry *et al.*<sup>21,26</sup> and Eichler *et al.*<sup>27</sup>

The influence of the  $\beta$ - and  $\gamma$ -vibrational bands, as well as the  $K^{\pi}=0^{-}$ , 1<sup>-</sup>, and 2<sup>-</sup> octupole bands, on the gamma-intensity ratios was also studied. The spin dependence of intraband and interband matrix elements was computed by using the Alaga rule.<sup>28</sup> For the coupling of the  $\gamma$ -vibrational band to the ground band, the rotational model of Bohr and Mottelson<sup>29</sup> was used, since gammaray branching ratios from levels of the  $\gamma$  band to the ground band, as determined from the present data (see below), indicate that this model provides a better description than the rigid rotor. The intrinsic E2 and E3 interband matrix elements were taken from Ref. 5, while the quadrupole moments of the side bands were assumed to be equal to that of the ground band. Variations in the intrinsic moments were found to change the gamma-intensity ratios within the ground band by no more than 2% as long as the calculated interband intensities reproduced the experimental data.

#### B. The $\gamma$ -vibrational band

In this subsection, the method used to determine interband E2 matrix elements between  $\gamma$ - and ground-band levels will be discussed. Since transitions within the  $\gamma$ band cannot be observed experimentally, their strength was calculated using the rigid-rotor relations. This assumption appears to be justified, because the energies of the  $\gamma$ -band levels follow very well the I(I+1) spacing. Since the phase relation between intraband and interband matrix elements cannot be deduced from the present data, we chose to use the same sign for all intrinsic moments. The particle-gamma angular correlations show that M1admixtures are insignificant.

With these assumptions, the interband gamma intensities depend exclusively on ground-band and interband matrix elements. Since the former have been fixed by the method described in the preceding section, the latter can now be determined from the data. In general, the interband gamma intensities can be normalized in different ways to calculate intensity ratios according to Eq. (2). Any transition, either within the ground band or from the  $\gamma$  band head to any ground-band level may be used for this purpose. The structure of the "sensitivity matrix" suggests that the interband transitions are only weakly correlated to each other, except for transitions originating

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TABLE II. Reduced E2 transition matrix elements within the ground band of <sup>230</sup>Th in comparison with rigid-rotor predictions.

	$\langle I+2  M(I)\rangle$	$\langle I+2  M(E2)  I\rangle$ (e b)		
Ι	Experiment	Rigid rotor		
4	6.1±0.9	5.7		
6	$7.2 \pm 1.1$	6.7		
8	7.6±1.4	7.6		

from the  $I_{\gamma}^{\pi} = 2^+$  band head. Therefore, the  $\langle I_g || M(E2) || 2_{\gamma}^+ \rangle$  matrix elements were determined first by employing the strong  $6_g^+ \cdot 4_g^+$  transition as normalization. In a second step, other interband matrix elements were computed relative to the  $2^+_{\gamma} - 2^+_g$  transition.

It was interesting to investigate to what extent the interband matrix elements are influenced by other vibrational bands, in particular the  $\beta$  band. As long as the interaction between the  $\beta$  and  $\gamma$  bands is not stronger than the coupling between the  $\gamma$  and ground-state band, the influence was found to be negligible. If the interaction exceeds this limit, then the impact-parameter dependence of the gamma intensities is noticeably affected. This effect has been used to estimate the  $\beta$ - $\gamma$ -interaction strength, as will be discussed below.

#### IV. RESULTS AND DISCUSSION

## A. The ground state band

The E2 transition matrix elements within the ground band as obtained from the present experiment are listed in Table II. The errors quoted include the experimental uncertainties of the gamma intensities, uncertainties due to correlations with other ground-band matrix elements, as well as with matrix elements connecting to vibrational levels, and the error associated with the E4 matrix elements. Table II also includes rigid-rotor predictions, which are normalized to the  $\langle 2^+ | | M(E2) | | 0^+ \rangle$  matrix element as obtained from the precise Coulomb-excitation experiment of Bemis et al.<sup>4</sup> As in other actinide nuclei,<sup>25</sup> the rigidrotor values are found to agree with the data within the quoted errors

The excitation energies of the ground-band are well reproduced by the relation  $E(I) = AI(I+1) + BI^2(I+1)^2$ with coefficients A = 8.88 keV and B = -8.27 eV. The occurrence of the second-order term in this expansion can be attributed to changes in the internal pairing structure<sup>30</sup> caused by the Coriolis force or a centrifugal stretching of the rotating nucleus as accounted for by the rotationvibration interaction.<sup>31</sup> Associated with the stretching of the core would be an increase of the E2 transition rates with respect to the rigid-rotor prediction. However, such an increase could hardly be detected because of the limited accuracy achievable in the determination of transition matrix elements.

