Off-shell and nonlocal effects in proton-nucleus elastic scattering

A. Picklesimer
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
and Los Alamos National Laboratory, Los Alamos, New Mexico 87545

P. C. Tandy
Department of Physics, Kent State University, Kent, Ohio 44242
and Los Alamos National Laboratory, Los Alamos, New Mexico 87545

R. M. Thaler
Department of Physics, Case Western Reserve University, Cleveland, Ohio 44106
and Los Alamos National Laboratory, Los Alamos, New Mexico 87545

D. H. Wolfe
Department of Physics, Kent State University, Kent, Ohio 44242
(Received 22 December 1983)

The influence of off-shell and nonlocal effects in the first-order nonrelativistic microscopic optical potential is investigated for elastic proton scattering above 100 MeV. With the free nucleon-nucleon t matrix taken from the model of Love and Franey, these effects are significant only for scattering angles greater than about 60° and energies below about 300 MeV. The inadequacy of the standard first-order theory for predictions of spin observables at forward scattering angles remains unchanged when these effects are included and the need for higher order processes including medium and relativistic effects is reinforced.

The microscopic optical potential for nonrelativistic calculations of elastic proton scattering at intermediate energies is usually based on the multiple scattering theory of Kerman, McManus, and Thaler (KMT)[1] in which the first-order term for the optical potential is expressed as the diagonal matrix element of the free nucleon-nucleon (NN) t matrix with respect to the ground state of the target nucleus. Most calculations are performed under the assumption that this matrix element can be adequately represented by the following factorized form:

$$U'(\vec{k}',\vec{k}) = (A-1)\langle \vec{k}',\phi_0|t|\phi_0,\vec{k}\rangle \approx \left(\frac{A-1}{A}\right)\sum_{i=n,p} t_i(\epsilon,\vec{q})\rho_i(q) \quad . \tag{1}$$

Here ρ_n and ρ_p are the point-neutron and point-proton ground-state densities normalized to the number of particles, and t_n and t_p are the on-shell pn and pp t matrices which describe NN scattering data at energy ϵ. The latter is usually taken to be the beam energy. The above factorization prescription ignores the general nonlocalities that arise from the exact matrix element and implicitly assumes that the off-shell NN t matrix is equal to the on-shell t matrix at the relevant momentum transfer q. Knowledge of the extent to which theoretical predictions depend on these assumptions is crucial for calibrating the successes and deficiencies of the standard first-order theory.

In this Rapid Communication, we present results which address this point. It is important to provide an improved calibration so that, starting from energies near 500 MeV and small scattering angles where the first-order theory works quite well, the deficiencies that arise as the energy is lowered or the scattering angle is increased may be more confidently identified. Judgments concerning the role of relativistic effects,[2,3] nuclear medium effects,[4] and second-order (correlative) multiple scattering effects[1] can then be considered in better light. The first-order calculations presented here have no adjustable parameters and we compare with data to set the size of the effects under investigation into context.

We present some of the results from detailed investigations of the influence of off-shell and nonlocal effects through comparisons of the performance of the optimally factorized optical potential[5,6] with that of several other factorized approximations. Differences in the calculated results for elastic scattering observables are used to provide estimated theoretical error bars from these particular microscopic aspects of the first-order theory. The exact first-order optical potential may be expressed in the symmetric form

$$U'(\vec{k}',\vec{k}) = \left(\frac{A-1}{A}\right)\sum_{i=n,p}\int d^3p\,\langle \vec{k}',\vec{p}-\tfrac{1}{2}\vec{q}|t_i|\vec{k},\vec{p}+\tfrac{1}{2}\vec{q}\rangle\rho_i\left(\vec{p}-\frac{A-1}{2A}\vec{q}+\frac{1}{A}\vec{\kappa};\vec{p}+\frac{A-1}{2A}\vec{q}+\frac{1}{A}\vec{\kappa}\right) \quad , \tag{2}$$

where $\vec{k}'$ and $\vec{k}$ are the final and initial momenta of the projectile in the frame of zero total nucleon-nucleus momentum, $\vec{q}=\vec{k}'-\vec{k}$, $\vec{\kappa}=\frac{1}{2}(\vec{k}'+\vec{k})$, and $\vec{p}\pm\frac{1}{2}\vec{q}$ are the initial and final momenta of the struck nucleon in this frame. The single nucleon density matrix $\rho_i(\vec{p}',\vec{p})$ describes the ability of the target ground state to absorb a change in the intrinsic (relative to the core) momentum of a nucleon from $\vec{p}$ to $\vec{p}'$. The method of optimum factorization[5,6] is to expand the t-matrix element as a function of the integration variable $\vec{p}$ about a fixed value chosen so that the contribution of the next term in the Taylor expansion is minimized. In the present case, this minimum value is zero and the optimum momentum is $\vec{p} = -1/A\,\vec{\kappa}$. The optimum factorization result is then obtained by integrating the density matrix to produce the diagonal density. The result is

