γ-rigid model for heavy transitional nuclei ^{186,188,190,192}Os

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The level energies and the electromagnetic properties of heavy transitional nuclei ^{186,188,190,192}Os are studied using a semimicroscopic model. In this model the nucleus is assumed to be γ rigid but soft in the β degrees of freedom. The asymmetry parameter γ is determined from the ratio of the experimental energies of the first 2^+ and second 2^+ states assuming a rigid asymmetric rotor model. The symmetry parameter β for each J is obtained from the minimization of energy approximately projected from a triaxially symmetric intrinsic wave function. The pairing $+Q\cdot Q$ interaction of Baranger and Kumar is employed to obtain the intrinsic wave function. Within our γ-rigid model, and without explicitly introducing the hexadecapole term in the Hamiltonian, it has been possible to get good agreement with experiment for the calculated level energies of the ground band, γ band, and $K=4^+$ band, the $B(E2)$ values, electromagnetic moments, and $E2$ and $E4$ matrix elements. The success in describing the properties of each of these nuclei taking only one value of γ suggests that these nuclei may have rather stable triaxial shapes.

[NUCLEAR STRUCTURE Variation after angular momentum projection, ^{186,188,190,192}Os, level energies, $B(E2)$ values, electromagnetic moments, $E2$ and $E4$ matrix elements, stable rigid triaxial shape.]

I. INTRODUCTION

The Os nuclei span a complex shape transition region between the well deformed rare-earth nuclei and the doubly closed shell nucleus ^{208}Pb. Hence this region has been the subject of many experimental and theoretical studies. These nuclei have a large number of low spin states below 2 MeV excitation energy. Of particular interest are the low-lying $K=2^+$ and 4^+ bands which lie very low in energy for these nuclei. It would be worthwhile to study the properties of the ground band and these excited bands, especially their electromagnetic properties, which particularly have been of recent interest.[1–3] Another interesting aspect that is worth investigating is the nature of triaxiality for these nuclei. Different models predict them to have varying degrees of triaxiality. A brief discussion of some of these models will be given.

Collective model calculations with the complete Bohr Hamiltonian have been performed by Baranger and Kumar.[4] In their model, the six inertial parameters and the deformation potential energy are calculated within the pairing $+Q\cdot Q$ interaction. Their model has been quite successful in describing the low spin states ($J\leq 4$) of the ground band, γ band, and β band with few adjustable parameters. They predict a prolate to oblate shape transition in this region. They found the deformation potential to be shallow and soft to γ deformation. Similar shallow deformation potentials have also been obtained from other semimicroscopic calculations[5] and the microscopic Hartree-Fock-Bogoliubov (HFB) calculation.[6]

Recently the interacting boson approximation (IBA) model of Arima and Iachello was applied to these nuclei[1,2] and found to be remarkably successful in predicting their level energies and $E2$ properties. This model predicts a rigid rotor (SU3) to O(6) transition from Os to Pt nuclei. Since the γ-vibrational model of Wilets and Jeans has the closest geometrical correspondence to the O(6) limit of the IBA, it has been argued[3] that these nuclei are γ soft.

There is equally convincing evidence in favor of nonaxial collective motion. Lee *et al.*[7] have found that the results of Coulomb excitation of ^{192,194}Pt are consistent with the asymmetric rotor model description, whereas the γ-soft models predict higher population for the γ band. Hence they concluded that these nuclei behave like rigid triaxial rotors. Meyer-ter-Vehn[8] and Toki and Faessler[9] have also arrived at similar conclusions from the study of odd A nuclei. They found that the unique parity spectra of odd A nuclei in the transitional region can be satisfactorily explained by taking one value of γ for the neighboring even-even core, which implies that the core is γ rigid.

Our present calculation complements the work of Meyer-ter-Vehn. Here we show that the level spectra and the electromagnetic properties of the even-even Os nuclei can be well explained by taking one value of γ for each nucleus. We take into account the softness of the nucleus in the β degrees of freedom by varying β for each J until the projected energy is minimum. In Sec. II, we give a brief outline of the model. The results of our model are given in Sec. III. Section IV contains the conclusions of the present study.

II. THE MODEL

A. The interaction and approximate angular momentum projection

The details of the model have already been published.[10] The only difference here is that the nucleus is assumed to be γ rigid and hence the value of γ for each nucleus is held fixed. For completeness we give here a brief outline of the model. We take the pairing $+Q\cdot Q$ Hamiltonian of Baranger and Kumar having the form

$$H=\sum_{\alpha\tau}\epsilon_\alpha^\tau c_\alpha^\dagger c_\alpha-\tfrac{1}{4}\sum_\tau G_\tau\sum_{\alpha\gamma}c_\alpha^\dagger c_{\bar\alpha}^\dagger c_{\bar\gamma}c_\gamma -\tfrac{1}{2}\chi\sum_{\substack{\tau\tau'\\ \mu}}\alpha_\tau\alpha_{\tau'}\sum_{\alpha\beta\gamma\delta}\langle\alpha\,|\,Q_\mu^{2(\tau)}\,|\,\gamma\rangle\langle\delta\,|\,Q_\mu^{2(\tau')}\,|\,\beta\rangle \times c_\alpha^\dagger c_\beta^\dagger c_\delta c_\gamma \;. \tag{1}$$

The symbols have the same meaning as in Ref. 10. In order to treat the field effects and the pairing effects self-consistently and on the same footing, one should solve the general HFB equations. But because of the simple form of the interaction taken above, essentially one has to perform the Nilsson-BCS calculation which is described below.