A different method to study the rotation-vibration interaction is the analysis of gamma-ray branching ratios of interband transitions from vibrational bands to the ground band. These ratios provide a rather sensitive measure of the strength of the rotation-vibration coupling.

## B. The $\gamma$ -vibrational band

In Table III experimental B(E2) ratios are compared with results of nuclear-model calculations. From this table it is seen that B(E2) ratios of interband transitions from the  $\gamma$ -vibrational band to ground-band levels are at variance with both the Alaga rule and results<sup>32</sup> of the rotation-vibration model. The rotational model of Bohr and Mottelson,<sup>29</sup> on the other hand, describes the data very well. The latter theoretical values were calculated assuming identical intrinsic quadrupole moments within the ground-state band and the  $\gamma$  band by using the following equation:

$$B(E2;I_{\gamma} \to I_g) = 2(I_{\gamma}22 - 2 \mid I_g 0)^2 \{M_1 + M_2[I_g(I_g + 1) - I_{\gamma}(I_{\gamma} + 1)]\}^2.$$
(5)

		$B(E2;I_{\nu} \to I_{\rho})/B(E2;I_{\nu} \to I_{\sigma}')$					
Iγ	$I_g$	$I_g'$	Experiment	Alaga ratio	Rotation-vibratio model <sup>a</sup>	n Bohr and Mottelson <sup>b</sup>	
2	2	0	2.1±0.2	1.4	1.6	2.1	
4	4	2	$8.1 \pm 1.2$	2.9	2.2	8.3	
6	6	4	> 20	3.7	0.9	28.3	
				$B(E2;I_{\gamma})$	$\rightarrow I_{\beta})/B(E2;I_{\gamma}\rightarrow I_{g})$	A <u>a </u>	
				Rotation-	vibration I	interaction-boson	
Iγ	$I_{\beta}$	$I_g$	Experiment	moo	del <sup>a</sup> mo	del [SU(3) broken] <sup>c</sup>	
2	0	0	$1.1^{+6.2}_{-1.1}$	2	9	>>1	
aRefer	rence 32.						
<sup>b</sup> Refe	rence 29						

TABLE III. B(E2) ratios of interband transitions in <sup>230</sup>Th in comparison with various nuclearmodel predictions.

<sup>c</sup>Reference 33.

Ig	Iγ	Experiment	Alaga value	Rotation-vibration model <sup>a</sup>
2+	2 <sup>+</sup>	0.43±0.11	0.42	0.71
4+	$4_{\gamma}^{+}$	$1.03 \pm 0.27$	0.62	0.87
6+	$6_{\gamma}^{+}$	2.3 ±1.9	0.76	0.65

TABLE IV. Reduced E2 interband matrix elements in <sup>230</sup>Th connecting levels of the  $\gamma$ -vibrational band with the ground band.

Here,  $M_1$  is the intrinsic interband  $E_2$  matrix element, and  $M_2$  contains the rotation-vibration coupling strength. The coupling constant  $a_{\gamma} = -M_2/M_1$  was found from the experimental branching ratios to be  $a_{\gamma} = 0.029^{+0.003}_{-0.005}$ . With this single parameter all experimental  $E_2$  ratios are reproduced. It is interesting to note that, within the uncertainties, the same value,  $a_{\gamma} = 0.030^{+0.003}_{-0.006}$ , was found<sup>9</sup> previously for <sup>232</sup>Th.

<sup>a</sup>Reference 32.

In Sec. III a method was outlined to deduce interband E2 matrix elements between the  $\gamma$  band and the ground band on the basis of certain model assumptions. The results of such an analysis are presented in Table IV and are compared to predictions based on the Alaga rule and the rotation-vibration model. The errors quoted have been obtained under the same aspects as those of the ground-state band. No E4 contribution has been taken into account, however. Table IV shows that, in view of the experimental uncertainties, both theories provide an equally good description of the experimental results.