$$U'(\vec{k}',\vec{k}) = \left[\frac{A-1}{A}\right]\eta(\vec{k}',\vec{k}) \sum_{i=n,p} t_i\left[\epsilon,\vec{q},\frac{A+1}{A}\vec{\kappa}\right]\rho_i(q) \ , \tag{3}$$

where $t_i(\epsilon, \vec{\mathcal{K}}'-\vec{\mathcal{K}}, \vec{\mathcal{K}}'+\vec{\mathcal{K}})$ is the NN t-matrix element in the zero total momentum frame of the two nucleons corresponding to initial and final nucleon momentum of $\vec{\mathcal{K}}$ and $\vec{\mathcal{K}}'$, respectively. The Möller frame transformation factor $\eta(\vec{k}',\vec{k})$ transforms this t-matrix element to the nucleon-nucleus frame of zero total momentum by building in the constraint of Lorentz invariant of flux.[7] Spin precession effects[8] in the frame transformation are ignored.

The result in Eq. (3) is schematic in that the spin dependence is not made explicit. The optimally factorized spin-dependent KMT optical potential for spin-saturated nuclei can be expressed as[6]

$$U'(\vec{k}',\vec{k}) = U_c'(\vec{k}',\vec{k}) + \frac{i}{2}\vec{\sigma}\cdot\vec{q}\times\vec{\kappa}\,U_{LS}'(\vec{k}',\vec{k}) \ , \tag{4}$$

where

$$U_c'(\vec{k}',\vec{k}) = \left[\frac{A-1}{A}\right]\eta(\vec{k}',\vec{k}) \sum_{i=n,p} t_i^c\left[\epsilon,\vec{q},\frac{A+1}{A}\vec{\kappa}\right]\rho_i(q) \ , \tag{5}$$

and

$$U_{LS}'(\vec{k}',\vec{k}) = \left[\frac{A-1}{A}\right]\eta(\vec{k}',\vec{k})\left[\frac{A+1}{2A}\right] \times \sum_{i=n,p} t_i^{LS}\left[\epsilon,\vec{q},\frac{A+1}{A}\vec{\kappa}\right]\rho_i(q) \ . \tag{6}$$

The nonlocality of this factorized first-order optical potential is evident in the dependence upon $\vec{\kappa}$ $[=\frac{1}{2}(\vec{k}'+\vec{k})]$ in Eqs. (5) and (6). Of course, the nonlocality of the factor $\frac{1}{2}i\,\vec{\sigma}\cdot\vec{q}\times\vec{\kappa}$ comes from the momentum dependence of the spin-orbit operator $\vec{L}\cdot\vec{S}$ and is always included in optical potentials which are referred to as local. We include the more general nonlocality by solving the Lippmann-Schwinger equation in momentum space with the computer code WIZARD1.[6] Off-shell effects enter in Eqs. (5) and (6) because the three arguments of t_i^c and t_i^{LS} are completely independent. With an on-shell constraint, the third argument is completely determined by ϵ and $\vec{q}$ and a local form is obtained.

The model we have employed for the NN t matrix is that of Love and Franey[9] in which the parametrization is in terms of Yukawa form factors in the momentum variables required in Eqs. (5) and (6). Although the model parameters of Ref. 9 are fitted to NN data under the on-shell constraint, the off-shell extension is well defined. The resemblance of the off-shell behavior inherent in this model with that obtained for t matrices generated from phenomenological potentials is not known. We view the results described below as qualitatively indicative of the nature and importance of off-shell and nonlocal effects. A more complete description of the details of these calculations will be presented in a future publication.[10]