The Nilsson Hamiltonian for a triaxial system is given by

$$h_D^\tau=h_s^\tau-\alpha_\tau\hbar\omega\beta\cos\gamma r^2Y_{20}(\mathscr{O},\phi) -\frac{1}{\sqrt{2}}\alpha_\tau\hbar\omega\beta\sin\gamma r^2[Y_{22}^\tau(\mathscr{O},\phi)+Y_{2-2}^\tau(\mathscr{O},\phi)]\;, \tag{2}$$

$\hbar\omega$ being the oscillator energy and h_s^τ the spherical single particle energy. The deformed eigenvalues and eigenfunctions are obtained by solving the eigenvalue equation

$$h_D^\tau\,|\,i,\tau\rangle=\eta_i\,|\,i,\tau\rangle\;. \tag{3}$$

The deformed eigenfunctions have the structure

$$|\,i,\tau\rangle=\sum_{jm}c_{jm}^{i,\tau}\,|\,jm\rangle\;. \tag{4}$$

Then a BCS transformation is performed over these deformed eigenvalues by solving self-consistently the appropriate gap equation and the number equation for protons and neutrons separately. This gives the wave functions U_i^τ and V_i^τ and hence the intrinsic wave function $\phi(\beta,\gamma)$. The intrinsic wave function has the form

$$\phi(\beta,\gamma)=\prod_{i\tau}(U_i^\tau+V_i^\tau a_{i\tau}^\dagger a_{\bar i\tau}^\dagger)\,|\,0\rangle\;. \tag{5}$$

$a_{i\tau}^\dagger$ creates a particle in the Nilsson state $|\,i,\tau\rangle$.

In order to project out good angular momentum states, we assume the total Hamiltonian to be separable into an intrinsic part H_i and a rotational part H_r,

$$H=H_i+H_r\;. \tag{6}$$

The rotational Hamiltonian has the usual form

$$H_r=A_xJ_x^2+A_yJ_y^2+A_zJ_z^2\;. \tag{7}$$

Here A_x,A_y,A_z are, as usual, the inverse of twice the moment of inertia. Substituting $A_x=A-\delta$ and $A_y=A+\delta$, we have

$$H=H_i+AJ^2+(A_z-A)J_z^2-\tfrac{1}{2}\delta(J_+^2+J_-^2)\;. \tag{8}$$

The nuclear stationary states $|\,\alpha JM\rangle$ can be written as

$$|\,\alpha JM\rangle=\sum_{K\geq 0}A_{\alpha JK}(\beta,\gamma)\psi_{MK}^J\;, \tag{9}$$

where

$$\psi_{MK}^J=\phi(\beta,\gamma)\left[\frac{2J+1}{16\pi^2(1+\delta_{K0})}\right]^{1/2} \times[D_{MK}^{J*}+(-1)^JD_{M-K}^{J*}]\;. \tag{10}$$

The energy eigenvalue equation we have to solve is

$$\sum_{K'}\langle\psi_{MK'}^J\,|\,H\,|\,\psi_{MK}^J\rangle A_{\alpha JK'}=E_JA_{\alpha JK}\;. \tag{11}$$

Using Eq. (8), the matrix element of the Hamiltonian can be written as

$$\langle\psi_{MK'}^J\,|\,H\,|\,\psi_{MK}^J\rangle=\langle\phi\,|\,H_i\,|\,\phi\rangle\delta_{KK'} +\langle\,\psi_{MK'}^J\,|\,AJ^2+(A_z-A)J_z^2 -\tfrac{1}{2}\delta(J_+^2+J_-^2)\,|\,\psi_{MK}^J\rangle\;. \tag{12}$$

Since $H_i=H-H_r$ from Eq. (6), we have

$$\langle\phi\,|\,H_i\,|\,\phi\rangle=\langle\phi\,|\,H\,|\,\phi\rangle-A_x\langle\phi\,|\,J_x^2\,|\,\phi\rangle -A_y\langle\phi\,|\,J_y^2\,|\,\phi\rangle-A_z\langle\phi\,|\,J_z^2\,|\,\phi\rangle\;. \tag{13}$$

Using Eqs. (10), (12), and (13), we finally get

$$\langle\psi_{MK'}^J\,|\,H\,|\,\psi_{MK}^J\rangle=[\langle\phi\,|\,H\,|\,\phi\rangle-A_x\langle\phi\,|\,J_x^2\,|\,\phi\rangle-A_y\langle\phi\,|\,J_y^2\,|\,\phi\rangle-A_z\langle\phi\,|\,J_z^2\,|\,\phi\rangle+AJ(J+1)+(A_z-A)K^2]\delta_{KK'} -\tfrac{1}{2}\delta\frac{1}{[(1+\delta_{K0})(1+\delta_{K'0})]^{1/2}}[A_1\delta_{K',K+2}+(-1)^JA_2\delta_{K',-K+2}+A_2\delta_{K',K-2}+(-1)^JA_1\delta_{K',-K-2}]\;, \tag{14}$$

where

$$A_1=[(J-K)(J+K+1)(J-K-1)(J+K+2)]^{1/2}\;,$$

$$A_2=[(J+K)(J-K+1)(J+K-1)(J-K+2)]^{1/2}\;.$$

B. Scheme of calculation

(i) For a given nucleus the value of γ is extracted using the experimental energies of the first 2^+ and second 2^+ states assuming the asymmetric rotor model of Davidov and Filippov.[12] According to this model, the value for γ is given by the expression

$$\gamma=\tfrac{1}{3}\sin^{-1}\left[\frac{3}{(y+1)}\sqrt{y/2}\right], \quad (15)$$

where $y=E_{2'}/E_2$, E_2 and $E_{2'}$ being the energies of the first 2^+ and second 2^+ states.

(ii) Then for a given value of β, the Nilsson Hamiltonian [Eq. (2)] is diagonalized within the model space and then the BCS transformations are performed by solving the gap equation and the number equation self-consistently. This gives the BCS wave functions U_i and V_i and hence the intrinsic wave function $\phi(\beta,\gamma)$.

(iii) The moments of inertia are calculated using the cranking formula. Then the Hamiltonian matrix [Eq. (14)] is set up and diagonalized.

(iv) The value of β is then changed and steps (ii) and (iii) are repeated until the energy is minimum for each J state of the ground band. The levels of the γ band and the $K=4^+$ band are the higher states coming from the diagonalization of the energy matrices at the above minimum. This is done to guarantee the mutual orthogonality of the wave functions.