As stated in Sec. III, interband E2 matrix elements connecting levels of the  $\beta$  and  $\gamma$  bands influence the impactparameter dependence of the  $I_{\gamma}$ - $I_g$  transitions. The ratio  $B(E2;2^+_{\gamma}\rightarrow 0^+_{\beta})/B(E2;2^+_{\gamma}\rightarrow 0^+_g)$  deduced from that dependence is listed in Table III and is compared with model predictions. In spite of the large uncertainties associated with this method we consider this an important result. Rotation-vibration model calculations<sup>32</sup> suggest a rather strong coupling between the  $\beta$  and  $\gamma$  bands. The interacting-boson model, even with slightly broken SU(3) symmetry, predicts a still stronger interaction,<sup>33</sup> a measure of which is provided by the ratio  $B(E2;2^+_{\gamma}\rightarrow 0^+_{\beta})/B(E2;2^+_{\gamma}\rightarrow 0^+_{g})$ . As can be seen in Table III, such a strong coupling is in marked disagreement with our findings. A similar result was recently obtained<sup>11</sup> for <sup>232</sup>Th.

## C. The $K^{\pi} = 0^{-}$ octupole band

In Table V experimental B(E1) ratios of transitions from the lowest octupole band to the ground band are shown. It is seen that there is disagreement with the Alaga ratios calculated for a pure  $K^{\pi}=0^{-}$  configuration. This is not unexpected, however, since the octupole bands in the heavier actinides are known to be strongly coupled with each other by the Coriolis force.<sup>34</sup> With this band mixing taken into account the E1 transition strength may be written<sup>35</sup>

$$B(E1;I_i \to I_f) = \langle K_f = 0^+ | M(E1,0) | K_i = 0^- \rangle^2 [c(I_i,0,0)(I_i,010) | I_f(0) + \sqrt{2}c(I_i,1,0)(I_i,11-1) | I_f(0)Z]^2.$$
(6)

The coefficients  $c(I,K,\alpha)$  are the amplitudes in the expansion of the mixed wave function in terms of pure configurations having different K-quantum numbers. The signature  $\alpha$  is related to the K value of the strongest component in the wave function. In this sense, K=0 is assigned to the lowest octupole band. The parameter Z in Eq. (6) is a ratio of intrinsic E1 matrix elements,

$$Z = \langle K_f = 0^+ | M(E_1, -1) | K_i = 1^- \rangle / \langle K_f = 0^+ | M(E_1, 0) | K_i = 0^- \rangle .$$
<sup>(7)</sup>

To determine the coefficients  $c(I,K,\alpha)$ , Coriolis calculations were performed, which included levels of the  $K^{\pi}=0^{-}$ , 1<sup>-</sup>, and 2<sup>-</sup> octupole bands. The energies of the  $K^{\pi}=2^{-}$  band members, which were not observed in the present experiment, were taken from Ref. 12. Theoretical level energies were computed for the mixed bands and adjusted to fit the experimental data by varying six parameters, viz., two coupling matrix elements, three band-head

TABLE V. B(E1) ratios of transitions from the  $K^{\pi}=0^{-}$  octupole band to the ground band in <sup>230</sup>Th.

Ι	Experiment	Alaga ratio $(K^{\pi}=0^{-})$	Coriolis fit <sup>a</sup>
1	2.44±0.15	2.00	2.44
3	$1.95 \pm 0.21$	1.33	2.12
5	3.17±0.23	1.20	2.63
7	$3.01 \pm 0.32$	1.14	3.38

<sup>a</sup>Calculated using Z=4.4 [see Eq. (6) and the text].

TABLE VI. Energies of levels belonging to the  $K^{\pi}=0^{-}$ , 1<sup>-</sup>, and 2<sup>-</sup> octupole bands in <sup>230</sup>Th in comparison with results of a Coriolis band-mixing calculation.

