Long range dipole correlations and electron scattering sum rules

S. Stringari
Departimento di Fisica, Università degli Studi di Trento, 38050 Povo (TN), Italy
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We investigate the effects of long range correlations on the longitudinal and transverse integrated electron scattering cross sections. We use a schematic model based on the random phase approximation in which the center of mass motion, as well as the giant dipole resonance, is properly described. We show that dipole correlations significantly lower the total electron scattering inelastic cross section in the region of low momentum transfer (0.5—1 fm^{-1}), in agreement with recent experimental data for ^{12}C.

I. INTRODUCTION

The possibility of getting direct information on the two-body correlation function in nuclear systems from measurements of integrated inelastic electron scattering cross sections was suggested a long time ago.[1] However, only recently have such measurements become available.[2–5] The experiments of Refs. 2—4 have focused on the momentum transfer range $q=1$—$2\ fm^{-1}$. This region is expected to be sensitive to the presence of short range as well as tensor correlations in the nuclear wave function. However, additional effects, like relativistic corrections, mesonic degrees of freedom etc., strongly affect the electron scattering cross sections in this range of momentum transfer. The present theoretical uncertainties about such effects raise doubt about the possibility of extracting, from these experiments, quantitative information on the amount of correlations in the wave function.

In the region of lower momentum transfer ($q<1\ fm^{-1}$), the electron scattering cross sections are expected to depend less critically on relativistic and mesonic effects and consequently more explicitly on the presence of nuclear correlations. Such correlations are mainly of a long range nature and are responsible for the collective phenomena (giant resonances) exhibited by nuclei. It is the aim of this work to investigate this range of momentum transfer using a microscopic description of nuclear excitations which takes into account the effects of long range correlations.

II. ELECTRON SCATTERING SUM RULES AND THE SCHEMATIC MODEL

The cross section for electron scattering on nuclei is written, in the plane-wave Born approximation, as:

$$\frac{d^2\sigma}{d\Omega\, d\omega}=\sigma_0\left[\left[\frac{q_\mu^2}{q^2}\right]^2 F_C^2(q,\omega)+\left[\frac{1}{2}\frac{q_\mu^2}{q^2}+\tan^2\frac{\theta}{2}\right]F_T^2(q,\omega)\right], \qquad (1)$$

where σ_0 includes the single particle Mott cross section and the kinematical recoil factor, F_C and F_T are the Coulomb and transverse form factors, and $q_\mu^2=q^2-\omega^2$. The quantities more directly connected with the two-body correlations are the longitudinal and transverse inelastic sums defined by

$$S^C(q)=\int_\epsilon^q F_C^2(q,\omega)d\omega$$

and (2)

$$S^T(q)=\int_\epsilon^q F_T^2(q,\omega)d\omega ,$$

respectively (the lower limit ϵ on the energy integral is chosen to exclude the elastic peak).

The analysis we present in this work is based on the random phase approximation (RPA) which is, at present, the most elaborate and complete framework in which long range two-body correlations are taken into account. The aim is to overcome the limits of independent particle models[6] which ignore the presence of dynamic correlations and consequently cannot provide a correct treatment of nuclear collective phenomena. The dipole case is particularly illustrative. In an independent particle model one expects both the isoscalar and the isovector dipole excitations to occur at $\sim 1\hbar\omega_0$. While the center of mass motion is correctly pushed down to zero energy by RPA correlations,[7] the frequency of the isovector dipole resonance is strongly increased and its strength quenched, in agreement with experiments. Such effects have crucial consequences on the behavior of the longitudinal inelastic sum rule (2) at low values of momentum transfer, the dipole being the dominant excitation in this region.

In the following we will evaluate the inelastic sum rules (2) using a separable interaction of a schematic type. Such a model is particularly simple to handle and, at the same time, provides important features of more realistic RPA calculations. The starting point is a Hartree-Fock ground state for which we choose a Slater determinant built up with harmonic oscillator (HO) wave functions (the value of the HO parameter $\alpha=\sqrt{m\omega_0}$ is fixed to reproduce the nuclear root mean square radius). The particle-hole interaction is chosen to be of a separable type[8] and, in view of the above discussion, to contain only dipole terms

$$V_{kilj} \equiv \langle k i^{-1} | V | l j^{-1} \rangle = -m\omega_0^2 \left\langle k \left| \sum_k \vec{r}_k \right| i \right\rangle \cdot \left\langle j \left| \sum_k \vec{r}_k \right| l \right\rangle + \chi_1 \left\langle k \left| \sum_k \vec{r}_k \tau_k^z \right| i \right\rangle \cdot \left\langle j \left| \sum_k \vec{r}_k \tau_k^z \right| l \right\rangle . \tag{3}$$