The $B(E2)$ values and the electromagnetic moments are calculated using the same formula as in Ref. 10. The $E2$ and $E4$ matrix elements are calculated using the expression

$$M^{\lambda}_{I_i,I_f}=i^{\lambda}(2I_f+1)^{1/2}\langle\alpha_f I_f\,||\,\mathscr{M}(E2)\,||\,\alpha_i I_i\rangle\ . \quad (16)$$

III. RESULTS

The different force parameters like single particle energies, pairing force strength, quadrupole strength, etc., are taken from Ref. 11. Following Baranger and Kumar, we have chosen a core consisting of 40 protons and 70 neutrons. $N=4,5$ harmonic oscillator shells are taken as active shells for protons and $N=5,6$ oscillator shells are taken for neutrons. A multiplying factor is used to renormalize all the calculated level energies. This renormalization factor is obtained by fitting the calculated $J=4^+$ level with experiment. Such a prescription has been suggested by Warke and Gunye[13] and has been used by us[10] before. However, there are some differences of opinion regarding the physical interpretation of this factor. Whether the renormalization factor takes care of the effect of the core or whether it takes care of some additional effects not included in the model has been the main cause of the difference. To test whether the renormalization factor takes care of the effect of the core, Kumar and Gunye[19] performed calculations for ^{170}W and ^{192}Pt with and without the assumption of the inert core. Their calculation was based on the microscopic variational projection formalism employing a pairing plus $Q\cdot Q$ interaction Hamiltonian. The energy spectra obtained with the assumption of the inert core were found to be quite spread out compared to the experiment. One has to use a renormalization factor to compress the spectra. Then they carried out the investigation in a large configuration space by considering all the nucleons in the nucleus explicitly without the assumption of an inert core. They found the energy spectra to be compressed, in agreement with experiment. From this they concluded that the renormalization prescription adopted is justified and that the renormalization factor approximately simulates the effect of the core. They also found that in the calculation without the assumption of the core, the electromagnetic properties are reproduced taking only the bare nucleon charges. On the other hand, Pomorski *et al.*[20] have studied the properties of the inertial functions using the cranking approximation based on a modified harmonic oscillator potential and have found that the renormalization factor cannot be interpreted as being due to the core contribution. They suggested that it should be ascribed to interactions not included in the model. This is in disagreement with the results obtained by Kumar and Gunye described above. It has been suggested that this conflict between the two interpretations arises because of the large differences between a stretched and an unstretched basis. In our calculation the values of the renormalization factor for ^{186}Os, ^{188}Os, ^{190}Os, and ^{192}Os are 0.59, 0.53, 0.52, and 0.48, respectively. In the above calculation of Guyne and Kumar[19] a similar factor ($\sim$0.55) is required for ^{192}Pt in order to compress the spectra obtained with the assumption of the inert core. We have taken the effective charge of the proton to be $e_p=1+1.7Z/A$ and of the neutron to be $e_n=1.7Z/A$. The experimental values are taken from Refs. 2–4 and 14–18.

A. Level energies and wave functions

In Tables I–IV, we present for the four nuclei the experimental and theoretical energies, the equilibrium values of β, the component of the wave functions $A_{\alpha JK}$, and the static quadrupole moment and magnetic moment for the levels of the ground band, the γ band, and the $K=4^+$ band. For ^{192}Os, the value of γ is taken to be 25.21; for ^{190}Os, $\gamma=22.28$; for ^{188}Os, $\gamma=19.16$; and for ^{186}Os, $\gamma=16.52$. We have calculated the level energies up to 2.5 MeV excitation energy because at higher excitation energies, the adiabatic assumption [Eq. (6)] is expected to be less and less valid. In the diagonalization of the energy matrices [Eq. (14)], the lowest energy level for each J is identified as belonging to the ground band. The higher spin states are classified into the γ band and the $K=4^+$ band on the basis of their energies. For example, the diagonalization for the $J=4^+$ state generates three levels. We identify the lowest 4^+ state as belonging to the ground band, the second 4^+ to the γ band, and the third 4^+ to the $K=4$ band. The levels of the γ band are represented by primes and those of the $K=4^+$ band by double primes. An analysis of the different components $A_{\alpha JK}$ of the wave functions shows that the low spin states of the ground band have $K=0$ as the dominant component. However, as we go to higher spin values, the $K=0$ component goes on decreasing and the mixing increases. For example, in ^{192}Os, the $J=2^+$ state has 93.5% of the $K=0$ and only 6.5% of the $K=2$ component. For the $J=4^+$ state, the $K=0$ component decreases to 69%. The decrease in the $K=0$ component at higher spin values for ^{192}Os is so

TABLE I. ^{192}Os: level energies (MeV) of the ground band, γ band, and $K=4^+$ band, the corresponding equilibrium values of β, the components $A_{\alpha JK}$ of the wave functions [Eq. (9)], the quadrupole moment (QM) in e b and the magnetic moment (MM) in μ_N of various states. E_{exp} and E_{th} refer to experimental and theoretical energies. The quantities in the parentheses give the experimental values. The levels of the γ band are represented by primes and of the $K=4^+$ band by double primes. For this nucleus the value of γ has been taken to be 25.21.

