Δ-isobar production in proton-nucleus collisions

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(Received 28 November 1983)

The (p,Δ) reaction has been investigated in the general framework of the distorted-wave Born approximation. By the use of various choices for the spin-isospin transition interaction, $V_{\sigma\tau}(\mathrm{NN}\rightarrow\Delta\mathrm{N})$, it has been shown that the data on ^{6}Li(p,Δ^{++})^{6}He at 1.04 GeV can be reproduced well, both in magnitude and shape, by the one-pion—and—one-rho—exchange potential. Using only the one-pion-exchange potential with the Landau-Migdal-type repulsive term it is found that the magnitude of the parameter g'_Δ at high momentum transfer lies below 0.4.

I. INTRODUCTION

In recent years there have been increasing efforts to go beyond the conventional approach to the atomic nucleus. These efforts have been concerned with the investigation of the relevance of and the role played by the non-nucleonic degrees of freedom, like mesons, baryonic excitations, and quarks, in the nuclear dynamics and the correlations associated with the spins and isospins of nucleons themselves. Out of them the knowledge about the role of quarks in nuclei is, at present, very much in its infancy. It is expected, that, due to their confinement in nucleon bags of radius $\lesssim 1$ fm, quark effects should show up in the process associated with short range nucleon-nucleon separations, or equivalently high momentum transfer. However, there does not yet seem to exist any compelling unambiguous results to suggest such effects. The impact of the quark structure of nucleons in nuclear physics, therefore, currently lies more in new approaches than in new results.[1] On the other hand, the role played by mesons in nuclei is very well established.[2] A most convincing example of their importance is the electrodisintegration of the deuteron.[3] Consequently, it is now a common feature to consider the effect of meson-exchange currents (MEC)'s in the nuclear response to the electromagnetic probes at medium and low energies. In between these two fields lies the field of spin-isospin correlations and Δ(1232) isobars in nuclei. This is a developing and hence most fascinating field.[4,5]

Experimentally, the spin-isospin correlations are dramatically manifested through the strong excitation of the isovector Gamow-Teller (GT) mode in the intermediate energy (p,n) reaction. The strengths of these and the magnetic spin transitions at low momentum transfers are also found to be quenched systematically by about 60% in heavy nuclei. The source of this quenching, though under considerable debate at present, seems to be intimately connected with the Δ-isobar degrees of freedom in the nucleus.[5] These virtual isobars also show up in other processes, such as the narrow width of the Σ hypernuclei,[6] through the renormalization of the pion propagator due to Δ-hole excitations in nuclei. The magnitude of this effect, of course, depends upon the knowledge, which is rather poor at the moment, of the NN→ΔN coupling potential. The real Δ-isobar excitation is seen in the pion-nucleus scattering and in the real or virtual photon-induced reactions. The data on these reactions, therefore, have been used to learn about the Δ-nucleus interaction in the nucleus and to shed light on the dynamical mechanisms responsible for it.[7]

Yet another reaction, which we study in the present paper and whose potential for the study of the non-nucleonic aspect of the nucleus has been recognized only very recently,[8,9] is the (p,Δ^{++}) reaction. Though experimentally this reaction is difficult to investigate, it is very rich theoretically. Because the Δ^{++} corresponds to the $S=\frac{3}{2}$, $T=\frac{3}{2}$, and $T_Z=\frac{3}{2}$ state of the baryon, like the (p,n) reaction, it can transfer $\Delta S=1$ and $\Delta T=1$ to the nucleus. In addition, it also transfers large linear momentum ($\gtrsim 250$ MeV/c) and can excite nuclear states differing from the ground state by even two units of spin and/or isospin. Since the Δ^{++} is free in the final state, the reaction provides an opportunity to learn about the Δ-nucleus potential and the NN→$N\Delta$ coupling potential. The first reaction of this type was detected recently on ^{6}Li at 1.04 GeV incident energy,[8] where the differential cross section was measured as a function of four-momentum transfer squared (t) corresponding to the transition to the ground state of ^{6}He. The data were analyzed using the Glauber framework for the description of multiple scattering. For some sets of parameters a fair fit to the data was obtained.[8] However, with its large number of free parameters this approach does not seem appropriate for exploiting the full potential of the (p,Δ^{++}) reaction. In this work we use an alternate theoretical framework. Since at intermediate energies the measured cross section for the (p,Δ^{++}) reaction on a nucleus is very small ($\lesssim 100$ μb) in comparison to the total cross section of a few hundred mb for protons on a nucleus, it should be adequate to describe this reaction in terms of the distorted-wave Born approximation (DWBA). The validity of the DWBA for this reaction is further suggested by the success of a similar approach for the (p,n) reaction and the dominance of the Born approximation term in generating the spin-isospin dependent forces in the boson-exchange model.[10] In our approach, the expression for the cross section consists, in general, of three unknown ingredients: (i) the NN→ΔN coupling potential; (ii) the Δ-nucleus optical potential; and

(iii) the spin-isospin nuclear transition density. Using the data on ^{6}Li(p,Δ^{++})^{6}He, where the relevant nuclear transition density is known from the inelastic electron scattering at high momentum transfer on ^{6}Li,[11] we show that this reaction can be used effectively to extract the other two components pertaining to the Δ-nucleus dynamics. Since the sensitivity of the calculated results to the Δ-nucleus optical potential is found to be weak, the experiment determines the spin-isospin transition potential. Using the longitudinal and transverse form for the coupling interactions, in Sec. II we present the formalism. Section III contains its application to the ^{6}Li(p,Δ^{++})^{6}He reaction and a discussion of various physical parameters extracted from it.