(For the sake of simplicity we have written the particle-hole interaction for $N=Z$ nuclei. Generalization to the $N \neq Z$ case is straightforward.) Interaction (3) modifies the collective solutions associated with the center of mass motion and with the giant dipole mode, leaving all the other solutions of the equations of motion unchanged with respect to the predictions of the harmonic oscillator model. (Very similar results can be obtained also using the approach of Ref. 9, which consists of a proper modification of the harmonic oscillator Hamiltonian written in terms of center of mass and neutron-proton relative coordinates.) The isoscalar coupling constant is fixed to ensure the RPA solution to occur at exactly zero energy, while the isovector coupling constant is connected with the symmetry potential energy and its value will be chosen later.

The effect of interaction (3) on the inelastic sum rules is easily evaluated. In the region of low momentum transfer a very good approximation to the longitudinal sum rule $S^C(q)$ is given by

$$S^C(q) = f_N^2(q) \sum_{n \neq 0} \left| \left\langle 0 \left| \sum_i e_i e^{i\vec{q}\cdot\vec{r}_i} \right| n \right\rangle \right|^2 , \tag{4}$$

where the sum runs over a complete set of nuclear excited states and $f_N(q)$ is the nucleon form factor. Actually Eq. (4) also includes states which stay beyond the photon point $\omega = q$. The strength relative to these unphysical excitations is, however, negligible, except for extremely small values of q. The collective solutions $|D_k\rangle$ $(k=x,y,z)$ of a dipole type given by interaction (3) are characterized by the form factor

$$\left| \left\langle 0 \left| \sum_i e_i e^{i\vec{q}\cdot\vec{r}_i} \right| D_k \right\rangle \right| = a q_k |F_{HO}(q)| , \tag{5}$$

where F_{HO} is the elastic form factor $[F(0)=1]$ of the uncorrelated HO ground state. Result (5) is easily understood by recalling that the collective solutions D_k of our schematic model completely exhaust the dipole sum rule and are consequently characterized by a Tassie-type form factor.[10] The quantity a^2 is proportional to the dipole strength carried by the state $|D_k\rangle$. In the absence of correlations ($V_{kilj}=0$) one has

$$a_{HO}^2 = \frac{Z}{4\alpha^2} \tag{6}$$

for the isoscalar as well as for the isovector dipole mode. When the particle-hole interaction is turned on, the isoscalar dipole solution (center of mass motion) disappears from the excitation spectrum and hence its strength has to be subtracted from the inelastic sum $S_{HO}^C(q)$ of the HO model. The isovector dipole solution is pushed up in energy, while its strength is quenched by a factor ξ directly connected with the isovector coupling constant χ_1, where

$$\xi = \left[1 + \frac{A}{m\omega_0^2} \chi_1 \right]^{1/2} , \quad a_{RPA}^2 = \frac{1}{\xi} \frac{Z}{4\alpha^2} . \tag{7}$$

It is now possible to write the correlated expression for the longitudinal inelastic sum. One finds

$$S_{RPA}^C(q) = S_{HO}^C(q) + f_N^2(q) \tfrac{1}{4} Z \frac{q^2}{\alpha^2} |F_{HO}(q)|^2 \left[\frac{1}{\xi} - 2 \right] . \tag{8}$$

Equation (8) differs from the HO prediction $S_{HO}^C(q)$ owing to the center of mass motion and to the presence of isovector dipole correlations in the nuclear wave function. Some comments are in order here:

(i) When $\xi = 1$ $(\chi_1 = 0)$ the second term on the right-hand side of Eq. (8) gives the center of mass corrections to the inelastic sum rule.

(ii) The presence of dipole correlations is clearly exploited in the $q \to 0$ limit

$$S_{RPA}^C \underset{q\to 0}{=} \frac{1}{4} \frac{q^2}{\alpha^2} \frac{1}{\xi}$$

and is always responsible for quenching of the inelastic cross section $(\xi > 1)$. Conversely, its effect vanishes for large values of q.