Level	E_{exp}	E_{th}				Wave function A_{JK}				QM	MM
J	(MeV)	(MeV)	β	$K=0$	$K=2$	$K=4$	$K=6$	$K=8$	$K=10$	(e b)	(μ_N)
0^+	0.0	0.0	0.138	1.0						0.0	0.0
2^+	0.206	0.231	0.158	0.967	0.255					−0.729	0.590
										(−0.8±0.3)	(0.60)
4^+	0.580	0.580	0.180	0.827	0.559	0.055				−0.371	1.191
6^+	1.088	1.005	0.196	0.736	0.658	0.160	0.011			−0.296	1.814
8^+	1.708	1.507	0.208	0.679	0.689	0.250	0.042	0.002		−0.263	2.453
10^+		2.088	0.220	0.638	0.697	0.317	0.081	0.011	0.000	−0.241	3.111
$2'^+$	0.489	0.447	0.158	−0.255	0.967					0.729	0.588
										(0.8±0.3)	
$3'^+$	0.690	0.650	0.158	0.0	1.0					0.0	0.883
$4'^+$	0.910	0.924	0.180	−0.560	0.811	0.167				−1.685	1.224
$5'^+$	1.143	1.082	0.180	0.0	0.962	0.272				−0.601	1.501
$6'^+$	1.362	1.540	0.196	−0.610	0.540	0.576	0.068			−1.345	1.865
$7'^+$	1.713	1.609	0.196	0.0	0.871	0.485	0.076			−0.550	2.136
$8'^+$		2.195	0.208	−0.536	0.234	0.765	0.268	0.021		−0.799	2.506
$9'^+$		2.212	0.208	0.0	0.775	0.605	0.180	0.020		−0.465	2.785
$4''^+$	1.069	1.243	0.180	0.049	−0.169	0.894				2.055	1.178
$5''^+$	1.362	1.560	0.180	0.0	−0.272	0.962				0.601	1.493
$6''^+$		1.808	0.196	0.295	−0.524	0.786	0.148			−1.164	1.871
$7''^+$		2.159	0.196	0.0	−0.485	0.826	0.287			−0.776	2.163

much so that for the $J=10^+$ state, the wave function has only 40.7% of the $K=0$ but 48.6% of the $K=2$ component. Even then we identify it as belonging to the ground band. As we go from ^{192}Os to ^{186}Os, we find that for a given J the mixing of wave functions decreases. For example, the $J=2^+$ state of ^{186}Os has almost 100% of the $K=0$ component, whereas for ^{192}Os the $J=2^+$ state has 93.5% of $K=0$. Similarly, the $J=10^+$ state of ^{186}Os has 79% of $K=0$, whereas for ^{192}Os it is only 40.7%. Similar behavior is also obtained for the γ band and the $K=4$

TABLE II. ^{190}Os: same as Table I. The value of γ for this nucleus has been taken to be 22.28.

Level	E_{exp}	E_{th}				Wave function				QM	MM
I	(MeV)	(MeV)	β	$K=0$	$K=2$	$K=4$	$K=6$	$K=8$	$K=10$	(e b)	(μ_N)
0^+	0.0	0.0	0.160	1.0						0.0	0.0
2^+	0.187	0.198	0.172	0.991	0.135					−1.124	0.591
										(−0.98±0.08)	(0.56)
4^+	0.548	0.548	0.190	0.910	0.414	0.022				−0.907	1.202
6^+	1.050	0.986	0.206	0.811	0.578	0.086	0.008			−0.755	1.832
8^+	1.667	1.508	0.220	0.744	0.649	0.160	0.016	0.000		−0.716	2.477
10^+	2.358	2.115	0.228	0.698	0.679	0.225	0.036	0.003	0.000	−0.700	3.118
$2'^+$	0.558	0.496	0.172	−0.135	0.991					1.124	0.614
										(0.9±0.4)	
$3'^+$	0.756	0.681	0.172	0.0	1.0					0.0	0.904
$4'^+$	0.955	0.889	0.190	−0.415	0.907	0.074				−1.536	1.234
$5'^+$	1.204	1.092	0.190	0.0	0.990	0.142				−0.953	1.522
$6'^+$	1.474	1.485	0.206	−0.579	0.775	0.251	0.012			−2.325	1.894
$7'^+$		1.625	0.206	0.0	0.953	0.303	0.025			−1.145	2.168
$8'^+$		2.199	0.220	−0.621	0.583	0.517	0.083	0.008		−2.102	2.563
$9'^+$		2.251	0.220	0.0	0.889	0.451	0.076	0.004		−1.108	2.827
$4''^+$	1.163	1.521	0.190	0.011	−0.076	0.997				2.443	1.253
$5''^+$	1.446	1.804	0.190	0.0	−0.142	0.990				0.953	1.552
$6''^+$	1.836	1.945	0.206	0.079	−0.254	0.961	0.072			−0.127	1.898
$7''^+$		2.300	0.206	0.0	−0.304	0.943	0.135			−0.675	2.206
$8''^+$		2.595	0.220	0.246	−0.488	0.813	0.202	0.009		−1.773	2.588

TABLE III. ^{188}Os: same as Table I. The value of γ has been taken to be 19.16.

Level I	E_{exp} (MeV)	E_{th} (MeV)	β	Wave function $K=0$	$K=2$	$K=4$	$K=6$	$K=8$	$K=10$	QM (e b)	MM (μ_N)
0^+	0.0	0.0	0.178	1.0						0.0	0.0
2^+	0.155	0.160	0.188	0.998	0.062					−1.401	0.591
										(−1.33±0.09)	(0.5)
4^+	0.478	0.478	0.200	0.974	0.228	0.005				−1.529	1.188
6^+	0.940	0.896	0.220	0.912	0.408	0.030	0.000			−1.414	1.794
8^+	1.514	1.390	0.230	0.845	0.531	0.070	0.003	0.000		−1.312	2.394
10^+	2.170	1.967	0.238	0.790	0.602	0.115	0.009	0.000	0.000	−1.272	2.989
$2'^+$	0.633	0.579	0.188	−0.062	0.998					1.401	0.658
										(0.83±0.45)	
$3'^+$	0.790	0.731	0.188	0.0	1.0					0.0	0.936
$4'^+$	0.966	0.882	0.200	−0.228	0.973	0.032				−1.184	1.232
$5'^+$	1.181	1.091	0.200	0.0	0.998	0.063				−1.175	1.521
$6'^+$	1.425	1.328	0.220	−0.409	0.907	0.101	0.002			−2.355	1.835
$7'^+$	1.686	1.531	0.220	0.0	0.990	0.143	0.005			−1.637	2.127
$8'^+$		1.957	0.230	−0.533	0.823	0.198	0.012	0.000		−2.862	2.436
$9'^+$		2.121	0.230	0.0	0.971	0.238	0.018	0.000		−1.782	2.726
$4''^+$	1.279	1.956	0.200	0.002	−0.033	0.999				2.713	1.359
$5''^+$		2.191	0.200	0.0	−0.063	0.998				1.175	1.628

band. Almost all the levels of the γ band have $K=2$ as the dominant component, whereas $K=4$ is the dominant component for the $K=4^+$ band.