II. FORMALISM

Considering the Δ as an elementary particle, the differential cross section in the DWBA for the $A(\mathrm{p},\Delta^{++})B$ reaction is written as

$$\frac{d\sigma}{dt}=E_pE_AE_\Delta E_B/[4\pi(\hbar^3c^3E_ck_p)^2]\langle\,|\,T_{BA}\,|^2\rangle\ , \tag{1}$$

where the angular brackets around $|\,T_{BA}\,|^2$ denote the sum and average over the spins in the final and initial states, respectively. The transition amplitude, T_{BA}, is given by

$$T_{BA}=\left\langle\chi^-_{\vec{k}_\Delta},\left\langle B,\Delta^{++}\left|\sum_i V_{\sigma\tau}(i)\right|A,\mathrm{p}\right\rangle,\chi^+_{\vec{k}_p}\right\rangle\ , \tag{2}$$

where E_x ($\vec{k}_x$) is the total energy (wave vector) of the particle x, t is the four-momentum transfer squared, and E_c is the total energy in the center of mass. χ represents the distorted wave in the initial and final states. $V_{\sigma\tau}$ is the effective spin-isospin transition potential for pp→nΔ^{++}, summed over the "active" nucleons in the target nucleus. This potential, in principle, should depend on both the incident energy and the transferred energy momentum. However, for the present, since we are going to analyze the experimental data at a single energy (1.04 GeV) only, we will not worry about the incident energy dependence. For the rest, the general form of the interaction, neglecting the spin-orbit dependence, is

$$V^i_{\sigma\tau}(\omega,\vec{Q})=[V_L(\omega,Q)\vec{S}\cdot\hat{Q}\vec{\sigma}_i\cdot\hat{Q}+V_T(\omega,Q)(\vec{S}\times\hat{Q})\cdot(\vec{\sigma}_i\times\hat{Q})]\vec{T}\cdot\vec{\tau}_i\ , \tag{3}$$

where ω (Q) is the energy (momentum) transfer from the incident proton to the ith nucleon in the nucleus, $\vec{S}$ ($\vec{T}$) is the spin (isospin) transition operator for p→Δ^{++}, $\hat{Q}$ denotes a unit operator, and V_L and V_T represent the (ω,Q) dependence of the longitudinal and transverse part of the interaction, respectively.

Using the identities

$$\begin{aligned}&\vec{S}_1\cdot\hat{Q}\vec{\sigma}_2\cdot\hat{Q}=\tfrac{1}{3}\vec{S}_1\cdot\vec{\sigma}_2+\tfrac{1}{3}S_{12}(\hat{Q})\ ,\\&(\vec{S}_1\times\hat{Q})\cdot(\vec{\sigma}_2\times\hat{Q})=\tfrac{2}{3}\vec{S}_1\cdot\vec{\sigma}_2-\tfrac{1}{3}S_{12}(\hat{Q})\ ,\end{aligned} \tag{4}$$

with the tensor operator

$$S_{12}(\hat{Q})=3\vec{S}_1\cdot\hat{Q}\vec{\sigma}_2\cdot\hat{Q}-\vec{S}_1\cdot\vec{\sigma}_2\ ,$$

the interaction [Eq. (3)] can also be written in an alternative form,

$$V^i_{\sigma\tau}(\omega,Q)=[V_C(\omega,Q)\vec{S}\cdot\vec{\sigma}_i+V_{\mathrm{NC}}(\omega,Q)S_{12}(\hat{Q})]\vec{T}\cdot\vec{\tau}_i\ , \tag{5}$$

where V_C and V_{NC} represent the central and noncentral part of the interaction, respectively. In terms of V_L and V_T they are defined as

$$\begin{aligned}&V_C(\omega,Q)=\tfrac{1}{3}V_L(\omega,Q)+\tfrac{2}{3}V_T(\omega,Q)\ ,\\&V_{\mathrm{NC}}(\omega,Q)=\tfrac{1}{3}V_L(\omega,Q)-\tfrac{1}{3}V_T(\omega,Q)\ .\end{aligned} \tag{6}$$

Since $V_{\sigma\tau}$ in Eq. (2) has been identified as an effective coupling interaction, in the evaluation of the transition matrix T_{BA}, only the direct term need be considered. Then the energy transfer ω appearing in $V_{\sigma\tau}$ [Eq. (3)] is going to be of the order of the nuclear excitation (i.e., ~10 MeV or less), which, in comparison to the incident energy (~1 GeV), is very small. Of course, to this excitation energy one should add the energy corresponding to the recoil, which, for light nuclei, might not be small. However, because of the large momentum transfer involved this may not affect the behavior of $V_{\sigma\tau}$ much. Therefore, in $V_{\sigma\tau}$, we approximate $\omega=0$.

A. Evaluation of T_{BA}

For evaluating T_{BA}, we first write the distorted waves in Eq. (2) in momentum space, i.e.,

$$T_{BA}=\int d\vec{k}'_\Delta d\vec{k}'_p\chi^{-*}_{\vec{k}_\Delta}(\vec{k}'_\Delta)\chi^+_{\vec{k}_p}(\vec{k}'_p)\times\left\langle\vec{k}'_\Delta\Delta^{++},B\left|\sum_i V^i_{\sigma\tau}(\omega=0,Q)\right|A,\mathrm{p}\vec{k}'_p\right\rangle\ , \tag{7}$$

where $\vec{k}'$ denotes the local momentum of the subscripted particle, and $\vec{Q}'$, correspondingly, is the local momentum transfer. Analogous to $\vec{Q}$,

$$\vec{Q}'=\vec{k}'_p-\vec{k}'_\Delta\ .$$

Using the following definitions for the transition matrices, $\vec{S}$ and $\vec{T}$,

$$\begin{aligned}&\langle\tfrac{3}{2}m_\Delta\,|\,S_\mu\,|\,\tfrac{1}{2}m_p\rangle=(1\tfrac{1}{2}\mu m_p\,|\,\tfrac{3}{2}m_\Delta)\ ,\\&\langle\tfrac{3}{2},+\tfrac{3}{2}\,|\,T_\mu\,|\,\tfrac{1}{2},+\tfrac{1}{2}\rangle=(1\tfrac{1}{2}\mu+\tfrac{1}{2}\,|\,\tfrac{3}{2}+\tfrac{3}{2})\delta_{\mu,+1}\ ,\end{aligned} \tag{8}$$

the spin-isospin transition corresponding to p→Δ^{++} can be calculated in Eq. (7), yielding

$$\begin{aligned}T_{BA}=(-1)^{m+1}(1\tfrac{1}{2}mm_p\,|\,\tfrac{3}{2}m_\Delta)\int d\vec{k}'_p d\vec{k}'_\Delta\chi^{-*}_{\vec{k}_\Delta}(\vec{k}'_\Delta)\chi^+_{\vec{k}_p}(\vec{k}'_p)\\\times\left[\left\langle\vec{k}'_\Delta,B\left|V_C(Q')\sum_i\sigma_{-m}(i)\tau_{-1}(i)\right|\vec{k}'_p,A\right\rangle\right.\\\left.+\left[\frac{8\pi}{15}\right]^{1/2}\sum_\mu(-1)^\mu(11\mu m\,|\,2M)\left\langle\vec{k}'_\Delta,B\left|V_{\mathrm{NC}}(Q)Y_{2,-M}(\hat{Q}')\sum_i\sigma_\mu(i)\tau_{-1}(i)\right|\vec{k}'_p,A\right\rangle\right]\ .\end{aligned} \tag{9}$$