I	$K^{\pi}$	Excitation energy (keV)		
		Experiment	Coriolis fit <sup>a</sup>	
1	0-	508.4	509.1	
3	0-	571.7	572.3	
5	0-	686.7	686.1	
7	0-	852.2	850.5	
9	0-	1064.4	1065.4	
1	1-	951.7	953.8	
2	1-	971.6	974.8	
3	1-	1013.0	1007.2	
5	1-	1108.7	1106.8	
7	1-	1252.7	1254.9	
2	2-	1079.3	1081.7	
3	2-	1127.8	1125.8	

<sup>a</sup>Calculated using the intrinsic Coriolis matrix elements  $\langle K=1^{-} | j_{+} | K=0^{-} \rangle = 0.27$ ,  $\langle K=2^{-} | j_{+} | K=1^{-} \rangle = 1.67$ , the band-head energies  $E_0(K^{\pi}=0^{-})=496.41$  keV,  $E_0(K^{\pi}=1^{-})=941.04$  keV,  $E_0(K^{\pi}=2^{-})=1039.18$  keV, and the inertial parameter  $A = \hbar^2/2 \mathscr{I} = 6.35$  keV.

energies, and one inertial parameter, which was assumed to be equal in all of the three bands. As shown in Table VI, very good agreement was achieved by this fit for all 12 levels. With the coefficients  $c(I,K,\alpha)$  obtained in this way E1 branching ratios were calculated using Eq. (6). The Z parameter in this equation was determined to be  $Z=4.4\pm0.4$  by adjusting these branching ratios to the

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- <sup>1</sup>R. C. Thompson, J. R. Huizenga, and Th. W. Elze, Phys. Rev. C <u>12</u>, 1227 (1975).
- <sup>2</sup>T. Valkeapää, J. Heinonen, and G. Graeffe, Phys. Scr. <u>5</u>, 119 (1972).
- <sup>3</sup>J. L. C. Ford, Jr., P. H. Stelson, C. E. Bemis, Jr., F. K. McGowan, R. L. Robinson, and W. T. Milner, Phys. Rev. Lett. <u>27</u>, 1232 (1971).
- <sup>4</sup>C. E. Bemis, Jr., F. K. McGowan, J. L. C. Ford, W. T. Milner, P. H. Stelson, and R. L. Robinson, Phys. Rev. C <u>8</u>, 1466 (1973).
- <sup>5</sup>F. K. McGowan, C. E. Bemis, Jr., W. T. Milner, J. L. C. Ford, Jr., R. L. Robinson, and P. H. Stelson, Phys. Rev. C <u>10</u>, 1146 (1974).
- <sup>6</sup>S. Bjørnholm, M. Lederer, F. Asaro, and I. Perlman, Phys. Rev. <u>130</u>, 2000 (1963).
- <sup>7</sup>M. Schmorak, C. E. Bemis, Jr., M. J. Zender, N. B. Gove, and P. F. Dittner, Nucl. Phys. A <u>178</u>, 410 (1972).
- <sup>8</sup>J. V. Maher, J. R. Erskine, A. M. Friedman, R. H. Siemssen, and J. P. Schiffer, Phys. Rev. C <u>5</u>, 1380 (1972).
- <sup>9</sup>J. Gerl, Th. W. Elze, H. Ower, K. Ronge, H. Bohn, and T.

corresponding experimental values. Table V shows that very good overall agreement was obtained with this Z value.

### V. SUMMARY

In conclusion, a Coulomb-excitation experiment employing a setup which measures the scattering-angle dependence of the gamma-ray intensities over a wide angular range, allows us to study in detail electromagnetic properties of collective nuclear states up to intermediate spins. The present <sup>230</sup>Th experiment has shown that B(E2) values within the ground-state band are in agreement with the rigid-rotor predictions within their experimental uncertainties. Gamma-ray branching ratios of transitions from the  $\gamma$ -vibrational band to the ground band have been shown to be a sensitive measure of the rotation-vibration interaction and were used to determine its strength within the framework of the rotational model of Bohr and Mottelson. The direct coupling between the  $\beta$ - and  $\gamma$ -vibrational bands was found to be considerably smaller than predicted by both the rotation-vibration model in the version considered here and the interactingboson model. Branching ratios of the  $K^{\pi}=0^{-}$  octupole band are well reproduced by the rotational model including the Coriolis interaction.

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Faestermann, Phys. Lett. 120B, 83 (1983).