In Figs. 1 through 3, we show the results for differential cross section and analyzing power for elastic proton scattering from ^{16}O at 135 MeV, ^{12}C at 200 MeV, and ^{40}Ca at 300 MeV. The point-proton densities are obtained from three-parameter Fermi shape representations of charge densities extracted from electron scattering data.[13] The point-neutron densities are assumed to be equal to the point-proton densities. The solid curve in each case represents the calculation with the optimally factorized optical potential which includes nonlocal and off-shell effects. The dashed curve in each case displays the results obtained from the on-shell (local) factorization prescription of Eq. (1). In both cases, the energy of the NN t matrix is fixed at the value appropriate to NN scattering at the beam energy. This on-shell t matrix does not exist for the full range of momentum transfer accessible in nucleon-nucleus scattering. Simple kinematic considerations show that this point is reached at about 60° in on-shell nuclear scattering. In the calculations using on-shell factorization (dashed curves), we have set the on-shell t matrix to zero beyond the largest momentum transfer for which it exists. Of course, this extreme method for dealing with the t matrix outside the region where it is fixed by data is unphysical. However, the difference between solid and dashed curves beyond 60° is a dramatic illustration of the need for off-shell t-matrix elements and/or a reliable model

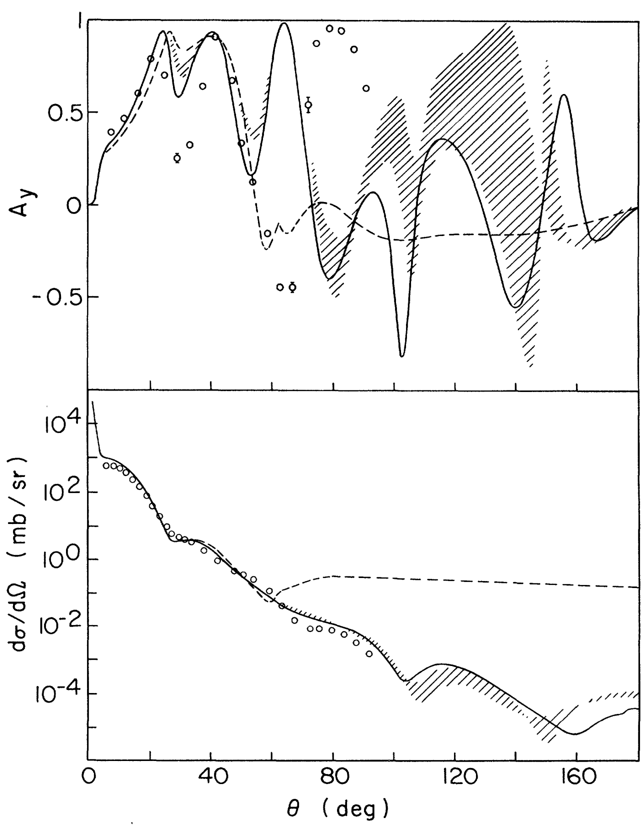

FIG. 1. Differential cross section and analyzing power for ^{16}O at 135 MeV calculated with the first-order microscopic optical potential treated in optimum factorization (solid curve) and on-shell factorization (dashed curve). The solid curve incorporates off-shell and nonlocal effects. The shaded band displays the uncertainties to be expected from approximate treatments of the folding integral of Eq. (2). The data are from Ref. 4.

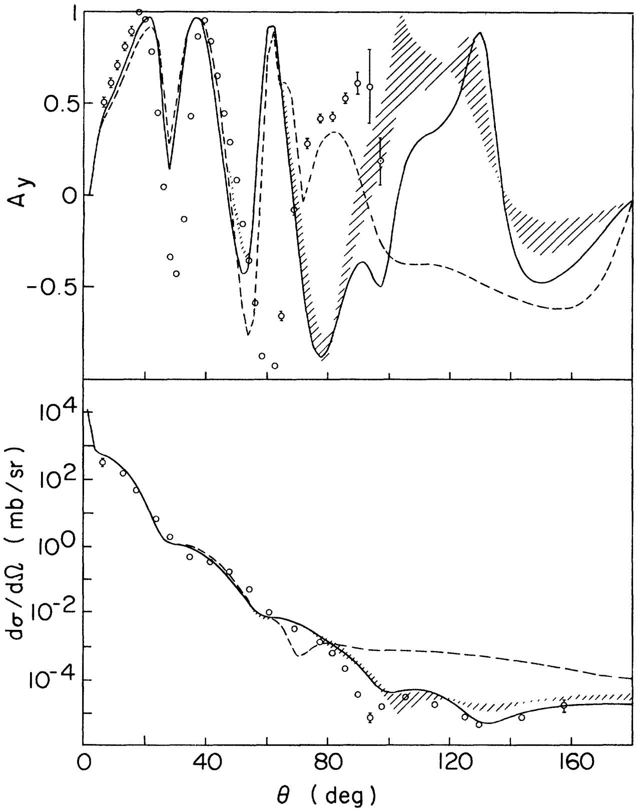

FIG. 2. Same as for Fig. 1 except for ^{12}C at 200 MeV. The data are from Ref. 11.