The calculated level energies of the ground band and γ band agree well with experiment. For almost all the levels, the experimental energies are reproduced within 10%. The γ bandhead energies are also well reproduced for all four nuclei. We have performed the calculation up to $J=10^+$ for the ground band and up to $J=9^+$ for the γ band. As is clear from Fig. 1, the relative spacings of the different levels of the ground band and γ band are excellently reproduced. As an example, we see that experimentally, the $2'^+$ state of the γ band lies above the 4^+ state of the ground band for ^{186}Os and ^{188}Os. For ^{90}Os, the two levels are almost degenerate, while for ^{192}Os, the $2'^+$ state lies below the 4^+ state. This is reproduced in our model, except for ^{190}Os, where the calculated $2'^+$ state lies 52 keV below the 4^+ level, whereas in experiment the $2'^+$ level lies 10 keV above the 4^+ level.

Experimentally a $K=4^+$ band starts between 1 and 1.6 MeV for all four nuclei. In our calculation, we also have a $K=4$ band which can be compared with the above band. For ^{192}Os we predict the $K=4$ band to start at 1.243 MeV, which is only 0.174 MeV higher than the experi-

TABLE IV. ^{186}Os: same as Table I. The value of γ for this nucleus has been taken to be 16.52.

Level I	E_{exp} (MeV)	E_{th} (MeV)	β	Wave function $K=0$	$K=2$	$K=4$	$K=6$	$K=8$	$K=10$	QM (e b)	MM (μ_N)
0^+	0.0	0.0	0.200	1.0						0.0	0.0
2^+	0.137	0.139	0.206	1.0	0.029					−1.604	0.555
											(0.63)
4^+	0.434	0.434	0.216	0.994	0.111	0.001				−1.962	1.103
6^+	0.869	0.845	0.234	0.974	0.228	0.008	0.000			−2.041	1.630
8^+	1.421	1.346	0.244	0.936	0.352	0.023	0.001	0.000		−1.968	2.154
10^+		1.929	0.252	0.889	0.455	0.046	0.002	0.000	0.000	−1.891	2.672
$2'^+$	0.767	0.749	0.206	−0.029	1.0					1.604	0.684
$3'^+$	0.910	0.883	0.206	0.0	1.0					0.0	0.929
$4'^+$	1.070	0.996	0.216	−0.111	0.994	0.015				−1.027	1.179
$5'^+$	1.275	1.197	0.216	0.0	1.0	0.029				−1.333	1.444
$6'^+$	1.491	1.357	0.234	−0.228	0.973	0.046	0.000			−2.149	1.673
$7'^+$		1.588	0.234	0.0	0.998	0.066	0.001			−1.921	1.946
$8'^+$		1.897	0.244	−0.353	0.932	0.088	0.002	0.000		−2.813	2.162
$9'^+$		2.140	0.244	0.0	0.994	0.114	0.004	0.000		−2.200	2.449
$10'^+$		2.576	0.252	−0.457	0.878	0.141	0.007	0.000	0.000	−3.220	2.646
$4''^+$	1.560	2.636	0.216	0.000	−0.015	1.0				2.989	1.427
$5''^+$		2.845	0.216	0.0	−0.029	1.0				1.333	1.647

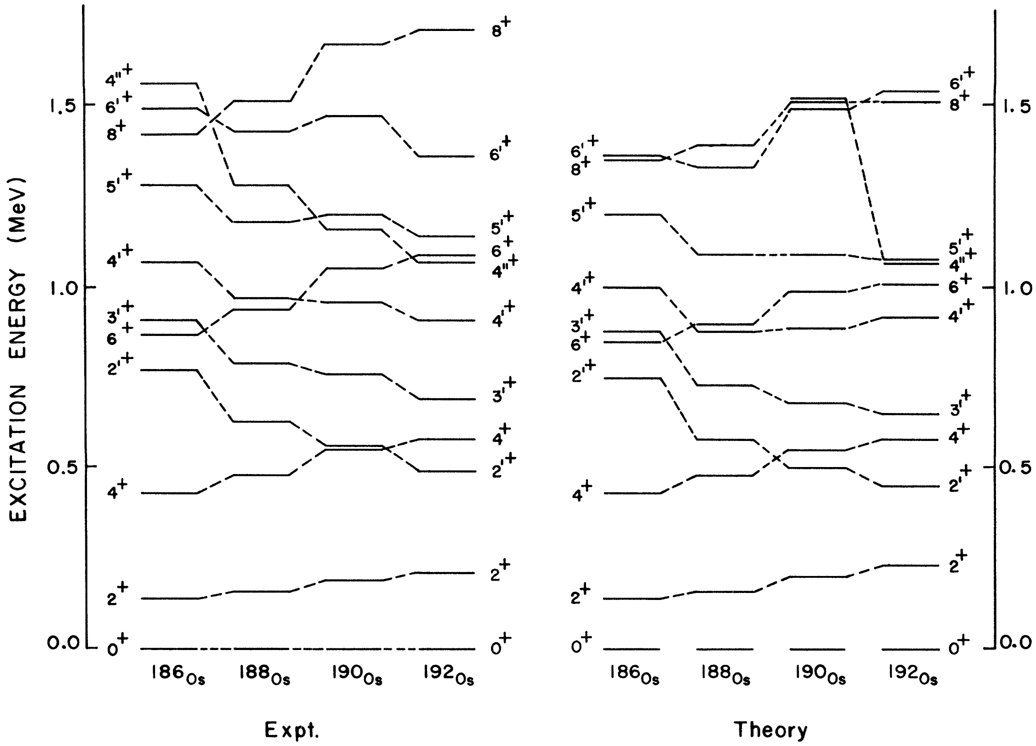

FIG. 1. Comparison of energy spectra between experiment and theory.