Here, m $(=m_\Delta - m_p)$ is the spin projection transferred to the nucleus by the $p \rightarrow \Delta^{++}$ transition. m_Δ (m_p) is the same quantum number for the delta (proton). $(ab\alpha\beta/c\gamma)$ is the Clebsch-Gordan coefficient. The first term inside the square brackets corresponds to the central part of the interaction and the second term to the noncentral part. In arriving at expression (9) we used the definition for the scalar product of two vectors given by

$$\vec{T}(1)\cdot\vec{T}(2)=\sum_{\mu}(-1)^{\mu}T_{\mu}(1)T_{-\mu}(2)\ . \tag{10}$$

The expression [Eq. (9)] for T_{BA} may be simplified because of the following observations:

(i) Since the energy region of interest is around 1 GeV, the local momenta $\vec{k}'_p$ and $\vec{k}'_\Delta$ of the continuum particles would not differ much from their corresponding asymptotic values.

(ii) The momentum transfer $\vec{Q}$ in the (p,Δ^{++}) reaction is around 250—700 MeV/c for the angular range up to about 40°.

(iii) The momentum dependence of the transition interaction $V_{\sigma\tau}$ in this high momentum range is weak (see Fig. 5.)

It is, therefore, reasonable to approximate the interactions $V(Q')$ in Eq. (9) by their values corresponding to the asymptotic momentum transfer $\vec{Q}$ $(=\vec{k}_p-\vec{k}_\Delta)$. This approximation then factorizes the expression for T_{BA} to

$$T_{BA}=(-1)^{m+1}(1\tfrac{1}{2}mm_p\,|\,\tfrac{3}{2}m_\Delta)\left[V_C(Q)F^{BA}_{-m,-1}(Q)+\left[\frac{8\pi}{15}\right]^{1/2}V_{NC}(Q)\sum_{\mu}(-1)^{\mu}(11\mu m\,|\,2M)Y_{2,-M}(\hat{Q})F^{BA}_{\mu,-1}(Q)\right], \tag{11}$$

where F^{BA} is the "distorted" nuclear structure factor. In configuration space it is given by

$$F^{BA}_{\nu,-1}(Q)=\left\langle B\left|\sum_i \chi^{-*}_{\vec{k}_\Delta}(\vec{r}_i)\chi^{+}_{\vec{k}_p}(\vec{r}_i)\sigma_\nu(i)\tau_{-1}(i)\right|A\right\rangle,$$

$$=\int d\vec{r}\,\chi^{-*}_{\vec{k}_\Delta}(\vec{r})\chi^{+}_{\vec{k}_p}(\vec{r})\rho^{BA}_{\nu,-1}(\vec{r})\ , \tag{12}$$

where ν is the general notation for μ and m.

$\rho^{BA}_{\nu,-1}(\vec{r})$ is the spin-isospin transition density and is defined as

$$\rho^{BA}_{\nu,-1}(\vec{r})=\left\langle B\left|\sum_i \delta(\vec{r}-\vec{r}_i)\sigma_\nu(i)\tau_{-1}(i)\right|A\right\rangle. \tag{13}$$

Corresponding to Eq. (11) for T_{BA}, $\langle\,|\,T_{BA}\,|^2\rangle$ is given by

$$\langle\,|\,T_{BA}\,|^2\rangle=[2(2J_A+1)]^{-1}\sum_{\substack{M_AM_B\\ m_pm_\Delta}}|\,T_{BA}\,|^2$$

$$=\tfrac{2}{3}(2J_A+1)^{-1}\sum_{M_AM_B}\sum_{m=-1}^{+1}\left|V_C(Q)F^{BA}_{-m,-1}(Q)\right.$$

$$\left.+\left[\frac{8\pi}{15}\right]^{1/2}V_{NC}(Q)\sum_{\mu}(-1)^{\mu}(11\mu m\,|\,2M)Y_{2,-M}(\hat{Q})F^{BA}_{\mu,-1}(Q)\right|^2. \tag{14}$$

As one may notice from the above expression, the contributions of the central and noncentral part of the interaction to the cross section are, in general, coherent. In certain cases, however, as we shall see later, they can become incoherent.

The evaluation of $\langle\,|\,T_{BA}\,|^2\rangle$, as is seen from Eq. (14), requires the knowledge of three factors: (i) the interaction $V_{\sigma\tau}$; (ii) optical potentials to generate distorted waves for protons and isobars; and (iii) the nuclear spin-isospin transition density $\rho_{\nu,-1}$ in the high momentum transfer region. The actual expression for $\rho_{\nu,-1}$, which might not be simple, depends upon the nuclear configurations of the states of the nuclei A and B. However, because of the high momentum transfer it is very important that these transition densities be evaluated very carefully. Since at present the experimental data exist only for ^{6}Li$(p,\Delta^{++})^6$He (g.s.), in the present paper we evaluate it only for $^6\text{Li}\rightarrow{}^6\text{He}$ (g.s.). This is done in the Appendix.

Using expression (A10) of the Appendix for the spin-isospin transition density, the "distorted" nuclear structure factor F^{BA} [Eq. (12)] for $^6\text{Li}\rightarrow{}^6\text{He}$ is given by

$$F^{BA}_{\nu,-1}(\vec{Q})=C_0\delta_{M_A,-\nu}G_{00}(Q)$$

$$+C_2(12M_A-M_L\,|\,1-\nu)G_{2M_L}(Q)\ , \tag{15}$$

where

$$G_{LM_L}(Q)=(4\pi)^{-1/2}\int d\vec{r}\,\chi^{-*}_{\vec{k}_\Delta}(\vec{r})\chi^{+}_{\vec{k}_p}(\vec{r})\rho_l(r)Y_{LM_L}(\hat{r})\ . \tag{16}$$

Here, $\rho_l(r)$ is the radial transition density and is normalized to unity. L is the transferred orbital angular momentum. For a $(1p)^2$ configuration its values are limited to 0 and 2. C_0 and C_2, the weights for the transfer of these two values of L, are determined by the configuration mixing coefficients for the description of the initial and final states of the nuclei (see the Appendix). For the following configurations,

$$\begin{aligned}&\Psi_{^6\mathrm{Li}}(1^+,T_A=0)=(\alpha_A\,{}^3S_1+\beta_A\,{}^1P_1+\gamma_A\,{}^3D_1)\,|\,T_A=0,M_{T_A}=0\rangle\ ,\\&\Psi_{^6\mathrm{He}}(0^+,T_B=1)=(\alpha_B\,{}^1S_0+\beta_B\,{}^3P_0)\,|\,T_B=1,M_{T_B}=-1\rangle\ .\end{aligned} \tag{17}$$