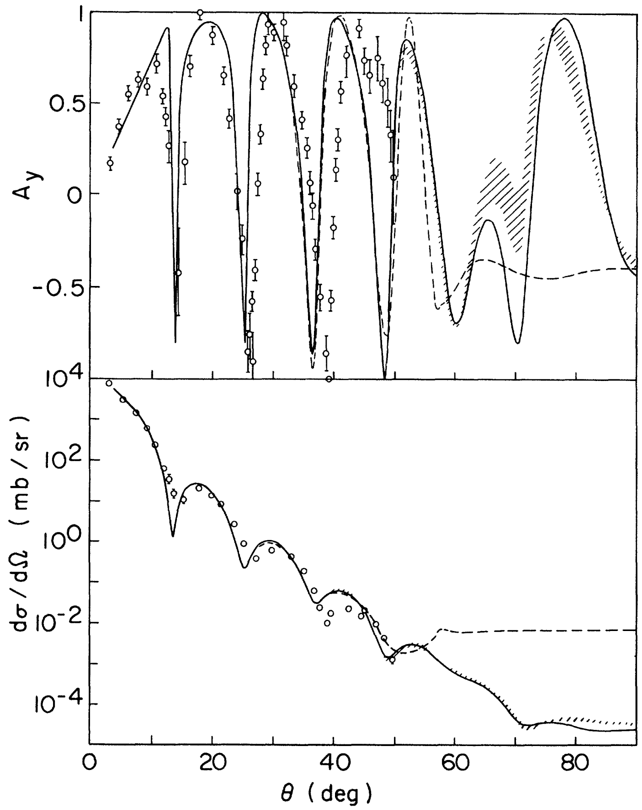

FIG. 3. Same as for Fig. 1 except for ^{40}Ca at 300 MeV. The data are from Ref. 12.

for the coupling of the effective NN energy to the kinematic variables of the reaction process.

The accuracy of the optimum factorization approximation to the folding integral given in Eq. (2) can be determined only by comparison with the exact result and work on this task is in progress. However, to gauge and illustrate the sensitivity to off-shell and nonlocal effects which might be encountered through the exact folding integral, we show as a shaded band in the figures the results bounded by two variations in the factorized treatment of the folding integral of Eq. (2). In the first variation, the nonlocality variable $[\vec{\kappa}=\frac{1}{2}(\vec{k}'+\vec{k})]$ in Eq. (3) is held fixed at its forward-angle on-shell value which is the beam momentum. This is the so-called asymptotic momentum approximation which has been found to be satisfactory in distorted-wave impulse approximation treatments of inelastic scattering.[14] The NN t matrix then becomes local but is still evaluated off-shell. In the second variation, the energy parameter of the t matrix is determined from the invariant mass of the NN system calculated from the NN momenta given in Eq. (2) with the optimum choice $\vec{p}=-1/A\,\vec{\kappa}$, and with $|\vec{k}'|=|\vec{k}|$. This leads to an NN energy $\epsilon(q)$ which increases significantly with increasing momentum transfer. The on-shell constraint is then imposed. The resulting NN t matrix $t[\epsilon(q),\vec{q}]$ exists for the full range of q accessible to physical nuclear scattering and the corresponding optical potential is local. We view the shaded bands in the figures as providing a sensible measure of the departures from the optimum factorization results that might be obtained if the folding integral in Eq. (2) were to be computed exactly.

The effects of variations in the treatment of the nonlocal and off-shell behavior are most evident in the analyzing powers beyond 60°. From these and other calculations we have performed, it is evident that these effects become less significant with increasing energy and/or increasing target mass. The calculated analyzing power for ^{40}Ca at 300 MeV shown in Fig. 3 contains all the features of the available data. In our opinion, an improved treatment of the folding integral of Eq. (2) is not likely to result in a significantly better description of the data. We have found that at this energy there is greater sensitivity to uncertainties in the employed density.[6] This is also the case for ^{40}Ca at 500 MeV. However, for the lighter nuclei at the two lower energies the situation is quite different. The calculated analyzing powers in Figs. 1 and 2 do not resemble the available data except at very forward angles. The estimated sensitivity to the treatment of the folding integral is significant at large angles where either data exists or can be measured with present experimental methods. This sensitivity is of the same size as sensitivities to the employed density.[6] For these lower energies and light targets, the standard first-order optical potential is clearly inadequate. However, it is also evident from Figs. 1 and 2 that this term must be treated with precision before it can reliably be combined with the necessary additional processes. The full-folding integral of Eq. (2) and a more realistic t-matrix model are apparently necessary ingredients in these cases even for work which is carried beyond first order. The extension of elastic data at 100–200 MeV to large scattering angles appears to be crucial for unravelling the many possible competing processes.