mental value. For ^{190}Os, our calculated bandhead energy of this band is about 0.358 MeV higher. But for ^{188}Os and ^{186}Os, our predicted bandhead energies are about 1 MeV higher than experiment. Possibly for these two cases the bandhead energy can be lowered by taking a slightly higher value of γ. Both for ^{192}Os and ^{190}Os, for which the $4''$ and $5''$ levels are experimentally identified, the relative spacing between these two levels is well reproduced. This $K=4^+$ band in ^{190,192}Os has often been described as a two phonon (four quasiparticle) γ-vibrational band. Bagnell *et al.*[14] recently observed that this level is very strongly populated by the single proton transfer reaction ^{193}Ir(t,α)^{192}Os. This is completely unexpected if the $4''^+$ state were a pure two phonon (four quasiparticle) state. They suggested that this state has a significant one phonon (two quasiparticle) component. In our calculation we interpret this band to be the $K=4$ band of a triaxial γ-rigid rotor.

B. Equilibrium values of β

We obtain the equilibrium values of β for each J by the minimization of energy following the variation after angular momentum projection (VAP) approach. These values are given in Tables I—IV. In our model, for a given J, the ground band, γ band, and $K=4^+$ band have the same intrinsic structure and hence the same value of β. This is a consequence of the adiabatic assumption [Eq. (6)]. As a result, the 2^+ level of the ground band and the $2'^+$ and $3'^+$ levels of the γ band have the same value of β. Similarly the 4^+ level of the ground band, the $4'^+$ and $5'^+$ levels of the γ band, and the $4''^+$ and $5''^+$ levels of the $K=4$ band have the same value of β. We see that the equilibrium value of β increases with the increase of spin. This is a result of the centrifugal stretching. Kumar and Baranger[4] have calculated the rms values of β and γ for different values of J in their dynamical calculation. Our values of β are usually smaller than the values given by them.

C. Electromagnetic moments and $B(E2)$ values

The static quadrupole moment of the different levels of the ground band, γ band, and $K=4^+$ band are calculated using the effective charge $e_p=1+1.7Z/A$ for protons and $e_n=1.7Z/A$ for neutrons. Their values are given in Tables I—IV for the four nuclei. Experimentally only the quadrupole moment of the first 2^+ and second 2^+ states are known. Baktash *et al.*[3] have given a comparison between different experimental and theoretical results for the quadrupole moments of these two states. There are large fluctuations between values given by different experiments with large error bars. For example, the experimental quadrupole moment of ^{192}Os for the 2^+ state is -0.97 ± 0.03 e b from muonic data, -0.53 ± 0.10 e b from Coulomb excitation data, and -0.7 ± 0.3 and -0.8 ± 0.3

e b from particle γ-coincidence data. We predict its value to be -0.729 e b, which is closer to the value obtained from particle γ-coincidence data. We have quoted in Tables I—IV those experimental results which are closer to our theoretical values. In our model we predict the quadrupole moment of the 2^+ and $2'^+$ states to be equal in magnitude and opposite in sign.

The magnetic moments of the different levels are given in Tables I—IV for the four nuclei. Our values agree quite well with those given by Bafanger and Kumar for the levels calculated by them. Experimentally only the magnetic moment of the $J=2^+$ state is known for each of these nuclei. The agreement with experiment is quite satisfactory.

The $B(E2)$ values are calculated using the same effective charge as for the quadrupole moment. Their values are given in Table V. The quantities in parentheses represent the experimental values. Though we have calculated all possible transition probabilities, we have given only 35 of them. We see that the agreement for both the interband and intraband transitions is satisfactory. Except for ^{192}Os, the $B(E2,0^+\rightarrow 2'^+)$ is well reproduced in our calculation. The rigid asymmetric model is often criticized[3,15] for its inability to fit the $B(E2,0^+\rightarrow 2'^+)$ value simultaneously with $B(E2,0^+\rightarrow 2^+)$. We see that our γ-rigid model is successful in describing these two interband and intraband transition probabilities simultaneously. The

TABLE V. $B(E2,i\rightarrow f)$ in e^2b^2 (quantities in the parentheses refer to experimental values).

J_i^π	J_f^π	^{186}Os	^{188}Os	^{190}Os	^{192}Os
0^+	2^+	3.236	2.654	2.226	1.800
		(3.11±0.11)	(2.75±0.15)	(2.55±0.25)	(2.21±0.22)
0^+	$2'^+$	0.227	0.223	0.178	0.086
		(0.244±0.024)	(0.250±0.022)	(0.220±0.020)	(0.215±0.019)
2^+	$2'^+$	0.117	0.177	0.292	0.434
		(0.107±0.011)	(0.146±0.013)	(0.245±0.022)	(0.361±0.032)
2^+	$3'^+$	0.116	0.117	0.096	0.051
2^+	4^+	1.769	1.513	1.293	1.106
		(1.69±0.12)	(1.41±0.11)	(1.07±0.10)	(0.98±0.09)
2^+	$4'^+$	0.016	0.002	0.007	0.016
		(0.026±0.006)	(0.020±0.004)	(0.019±0.004)	
2^+	$4''^+$	0.0001	0.0003	0.002	0.008
$2'^+$	$3'^+$	1.654	1.388	1.185	1.033
$2'^+$	4^+	0.014	0.029	0.046	0.032
$2'^+$	$4'^+$	0.694	0.549	0.415	0.346
		(0.99±0.35)	(1.05±0.35)	(0.878±0.180)	(0.312±0.062)
$2'^+$	$4''^+$	0.137	0.157	0.167	0.142
$3'^+$	4^+	0.103	0.192	0.337	0.407
$3'^+$	$4'^+$	1.068	0.778	0.444	0.187
$3'^+$	$4''^+$	0.066	0.089	0.127	0.182
$3'^+$	$5'^+$	1.012	0.854	0.737	0.650
$3'^+$	$5''^+$	0.071	0.072	0.058	0.029
4^+	$4'^+$	0.146	0.196	0.210	0.157
		(0.176±0.053)	(0.159±0.032)	(0.362±0.072)	(0.367±0.184)
4^+	$4''^+$	0.0001	0.001	0.004	0.009
4^+	$5'^+$	0.059	0.036	0.009	0.001
4^+	6^+	1.716	1.536	1.444	1.322
		(1.64±0.25)	(1.68±0.26)	(1.50±0.23)	(1.26±0.25)
4^+	$6'^+$	0.001	0.002	0.005	0.001
$4'^+$	$4''^+$	0.024	0.037	0.072	0.177
$4'^+$	$5'^+$	0.852	0.755	0.683	0.580
$4'^+$	$5''^+$	0.076	0.096	0.123	0.132
$4'^+$	6^+	0.041	0.058	0.033	0.007
$4'^+$	$6'^+$	1.114	0.863	0.704	0.583
$4''^+$	$5''^+$	1.385	1.161	0.995	0.861
$5'^+$	$6'^+$	0.463	0.257	0.082	0.005
$5'^+$	$7'^+$	1.298	1.127	1.029	0.935
6^+	$6'^+$	0.146	0.142	0.096	0.082
6^+	$7'^+$	0.033	0.011	0.002	0.012
6^+	8^+	1.872	1.749	1.619	1.478
$6'^+$	$7'^+$	0.540	0.506	0.474	0.513
$6'^+$	8^+	0.053	0.038	0.011	0.002
8^+	10^+	1.997	1.872	1.758	1.600