The C coefficients are given by

$$\begin{aligned}&C_0=-2\alpha_A\alpha_B+2(3)^{-1/2}\beta_A\beta_B\ ,\\&C_2=4(10)^{-1/2}\gamma_A\alpha_B+2(6)^{-1/2}\beta_A\beta_B+3(5)^{-1/2}\gamma_A\beta_B\ .\end{aligned} \tag{18}$$

If we restrict the description of ^{6}Li and ^{6}He ground states only to the dominant $^3S_1$ and $^1S_0$ configurations, the $L=2$ term in Eq. (15) would not be allowed. Consequently, the expression for $\langle\,|\,T_{BA}\,|^2\rangle$ gets reduced to a simple form with the central and noncentral terms of the interaction contributing incoherently, i.e.,

$$\langle\,|\,T_{BA}\,|^2\rangle=\tfrac{2}{3}[V_C^2(Q)+2V_{\mathrm{NC}}^2(Q)]\,|\,G_{00}(Q)\,|^2\ . \tag{19}$$

With full configuration mixing [Eq. (17)], on the other hand, the final expression for $\langle\,|\,T_{BA}\,|^2\rangle$ is quite complicated. It gets coherent contributions from both values of L as well as from both pieces of the interaction.

B. $G_{LM_L}(Q)$

Since we are interested in the energy region of continuum particles around 1 GeV, G_{LM} is evaluated by approximating distorted waves χ by the eikonal form. In addition, if we also take the distortion factor for Δ along the incident direction only,

$$\begin{aligned}G_{LM_L}(Q)&=(4\pi)^{-1/2}\int b\,db\,dz\,d\phi\,\exp(iQ_{||}z-i\vec{Q}_\perp\cdot\vec{b})D_{\vec{k}_p}(b,z)D_{-\hat{k}_\Delta=-\hat{k}_p}(b,z)\rho_l(r)Y_{LM_L}(\hat{r})\\&=i^{-M_L}\pi^{1/2}\int b\,db\,dz\exp(iQ_{||}z)J_{M_L}(Q_\perp b)D_{\vec{k}_p}(b,z)D_{-\hat{k}_p}(b,z)\rho_l(r)\Theta_{LM_L}(b,z),\end{aligned} \tag{20}$$

where the D's are the distortion factors. In terms of the distorting potentials V they are given by

$$D_{\pm\hat{k}}(\vec{r})=\exp\left[-\frac{iE}{\hbar^2c^2k}\int_0^\infty V(\,|\,\vec{r}\mp\hat{k}s\,|\,)ds\right], \tag{21}$$

where $Q_{||}$ and $Q_\perp$ are the longitudinal and transverse components of the momentum transfer $\vec{Q}$, and J_{M_L} is the cylindrical Bessel function. Θ_{LM_L} is given by

$$Y_{LM_L}(\theta,\phi)=\Theta_{LM_L}(\theta)e^{iM_L\phi}\ .$$

C. Choice of $V_{\sigma\tau}$

In order to learn about the $V_{\sigma\tau}(\mathrm{pp}\to\mathrm{n}\Delta^{++})$ from the data on the (p,Δ^{++}) reaction, to begin with, it is necessary that we start with some ansatz for it. For this purpose, we take guidance from the (p,n) reaction for the corresponding interaction in the NN→NN channel. For this interaction, it has been shown by Brown *et al.*[10] that the $V_{\sigma\tau}(\mathrm{pn}\to\mathrm{np})$ of Love and Petrovich,[12] which fit the intermediate energy (p,n) data , can be reproduced very well by the one-pion—and—one-rho—exchange potentials. It is also found that, in the spin-isospin channel, the dominant contribution to this interaction comes only from the first order Born term. Following this we have identified V_L and V_T of $V_{\sigma\tau}(\mathrm{pp}\to\mathrm{n}\Delta^{++})$ with the one-pion—and—one-rho—exchange potentials, respectively, i.e.,

$$V_L(\omega,q)=-\frac{4\pi\hbar c f_\pi f_\pi^*}{m_\pi^2}F_\pi(q)F_\pi^*(q)\frac{q^2}{q^2+m_\pi^2-\omega^2}\ , \tag{22}$$

$$V_T(\omega,q)=-\frac{4\pi\hbar c f_\rho f_\rho^*}{m_\rho^2}F_\rho(q)F_\rho^*(q)\frac{q^2}{q^2+m_\rho^2-\omega^2}\ , \tag{23}$$

where $f(F)$ and $f^*(F^*)$ are the coupling constants (form factors) at the πNN and πNΔ vertices, respectively. m is the mass of the exchanged boson. The structure of the form factors was chosen to be of the monopole type with the assumption that $F=F^*$,

$$F(q)=F^*(q)=\frac{\Lambda^2-m^2}{\Lambda^2+q^2}\ . \tag{24}$$

The values of the cutoff momenta Λ and the coupling constants in Eqs. (22) and (23) are taken from those required for the description of πN scattering, np and $\mathrm{p\bar{p}}$ charge-exchange scattering, one boson exchange nucleon-nucleon potentials, etc.[5,13] These values are

$$\begin{aligned}&f_\pi^2=0.081,\ \ f_\pi^{*2}=0.37,\ \ \Lambda_\pi=1.2\ \mathrm{GeV}\ ,\\&f_\rho^2=4.86,\ \ f_\rho^*=1.85f_\rho,\ \ \Lambda_\rho=2\ \mathrm{GeV}\ .\end{aligned} \tag{25}$$

The values for ρ exchange correspond to the so-called "strong ρ-exchange coupling."