(iii) Equation (8) contains two independent parameters: α and ξ. The former is fixed to reproduce the correct sizes of the system. The latter can be determined to reproduce measured quantities which are sensitive to the presence of dipole correlations in the nuclear system. The most natural quantity is the bremsstrahlung-weighted sum rule measured in photonuclear reactions:[11]

$$\sigma_{-1} = \int \frac{\sigma(\omega)}{\omega} d\omega , \tag{9}$$

which, in the dipole long wavelength approximation, is directly connected with the low q behavior of $S^C(q)$. From the analysis of Ref. 12 it emerges that dipole correlations considerably reduce (25—40%) the uncorrelated values of σ_{-1} and are crucial for reproducing the experimental results. The schematic model of Eq. (3) gives, neglecting higher multipole contributions and finite wavelength modifications, the following expression for σ_{-1}:

$$\sigma_{-1} = 60 \frac{NZ}{A} \frac{1}{\omega_0} \frac{1}{\xi} , \tag{10}$$

from which one concludes that a realistic value of ξ should be 1.4—1.6.

Figures 1 and 2 show the quenching effect produced by dipole correlations $(\xi = 1.5)$ on the longitudinal sum rule $S^C(q)$ for ^{12}C and ^{40}Ca, respectively. It is interesting to notice that for a given value of q such an effect is relatively less important in ^{40}Ca than in ^{12}C. This is due to the presence of higher multipolarities in the excitation spectrum which are more strongly excited in heavier nuclei.

RPA dipole correlations are responsible for a change of the transverse electron scattering cross section too. Using force (3) it is possible to find the following result for the

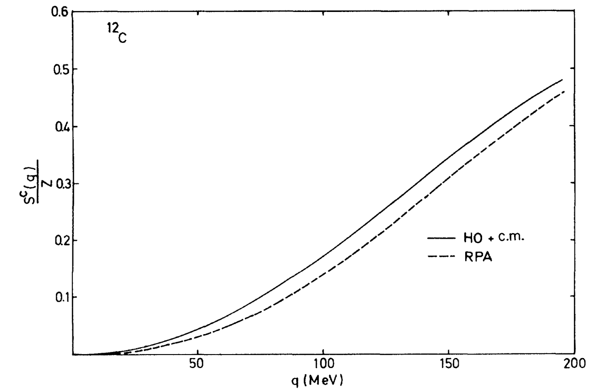

FIG. 1. Longitudinal electron scattering sum rule [see Eq. (2)] as a function of q for ^{12}C. The full line gives the harmonic oscillator prediction including center of mass (c.m.) corrections. The dashed line gives the RPA prediction obtained using the schematic force (3) ($\xi=1.5$). The value of α was 0.6 fm^{-1}.

transverse convection current sum rule:

$$S_{RPA}^{T_{conv}}(q)=\frac{f_N^2(q)}{m^2}\sum_n \left|\left\langle 0\left|\sum_i e_i \vec{p}_i\times\hat{q}e^{i\vec{q}\cdot\vec{r}_i}\right|n\right\rangle\right|^2$$
$$=S_{HO}^{T_{conv}}(q)+f_N^2(q)\tfrac{1}{2}Z\frac{\alpha^2}{m^2}F_{HO}^2(q)(\xi-2)\ , \tag{11}$$

where $S_{HO}^{T_{conv}}(q)$ is the current sum rule evaluated in the harmonic oscillator model. Result (11) shows that isovector dipole correlations tend to increase the transverse cross section differently than what happens for the longitudinal case [Eq. (8)]. Spin current terms also affect the transverse cross section. Their contribution has been evaluated in the present work using harmonic oscillator wave functions with j-j coupling and, in the case of spin unsaturated nuclei like ^{12}C, turns out to be important also at low values of q.

III. COMPARISON WITH EXPERIMENTAL DATA AND FINAL DISCUSSIONS

The above predictions for the inelastic sum rules can be used to analyze the experimental data of Ref. 5. Unfortunately, these data do not permit separation of the longitudinal and transverse response function, for which measurements at different angles should be necessary. As a consequence, comparison between theory and experiment can be made only for the total integrated cross section

$$S_{inel}=\int \frac{1}{\sigma_0}\frac{d\sigma}{d\omega}d\omega\ . \tag{12}$$

In order to evaluate S_{inel} from Eq. (1), one has to take into account the kinematic factor

$$\frac{q_\mu^2}{q^2}=1-\frac{\omega^2}{q^2}\ ,$$

which is usually estimated by replacing ω^2 by an average value $\langle\omega^2\rangle_{av}$. This average squared excitation energy is straightforwardly calculated in the framework of the schematic model and turns out to be rather close to the frequency of the giant dipole resonance in the region of low momentum transfer considered here. Figure 3 shows the experimental results of Ref. 5 together with our predictions for ^{12}C at $\theta=20°$. At these values of the scattering angle the cross section is dominated by the longitudinal term. The transverse term cannot, however, be ignored, its contribution being about 15% in the region $q=0.5$–$1\ fm^{-1}$. The figure clearly shows that inclusion of RPA dipole correlations is responsible for a sizeable quenching of the total strength and significantly improves the agreement with experimental data.