study of different $B(E2)$ values shows that the intraband transitions are larger than interband transitions by an order of magnitude, as expected. For ^{188}Os and ^{190}Os, we see that the calculated $B(E2,2^+ \rightarrow 4'^+)$ almost goes to zero. Such a sharp drop in $B(E2,2^+ \rightarrow 4'^+)$ has also been predicted by Baker,[16] which he explained as due to the $E2$ matrix element $M^2_{4'2}$ changing sign. In our calculation also we see that $M^2_{4'2}$ changes sign for ^{190}Os.

The transition probabilities provide a test of the nuclear wave functions and hence the model used. Because of our good agreement with experiment for the transition probabilities, it seems that our γ-rigid model is satisfactory.

D. $E2$ and $E4$ matrix elements

$E2$ and $E4$ matrix elements are calculated using formula (16). In the calculation the effective charge of the proton is taken to be $e_p = 1 + 1.7Z/A$ and of the neutron to be $e_n = 1.7Z/A$. To give the detailed matrix elements for all four nuclei would require a lot of space. Hence we present the detailed matrix elements only for ^{192}Os. For the other three nuclei we present only some selected matrix elements. The $E2$ and $E4$ matrix elements for ^{192}Os are given in Table VI. Experimental $E2$ matrix elements[2] are also given. As far as $E2$ matrix elements are concerned, we see that except for the $M^2_{02'}$ element, all other matrix elements are reasonably well reproduced in our calculation. Baker *et al.*[2] performed an IBA calculation for ^{192}Os and obtained $E2$ matrix elements in agreement with the experiment. Most of our calculated $E2$ matrix elements agree quite well with those given by them in their IBA calculation. However, there are some discrepancies. For example, the value of $M^2_{24'}$ in our calculation is -28.2 $e$fm^2, while the IBA calculation gives the value 17.02. Experimentally its sign has not been properly determined. Its experimental value is given to be ± 16.3. Baker *et al.* have found that this matrix element plays a very small role in the excitation of the $4'^+$ state. There is some disagreement with their predicted matrix elements involving the $4''$ state. However, as pointed out by them, the IBA model with only s and d bosons is not sufficient for the description of this level. The experimental value of $M^2_{44'}$ has been given as ± 182 $e$fm^2. It has been pointed out by Baker *et al.*[2] that this is an old value; more recent experimental work has shown that $M^2_{2'4'} \approx M^2_{44'}$. By taking $M^2_{2'4'} = -125$ $e$fm^2 and $M^2_{44'} = 125$ $e$fm^2, they found that the experimental data for the $2'$ level could be better reproduced. This experimental fact that $M^2_{2'4'} \approx M^2_{44'}$ is remarkably well reproduced in our calculation. We find the values of these two matrix elements to be $M^2_{2'4'} = -131.5$ efm^2 and $M^2_{44'} = 118.8$ efm^2. These are very close to the 125 $e$fm^2 taken by them.

The calculated $E4$ matrix elements are given in Table VI. For the cases involving 0^+, 2^+, $2'^+$, 4^+, and $4'^+$ states, the majority of our $E4$ matrix elements agree in trend with those given by Baker *et al.* in their IBA calcu-

TABLE VI. Calculated $E2$ matrix elements, calculated $E4$ matrix elements, and experimental $E2$ matrix elements for ^{192}Os.

	0^+	2^+	$2'^+$	4^+	$4'^+$	$4''^+$
		Calculated $E2$ matrix elements ($e$fm^2)				
0^+	0.0	-134.2	29.4	0.0	0.0	0.0
2^+		96.2	147.3	-235.2	-28.2	-20.4
$2'^+$			-96.2	40.0	-131.5	84.2
4^+				49.2	$+118.8$	-29.2
$4'^+$					223.4	126.3
$4''$						-272.5
		Calculated $E4$ matrix elements ($e$fm^4)				
0^+	0.0	0.0	0.0	-932.3	297.4	-109.9
2^+		-1281.4	-62.9	782.9	-728.4	-244.8
$2'^+$			-29.7	1382.7	-711.3	-32.1
4^+				-400.3	1175.3	355.0
$4'^+$					-167.3	668.3
$4''^+$						-880.3
		Experimental $E2$ matrix elements ($e$fm^2)				
0^+	0.0	-145 ± 1	46.4 ± 2.1	0.0	0.0	
2^+		105.5 ± 39.6	134.4 ± 6.0	-221.4 ± 21.2	± 16.3	
$2'^+$			-105.5 ± 39.6		$\pm(125 \pm 56)$	
4^+					$\pm(182 \pm 129)$	
$4'^+$						

TABLE VII. Hexadecapole deformation parameters for ^{192}Os.