III. RESULTS AND DISCUSSION

As mentioned earlier, results are presented for the differential cross section as a function of four-momentum transfer squared at 1.04 GeV incident energy for ^{6}Li(p,Δ^{++})^{6}He (g.s.). The best choice for the various parameters required in the spin-isospin transition density, $\rho^{BA}_{\nu,-1}$, comes from the inelastic scattering of medium energy electrons in the three-momentum transfer range 1.0–3.0 fm^{-1} from the ground state of ^{6}Li to its second excited state $(0^+,T=1)$ at 3.56 MeV. This excited state is the isobaric analog of the ground state $(0^+,T=1)$ of ^{6}He. From their measured experimental data Bergstrom *et al.*[11] extracted the radial transition density $\rho_I(r)$, which is parametrized as

$$\rho_I(r)=\exp(-r^2/b^2)[0.1063r^2-0.05091r^3+0.008433r^4-0.0001126r^6+0.000001407r^8]\ , \tag{26}$$

with $b=2.324$ fm. The configuration mixing parameters of Eq. (17) are chosen as

$$\alpha_A=0.924,\ \beta_A=0.2,\ \gamma_A=0.102\ ,\quad \alpha_B=1.00,\ \beta_B=0.08\ . \tag{27}$$

These parameters, apart from well reproducing the (e,e') form factors for the 3.56 MeV $(0^+,T=1)$ and 5.37 MeV $(2^+,T=1)$ states in ^{6}Li, also fit the ground state moments.

The required optical potential to generate the distortion factor for a proton on ^{6}Li at 1.04 GeV does not exist. This potential is therefore generated by use of the high-energy ansatz of folding in the elementary nucleon-nucleon scattering amplitude with the nuclear density. We write

$$V_p(r)=(i+\alpha)W_0\rho(r)/\rho(0)\ , \tag{28}$$

with W_0, the imaginary part of the potential, given by

$$W_0=-(\hbar^2c^2k/2E)\rho_0\sigma_T\ . \tag{29}$$

Here, σ_T is the total proton-nucleon cross section and ρ_0 the nuclear density. α is the ratio of the real to imaginary parts of the nucleon-nucleon scattering amplitude. The values of σ_T and α are taken to be 44 mb and -0.073, respectively, from the scattering of 1.05 GeV protons on nucleons.[14] The function $\rho(r)/\rho(0)$ describes the radial dependence of $V_p(r)$ in terms of the density. $\rho(r)$ is taken from the elastic electron scattering analysis on ^{6}Li of Li *et al.*[15]

No information exists on the Δ-nucleus potential (V_Δ) in the intermediate-energy range. At low energies, some estimates have been made,[7] but they are not of much relevance for the present purpose. In view of such a lack of knowledge about V_Δ, we take guidance from the fact that the Δ is an excited state of the nucleon and thus carries, intrinsically, about 300 MeV more energy than the proton/neutron at the same kinetic energy. Therefore it is plausible that W_0 for the Δ at a kinetic energy T may be close to that for the proton around $T+300$ MeV. This correspondence with the W_0 at an enhanced energy for protons to a certain extent takes care of the additional channels (like πN, etc.) which are open to the Δ. Hence, as a reasonable guess, to start with, we have taken W_0 for Δ to correspond to that for the proton at the incident energy. The energy of the incident proton approximately equals that of the Δ plus 300 MeV. For the real part of the Δ-nucleus potential we are again guided by that of protons. The proton optical potential (the real part) goes to zero around 600 MeV laboratory energy and becomes repulsive beyond it. Since the laboratory energy of Δ for a 1.04 GeV incident proton energy is around 700 MeV, we guess that the real part of the Δ-nucleus potential should be small. It is taken equal to zero (i.e., $\alpha_\Delta=0.0$). We would, however, investigate the dependence of the cross section on V_Δ later on.

The calculated cross sections (continuous curve) as a function of t, along with the measured values, are shown in Fig. 1. It is remarkable that the calculated results agree

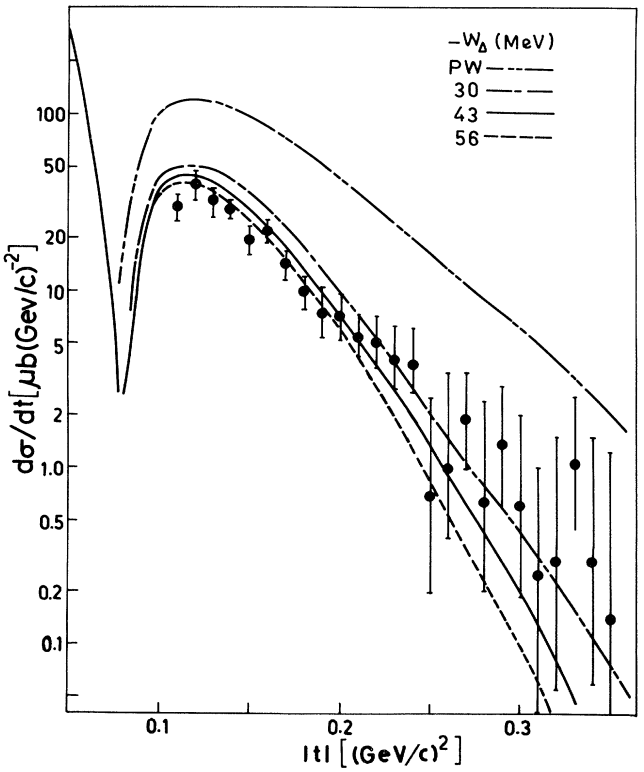

FIG. 1. Differential cross section $d\sigma/dt$ vs $|t|$ for ^{6}Li(p,Δ^{++})^{6}He for various values of W_Δ. The experimental points are from Ref. 8. The PW curve corresponds to the plane wave approximation for protons and deltas.

very well, in magnitude as well as in shape, with the experimental data. Since this agreement is practically parameter-free, it determines, in a direct way, that the effective spin-isospin coupling potential $V_{\sigma\tau}(\mathrm{NN}\rightarrow\mathrm{N}\Delta)$ can be correctly described by the one-pion—and—one-rho—exchange interaction.

The only source of uncertainty in the above conclusion arises from the uncertainty in the Δ-nucleus optical potential. In order to see the changes in the results due to the variation of this potential, we show in Fig. 1 two more curves. These curves correspond to W_0 for Δ^{++} increased and decreased by 30% from that used for the continuous curve. Sensitivity of the results to the variation in the real part of the potential, to the extent of ±10 MeV, is shown in Fig. 2. From these curves we observe that the results do not change much. This suggests that the above conclusion about the coupling potential can be accepted with confidence.

The different curves in Figs. 1 and 2 for various values of the Δ-nucleus optical potential also show that the accuracy in the experimental data has to be greater if one wants to use the (p,Δ^{++}) reaction to learn about this potential. For completeness, Fig. 1 also contains the result corresponding to the plane wave approximation for the continuum particles.