The analysis of the present work could be further improved by carrying out RPA calculations of electron-scattering cross sections with more appropriate effective interactions, taking into account high multipole correlations and possibly finite range potential effects. This would make it possible to investigate the effects of long range correlations at higher values of momentum transfer where the dipole is no longer the dominant excitation.

Another point that should be investigated is whether the use of RPA long range correlations provides a sufficiently correct description of electron scattering sum rules. While the longitudinal sum rule, being proportional in the $q\to 0$ limit to the bremsstrahlung weighted sum rule (9), is expected to be mainly affected by long range correlations,

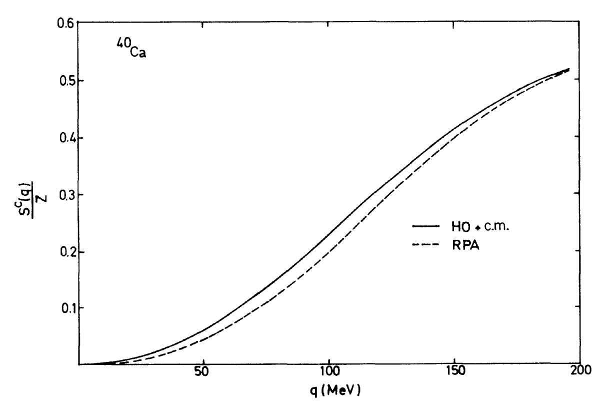

FIG. 2. Same as Fig. 1, but for ^{40}Ca ($\alpha=0.5\ fm^{-1}$).

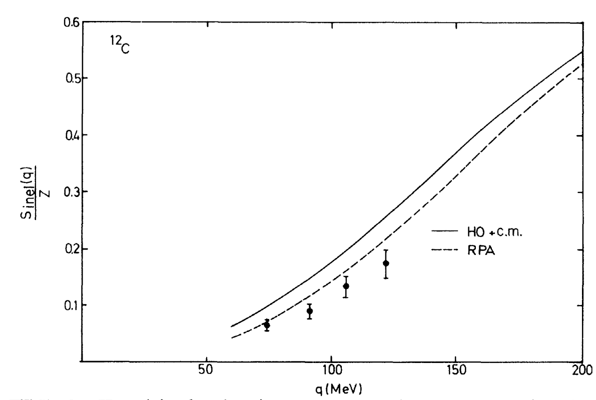

FIG. 3. Total inelastic electron scattering cross section (S_{inel}) as a function of q for ^{12}C at scattering angle $\theta=20°$. The full line gives the harmonic oscillator prediction including center of mass corrections. The dashed line gives the RPA prediction ($\xi=1.5$, $\alpha=0.6\ fm^{-1}$).

the situation for the transverse sum rule is different. In fact, in the low q limit, $S^T(q)$ is proportional to the dipole contribution to the photonuclear cross section

$$\sigma_{+1}=\int \sigma(\omega)\omega\, d\omega \ .$$

Such a sum rule is known to be dominated by intermediate and high energy effects which probably cannot be adequately taken into account by approaches based on the RPA and which are expected to significantly increase the value of σ_{+1}. The same effects are also responsible for an enhancement of the square average energy $\langle \omega^2 \rangle_{av}$, associated with the electron-scattering excitation spectrum, with respect to typical RPA predictions. For small scattering angles θ, where the cross section is dominated by the longitudinal term, the increase of $\langle \omega^2 \rangle_{av}$ produces a quenching of the total integrated strength $S_{inel}(q)$ owing to the kinematic factor (q_μ^2/q^2) multiplying the longitudinal contribution [see Eq. (1)].

Finally, one should investigate more carefully the spin contribution to the transverse sum rule. The present calculation, based on the HO model with j-j coupling, predicts a very large spin effect in ^{12}C (at $q=100$ MeV only $\frac{1}{3}$ of the transverse sum rule comes from the correction current term). However, it is well known that inclusion of dynamic correlations in shell model calculations strongly reduces the magnetic strength of the 15.11 MeV $M1$ collective state of ^{12}C. Such an effect is clearly expected to reduce the importance of the spin current contribution to the transverse sum rule.

It is evident that the theoretical investigation of electron scattering sum rules would be strongly stimulated by the availability of experimental data on separate longitudinal and transverse integrated cross sections for a wide range of momentum transfer and for different nuclei.

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