We note that the addition of relativistic pair-production and annihilation effects for the propagation of the projectile with the nucleus treated as an external field has a dramatic effect on the analyzing power for ^{16}O at 135 MeV and ^{12}C at 200 MeV.[3] The calculated minima are sharply deepened at the places indicated by the data. This effect has also been attributed to completely different physics through the use of a medium-dependent NN t matrix in the local-density approximation.[4] For the cross section shown in Fig. 2 for ^{12}C at 200 MeV, the mismatch between data and theory for the positions of the shoulders of the angular distributions at forward angles persists even when a variety of nuclear densities are employed. We note that the omitted spin-unsaturated effects,[15] which can be relatively important for this target, should be included and treated accurately before the standard first-order term can be considered complete. Correlative effects from a second-order multiple scattering term in the optical potential, while possibly important at large angles, are not expected to be significant at the very forward angles where clear discrepancies are evident. At the lower energies, Pauli exchange effects involving the external nucleon and the spectator nucleons may be required, but we have as yet not investigated the regime of importance of the external antisymmetry.

In summary, even with off-shell and nonlocal effects included, the standard first-order nonrelativistic optical potential remains inadequate for quantitative description of spin-dependent data. The finding that these effects are negligible for scattering angles forward of 60° is supportive of recent indications from these regions on the need for addition of relativistic and medium effects. However, as the energy is decreased below 300 MeV, it becomes increasingly important, especially for large angle analyzing powers, to treat the full-folding integral of the first-order term with precision before it can be combined with the necessary additional processes to form a reliable result.

The authors wish to acknowledge D. J. Ernst, M. V. Hynes, L. Ray, and F. Tabakin for helpful discussions during the course of this work. Three of us (A.P., P.C.T., and R.M.T.) wish to acknowledge the support, hospitality, and provision of computing facilities of the Los Alamos National Laboratory. Partial support was provided by the U.S. Department of Energy under Grant No. DE-AS05-76-ER05126, and the National Science Foundation under Grants No. PHY-83-05745, No. PHY-79-07511, and No. PHY-81-21956.

---

[1]A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (N.Y.) 8, 551 (1959).

[2]J. R. Shepard, J. A. McNeil, and S. J. Wallace, Phys. Rev. Lett. 50, 1443 (1983); B. C. Clark, S. Hanna, R. L. Mercer, L. Ray, and B. D. Serot, *ibid.* 50, 1644 (1983).

[3]M. V. Hynes, A. Picklesimer, P. C. Tandy, and R. M. Thaler, Phys. Rev. Lett. (to be published).

[4]J. Kelley *et al.*, Phys. Rev. Lett. 45, 2012 (1980).

[5]D. J. Ernst and D. L. Weiss, Phys. Rev. C 26, 605 (1982); D. J. Ernst, G. A. Miller, and D. L. Weiss, *ibid.* 27, 2733 (1983).

[6]D. H. Wolfe, Ph.D. dissertation, Kent State University, 1983 (unpublished).

[7]M. Goldberger and K. Watson, *Collision Theory* (Wiley, New York, 1964).

[8]J. A. McNeil, L. Ray, and S. J. Wallace, Phys. Rev. C 27, 2123 (1983).

[9]W. G. Love and M. A. Franey, Phys. Rev. C 24, 1073 (1981). We have verified that this model provides a good representation of the relevant NN scattering amplitudes constructed from phase-shift data. The on-shell factorized optical potential constructed from this model t matrix produces nuclear scattering observables which are in good agreement with those produced by using NN amplitudes constructed directly from phase-shift data.

[10]A. Picklesimer, P. C. Tandy, R. M. Thaler, and D. H. Wolfe (unpublished).

[11]H. O. Meyer, P. Schwandt, W. W. Jacobs, and J. R. Hall, Phys. Rev. C 27, 459 (1983).

[12]D. Hutcheon (private communication).

[13]C. W. DeJager, M. DeVries, and C. DeVries, At. Data Nucl. Data Tables 14, 479 (1974).

[14]F. Petrovich, H. McManus, V. A. Madsen, and J. Atkinson, Phys. Rev. Lett. 22, 895 (1969).

[15]A. Picklesimer, Ph.D. dissertation, Indiana University, 1979 (unpublished); W. G. Love, Phys. Rev. C 20, 1638 (1979); W. G. Love, D. Halderson, F. Petrovich, J. A. Carr, and H. McManus, *ibid.* 24, 2188 (1981).