In order to convince ourselves further that the $(1\pi+1\rho)$-exchange interaction is the correct representation of the spin-isospin coupling potential $V_{\sigma\tau}(\mathrm{NN}\rightarrow\Delta\mathrm{N})$, we have also tried some other prescriptions for it. For this purpose we consider the pion production (absorption) in two nucleons (deuteron), pion-nucleon scattering, and, again, the spin-isospin nucleon-nucleon interaction $V_{\sigma\tau}(\mathrm{NN}\rightarrow\mathrm{NN})$. In the physical interpretation of the empirically determined $V_{\sigma\tau}(\mathrm{NN}\rightarrow\mathrm{NN})$ in terms of the one-boson exchange model, one normally includes, in addition to the one-pion—and—one-rho—exchange interactions [analogous to Eqs. (22) and (23)], a correlation function $g(r)$.[10,16] The purpose of this function is to simulate the repulsion at high momenta due to ω-meson exchange. In the coupling interaction $V_{\sigma\tau}(\mathrm{NN}\rightarrow\Delta\mathrm{N})$, however, since the ω meson has zero isospin, this repulsion would not exist. Still, the inclusion of $g(r)$ in $V_{\sigma\tau}(\mathrm{NN}\rightarrow\Delta\mathrm{N})$ may be necessary to incorporate the repulsion due to multipion exchange and other dynamical processes, provided the same is not included to the required extent in the form factor $F(F^*)$. We have, therefore, calculated the cross section using one-pion—and—one-rho—exchange interactions [Eqs. (22) and (23)] with their central part modified to $\tilde{V}_C$ as

$$\tilde{V}_C(q)=(2\pi)^{-3}\int d\vec{k}\, g(\vec{q}-\vec{k})V_C(\vec{k})\,, \tag{30}$$

where g is the correlation function in momentum space. Following Brown *et al.*[16] this correlation function is generated through the Fourier transform of $[1-j_0(q_c r)]$. This yields

$$\tilde{V}_C(q)=V_C(q)-(4\pi q_c^2)^{-1}\times\int d\vec{k}\,\delta[\,|\vec{q}-\vec{k}\,|-q_c]V_C(\vec{k})\,, \tag{31}$$

where q_c is the correlation parameter. It is taken equal to 3.93 fm^{-1}. We have not modified the noncentral part of the interaction as it is already cut, at high momentum transfers, due to opposite signs of the one-pion—and—one-rho—exchange interactions [see Eq. (6)].

For the other choice of interaction we have taken the potential consisting of one-pion-exchange only. The effect of the other pieces of the interaction is included via the form factor with the appropriate choice of the cutoff momentum Λ. Values of Λ are taken equal to 2 and 4 fm^{-1}. The former value is consistent with the πN phase shifts,[17] while the latter, using single-nucleon and two-nucleon absorption/production vertices, provides a reasonable description of the total cross section for $\pi^+\mathrm{d}\rightarrow\mathrm{pp}$ and the inverse reaction $\vec{\mathrm{p}}\mathrm{p}\rightarrow\pi^+\mathrm{d}$.[18]

The results corresponding to these potentials are shown in Fig. 3. It is evident that, except for the $(1\pi+1\rho)$-exchange interaction [Eqs. (22) and (23)], the results due to all other interactions do not agree with the experimental data.

In context with the processes involving small momentum transfer in the literature, there also exists another approach for the description of $V_{\sigma\tau}(\mathrm{NN}\rightarrow\mathrm{NN})$. In this approach, the $V_{\sigma\tau}$ is written as a sum of the one-pion-exchange potential without any form factor and a purely phenomenological repulsive spin-spin term.[5,16] For the transition potential this interaction is

$$V_{\sigma\tau}(\mathrm{NN}\rightarrow\Delta N)=4\pi\hbar c\frac{f_\pi f_\pi^*}{m_\pi^2}\left[-\frac{\vec{\sigma}\cdot\vec{q}\vec{S}\cdot\vec{q}}{q^2+m_\pi^2}+g'_\Delta\vec{\sigma}\cdot\vec{S}\right]\times\vec{\tau}\cdot\vec{T}\,, \tag{32}$$

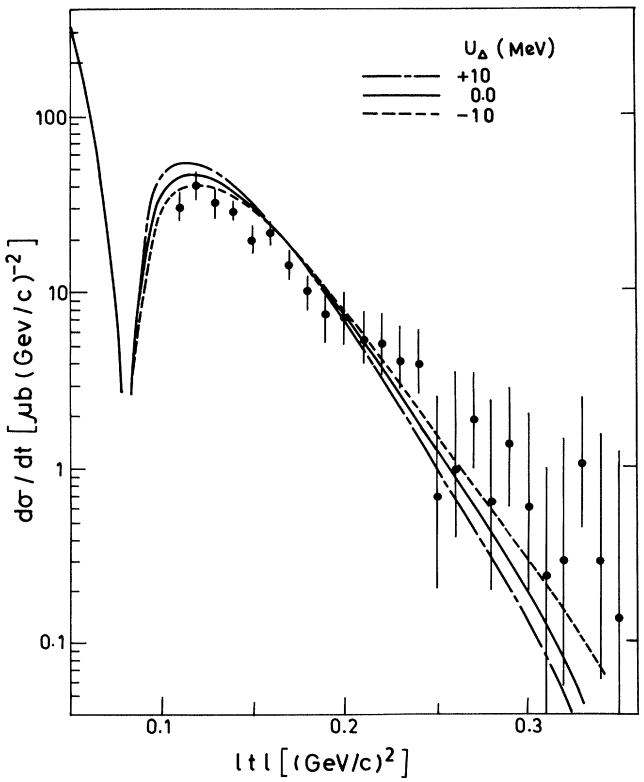

FIG. 2. $d\sigma/dt$ for $^6\mathrm{Li}(\mathrm{p},\Delta^{++})^6\mathrm{He}$ for various values of the real part (U_Δ) of the Δ potential.