State	Expt	Present result	IBA[a]	Other theoretical results
4^+	-0.02	-0.023	-0.031	-0.09[b]
$4'^+$		0.007	0.006	
$4''^+$	-0.014	-0.003	0.0002	-0.025[c]

[a]Reference 2.
[b]Reference 16.
[c]Reference 14.

lation. They found that the coupled channel data for the $4''$ state can be better fit by taking $M^4_{04''}=-485$ $e\mathrm{fm}^4$. We predict it to be -109.9 $e\mathrm{fm}^4$, while the IBA gives the value 7.6 $e\mathrm{fm}^4$.

Even though we have not taken the hexadecapole term in the Hamiltonian explicitly, we can still follow the usual practice[2] of parametrizing the $E4$ matrix elements by introducing a β_4-deformation parameter

$$M^4_{04}=\frac{3Ze}{4\pi}R^4\beta_4 \; . \tag{17}$$

The experimental values of β_4, our calculated values, and the values given by different theories are presented in Table VII. We see that β_4 for the first 4^+ state predicted by our theory agrees quite well with the experiment. The experimental result for the $4'$ state is not available. Our model as well as the IBA model predicts similar values of β_4 for this state. For the $4''$ state our calculated β_4 is ~ 4.4 times smaller than the experiment.

In Table VIII, we compare with experiment the matrix elements M^2_{02}, $M^2_{02'}$, M^2_{22}, and $M^2_{22'}$ and the hexadecapole deformation parameter β_4 extracted from the $E4$ matrix element M^4_{04} for all four nuclei. We see that there is overall agreement with experiment. As pointed out before, the element $M^2_{02'}$ for ^{192}Os shows deviation from experiment by 30%, whereas for the other three nuclei this element is very well reproduced. The asymmetric rotor model is usually critized[3,15] for its failure to fit the element $M^2_{02'}$ simultaneously with M^2_{02}. We see that in our γ-rigid model, except for ^{192}Os, there is fairly good agreement with experiment for both matrix elements. The result for ^{192}Os could possibly have been improved by starting the calculation with a slightly smaller value of γ. The extracted β_4 deformation parameters also agree quite well with experiment, except for ^{186}Os, where it is underestimated by a factor of 3.

In Ref. 10, we had performed a similar calculation for ^{188}Os and ^{188}Pt. In that calculation both β and γ for each J state were obtained by the minimization of energy. The equilibrium values of β for ^{188}Os are almost the same as in the present study. Only three members of the γ band and one member of the $K=4^+$ band were calculated. The $K=4^+$ bandhead was lying very high. However, the rest of the calculated levels, the electromagnetic moments, $B(E2)$ values, etc., were in good agreement with experiment, as in the present case. In that study also we had some evidence regarding the γ rigidity of these nuclei.

IV. CONCLUSION

We see that our γ-rigid model has been quite successful in describing the level energies and the electromagnetic properties of the four transitional nuclei ^{186,188,190,192}Os. The value of γ for each nucleus is extracted from the ratio of the first 2^+ and second 2^+ states assuming the rigid asymmetric rotor model of Davydov and Filippov.[12] The value of β for each J is obtained from the minimization of energy approximately projected from a triaxially symmetric intrinsic wave function. This variation of β from one J state to another takes into account the softness of the nucleus. The triaxial intrinsic wave functions are obtained using the pairing $+Q\cdot Q$ interaction of Baranger and Kumar. Good angular momentum states are approximately projected assuming the total Hamiltonian to be separable into an intrinsic part and a rotational part.

We have calculated the level energies up to $J=10^+$ for the ground band and up to $J=9^+$ for the γ band. The relative spacing of the different levels of these two bands

TABLE VIII. The $E2$ matrix elements and the hexadecapole deformation parameter. The quantities in parentheses represent experimental values. The experimental values are taken from Refs. 2, 15, and 16.

Nucleus	M^2_{02}	$M^2_{02'}$	M^2_{22}	$M^2_{22'}$	β_4
^{186}Os	-179.9	$+47.6$	$+211.6$	$+76.3$	-0.030
	(-177 ± 1)	(49.4 ± 2.5)	(217.7 ± 5.3)	(73.1 ± 3.8)	(-0.099)
^{188}Os	-162.9	$+47.2$	$+184.9$	$+94.02$	-0.029
	(-169 ± 1)	(50.0 ± 2.3)	(193.9 ± 5.3)	(85.4 ± 3.9)	(-0.031)
^{190}Os	-149.2	$+42.2$	$+148.2$	$+120.8$	-0.026
	(-157 ± 1)	(46.9 ± 2.2)	(155.7 ± 8.0)	(110.7 ± 5.1)	(-0.04)
^{192}Os	-134.2	29.4	96.2	147.3	-0.023
	(-145 ± 1)	(46.4 ± 2.1)	(105.5 ± 39.6)	(134.4 ± 6.0)	(-0.02)

are reasonably well reproduced. The γ-bandhead energy for each nucleus also compares well with experiment. The $K=4$ band is well described by ^{192,190}Os. But for ^{188,186}Os, the $K=4$ bandhead energy lies higher than the experiment by ~ 1 MeV. Quadrupole moments, magnetic moments, and $B(E2)$ values are calculated for the different levels and compared with experiment. Except for ^{192}Os, our γ-rigid model has been able to describe the transition probabilities $B(E2,0^+\rightarrow 2^+)$ and $B(E2,0^+\rightarrow 2'^+)$ simultaneously, in agreement with experiment. The calculated $E2$ and $E4$ matrix elements also compare well with experiment.

Thus our semimicroscopic model is able to describe the level energies and the electromagnetic properties of these four nuclei taking one value of γ for each nucleus. This suggests that these nuclei may have a rather rigid triaxial shape. This agrees with the experimental finding of Lee *et al.*[7]

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