- <sup>10</sup>J. Gerl, Th. W. Elze, H. Ower, H. Bohn, and T. Faestermann, in *Proceedings of the International Conference on Nuclear Physics, Florence, Italy, 1983* (Tipografia Compositori, Bologna, Italy, 1983), Vol. I, p. 189.
- <sup>11</sup>J. Gerl, Ph.D. thesis, Universität Frankfurt, 1983 (unpublished).
- <sup>12</sup>Y. A. Ellis, Nucl. Data Sheets <u>20</u>, 139 (1977).
- <sup>13</sup>K. Hardt, P. Schüler, C. Günther, J. Recht, K. P. Blume, and H. Wilzek, Nucl. Phys. (to be published).
- <sup>14</sup>J. de Boer, Ch. Lauterbach, Ch. Mittag, Ch. Schandera, Ch. Briancon, A. Lefebvre, C. F. Liang, J. P. Thibaud, R. J. Walen, A. Celler, R. Kulessa, H. Emling, and R. S. Simon, in *Proceedings of the International Conference on High Angular Momentum Properties of Nuclei, Oak Ridge, 1982*, edited by N. R. Johnson (Harwood, New York, 1982), p. 119.
- <sup>15</sup>K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. <u>28</u>, 432 (1956).
- <sup>16</sup>H. Ower, Universität Frankfurt, Institut für Kernphysik Technical Report, 1984 (unpublished).
- <sup>17</sup>A. Winther and J. de Boer, in *Coulomb Excitation*, edited by K. Alder and A. Winther (Academic, New York, 1966), p. 303.
- <sup>18</sup>R. S. Hager and E. C. Seltzer, Nucl. Data Tables A 4, 1

(1968).

- <sup>19</sup>D. C. Camp and A. L. Van Lehn, Nucl. Instrum. Methods <u>76</u>, 192 (1969).
- <sup>20</sup>K. Alder and A. Winther, *Electromagnetic Excitation* (North-Holland, Amsterdam, 1975), p. 292.
- <sup>21</sup>M. W. Guidry, E. Eichler, N. R. Johnson, G. D. O'Kelley, R. J. Sturm, and R. O. Sayer, Phys. Rev. C <u>12</u>, 1937 (1975).
- <sup>22</sup>A. Abragam and R. V. Pound, Phys. Rev. <u>92</u>, 943 (1953).
- <sup>23</sup>O. Häusser, H. Gräf, L. Grodzins, E. Jaeschke, V. Metag, D. Habs, D. Pelte, H. Emling, E. Grosse, R. Kulessa, D. Schwalm, R. S. Simon, and J. Keinonen, Phys. Rev. Lett. <u>48</u>, 383 (1982).
- <sup>24</sup>J. Gerl, Th. W. Elze, H. Bohn, and T. Faestermann (unpublished).
- <sup>25</sup>H. Ower, Th. W. Elze, J. Idzko, K. Stelzer, E. Grosse, H. Emling, P. Fuchs, D. Schwalm, H. J. Wollersheim, N. Kaffrell, and N. Trautmann, Nucl. Phys. A <u>388</u>, 421 (1982).

- <sup>26</sup>M. W. Guidry, P. A. Butler, P. Colombani, I. Y. Lee, D. Ward, R. M. Diamond, F. S. Stephens, E. Eichler, N. R. Johnson, and R. Sturm, Nucl. Phys. A <u>266</u>, 228 (1976).
- <sup>27</sup>E. Eichler, N. R. Johnson, R. O. Sayer, D. C. Hensley, and L. L. Riedinger, Phys. Rev. Lett. <u>30</u>, 568 (1973).
- <sup>28</sup>G. Alaga, K. Alder, A. Bohr, and B. R. Mottelson, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. <u>29</u>, No. 9 (1955).
- <sup>29</sup>A. Bohr and B. R. Mottelson, Nuclear Structure (Benjamin, Reading, Mass., 1975), Vol. II, p. 150.
- <sup>30</sup>B. R. Mottelson and J. G. Valatin, Phys. Rev. Lett. <u>5</u>, 511 (1960).
- <sup>31</sup>J. M. Eisenberg and W. Greiner, *Nuclear Theory* (North-Holland, Amsterdam, 1978), Vol. I, p. 127.
- <sup>32</sup>M. Seiwert (private communication).
- <sup>33</sup>A. Arima and F. Iachello, Ann. Phys. (N.Y.) <u>111</u>, 201 (1978).
- <sup>34</sup>K. Neergard and P. Vogel, Nucl. Phys. A <u>149</u>, 209 (1970).
- <sup>35</sup>L. Kocbach and P. Vogel, Phys. Lett. <u>32B</u>, 434 (1970).