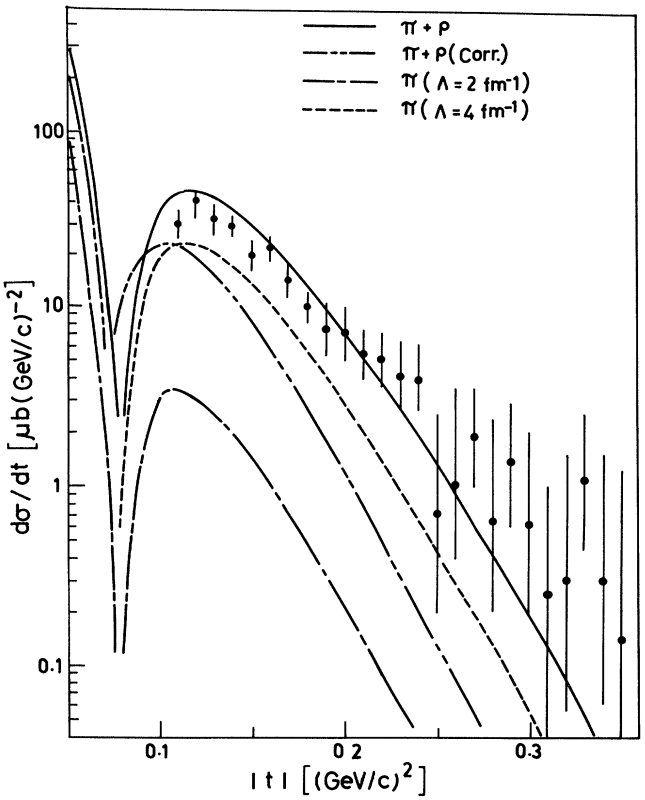

FIG. 3. $d\sigma/dt$ for ^{6}Li(p,Δ^{++})^{6}He for various choices of the coupling potential $V_{\sigma\tau}$(pp$\rightarrow\Delta^{++}$n).

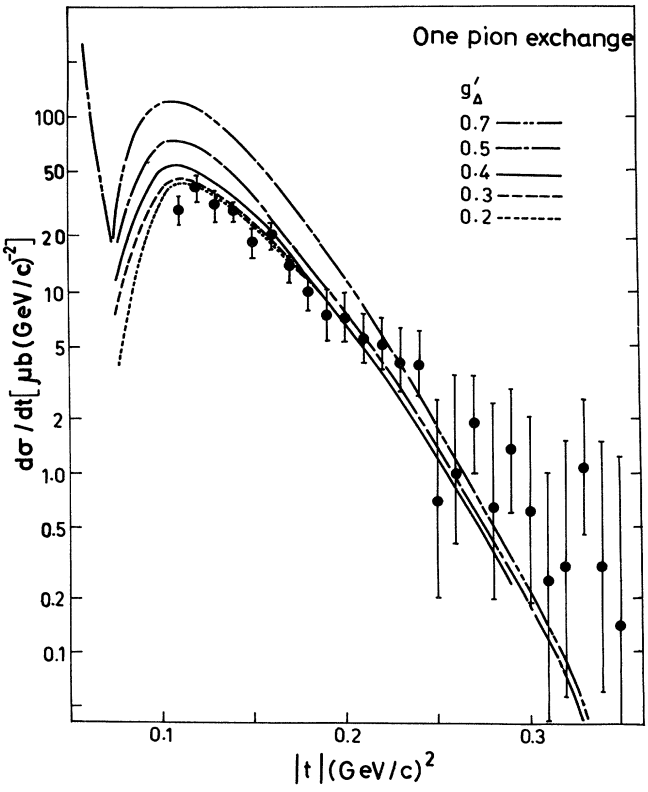

FIG. 4. Sensitivity of the ^{6}Li(p,Δ^{++})^{6}He differential cross section to g'_Δ using the one-pion—exchange potential (without the form factor) only.

where g'_Δ is called the Landau-Migdal parameter.[19]

The motivation for writing $V_{\sigma\tau}$ in this form arises from the basic philosophy that, in the boson-exchange theory of nuclear forces, the essential bosons which are exchanged are pions; ρ exchange is really the pole term in the exchange of a continuous mass spectrum of two interacting pions. Secondly, the repulsion in the interaction at short distances arises from several sources. Therefore, instead of putting the screening effects due to these sources separately, it is more economical to summarize them in one parameter, g'. If the repulsive effects are sufficiently short ranged, g' can be taken as a constant, independent of momentum and energy transfer. For the nucleon-nucleon case, the value of this parameter (g'_N) seems to be around 0.6$\sim$0.7 (Ref. 20). For $V_{\sigma\tau}$(NN$\rightarrow\Delta$N) in the low momentum transfer region, g'_Δ is often taken to be equal to g'_N (Ref. 21). In the present paper, however, we are concerned with $V_{\sigma\tau}$(NN$\rightarrow\Delta$N) in the high momentum transfer region, where the value of g'_Δ may be different and may also depend upon $\vec{q}$. It would, therefore, be of considerable interest if this parameter could be determined from the data on the (p,Δ^{++}) reaction. With this motivation, in Fig. 4, using the interaction given by Eq. (32), we present results for various values of g'_Δ, ranging from 0.2 to 0.7. In order to restrict the number of unknown quantities to a minimum, the parameter g'_Δ is taken as constant. Comparison of these results with the experimental data shows that the value of g'_Δ in the high momentum transfer region lies below 0.4. This value of g'_Δ is obviously lower than its value (0.5—0.7) normally thought to be at low momentum transfer (except for the recent conclusions of Arima *et al.*[22]).

The assumption of representing the effect of short range repulsion by a purely q-independent parameter, g'_Δ, can be partially corrected, without introducing additional unknown parameters, by starting with the one-pion—and—one-rho—exchange interaction.[23] Addition of the q-independent g'_Δ term to this interaction may then be better justified, as part of the q dependence of the repulsive term at short distances is simulated by the ρ-exchange term. This means that for the coupling interaction we write

$$V_{\sigma\tau}(\mathrm{NN}\rightarrow\Delta\mathrm{N})=V_{\sigma\tau}^{\pi+\rho}(\vec{q})+4\pi\hbar c\frac{f_\pi f_\pi^*}{m_\pi^2}g'_\Delta\vec{\sigma}\cdot\vec{S}\vec{\tau}\cdot\vec{T}\,, \tag{33}$$

where $V_{\sigma\tau}^{\pi+\rho}$ is the earlier defined one-pion—and—one-rho—exchange interaction [Eqs. (5) and (22)—(25)]. Using this interaction in Fig. 5 we show results for various values of g'_Δ. Comparison of these results with the experimental data shows that once we have set the one-pion—and—one-rho—exchange interactions explicitly, it is not necessary to have any additional repulsion. That is, with the interaction [Eq. (33)], the data on ^{6}Li(p,Δ^{++})^{6}He suggest that $g'_\Delta=0$. This, in fact, again reiterates our earlier conclusion of the adequacy of the one-pion—and—one-rho—exchange interaction for the description of the spin-isospin coupling potential.

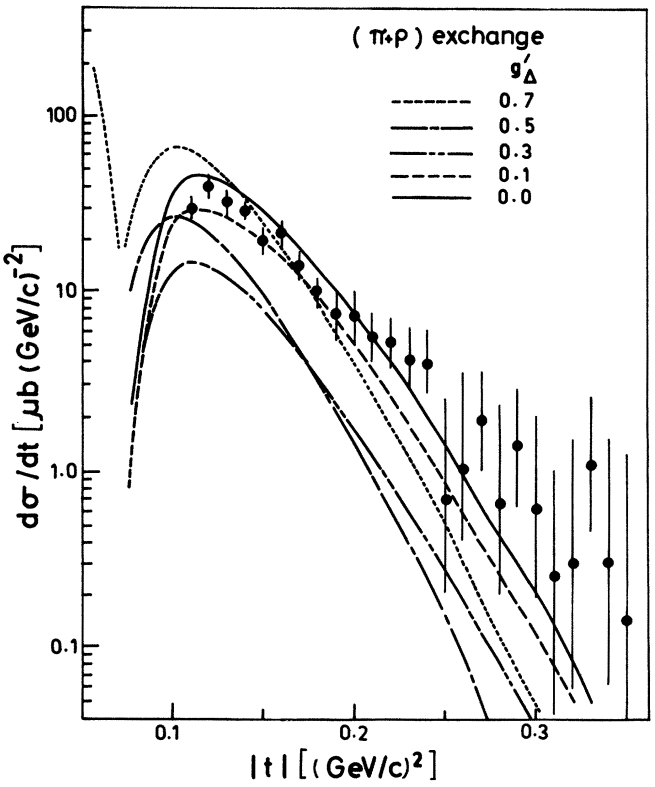

FIG. 5. Sensitivity of the ^{6}Li(p,Δ^{++})^{6}He differential cross section to g'_Δ using the one-pion—and—one-rho—exchange interaction.

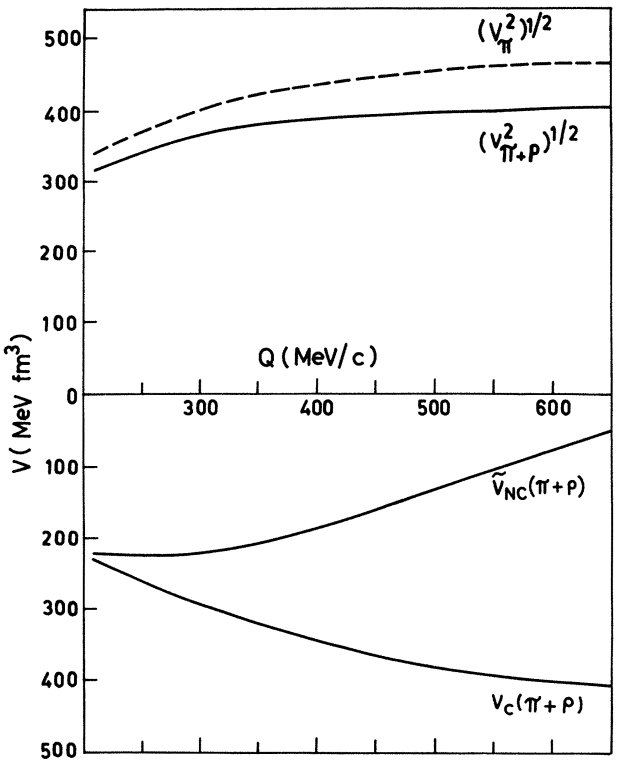

FIG. 6. Distribution of various pieces of the interaction as a function of the three-momentum transfer Q.

Finally, with the ultimate aim of using the (p,Δ^{++}) reaction for the study of the nuclear spin-isospin excitations in a most effective way, we explored the behavior of different ingredients in the transition matrix $\langle \mid T_{BA} \mid^2 \rangle$. For clarity we only consider the $^3S_1$ and $^1S_0$ configurations of ^{6}Li and ^{6}He. With this choice, as shown in Sec. II, the expression for $\langle \mid T_{BA} \mid^2 \rangle$ factorizes into an interaction term and the transition density term. In Fig. 6 we show the behavior of the central term, $V_C(Q)$, and the term $2^{1/2}V_{\rm NC}(Q)$ $(=\tilde{V}_{\rm NC})$ corresponding to the noncentral part of the interaction, as a function of Q. Also shown is the full interaction factor, $(V_C^2+V_{\rm NC}^2)^{1/2}$ $[=(V_{\pi+\rho}^2)^{1/2}]$. From these distributions one observes that, while around 200 MeV/c the contributions of the central and noncentral pieces of the interaction are about the same, beyond it the contribution of the noncentral term drops down to a small value and that, due to the central term, rises and settles to a constant value of about 400 MeV fm^3. The summed square of these terms, $V_{\pi+\rho}^2$, which finally determines the $\langle \mid T_{BA} \mid^2 \rangle$, of course remains more or less constant over the entire range of Q of interest (i.e., $250<Q<700$ MeV/c). This suggests that the shape of $\langle \mid T_{BA} \mid^2 \rangle$ is mainly determined by the transition density. The interaction term determines only the magnitude of the cross section. Therefore, from the point of view of simplicity, it can as well be approximated by a Q-independent term. Agreement with the ^{6}Li(p,Δ^{++})^{6}He data suggests that the magnitude of this constant interaction can be fixed around 375 MeV fm^3.

Since with fully configuration mixed wave functions the contributions of various terms corresponding to interaction and configuration become coherent [see Eq. (14)], it is not possible to say, *a priori*, what effect it will have if we neglect the noncentral part of the interaction. Therefore, using the fully configuration mixed wave functions [Eq. (17)] for ^{6}Li and ^{6}He, in Fig. 7 we show the cross sections including (σ) and excluding (σ_C), the noncentral term in $V_{\sigma\tau}$. It appears that the phase relationships among various contributions in this case are such that for the major part of the distribution the two results (σ and σ_C) do not differ much. However, this may not be the case in general, because, as we see in Fig. 5, the contribution of $V_{\rm NC}$ does remain significant up to $Q\approx400$ MeV/c.

IV. CONCLUSIONS

The findings of this paper demonstrate that the outlook for exploiting the full potential of the (p,Δ^{++}) reaction for the study of the new modes of nuclear excitation and the isobar-nucleus dynamics is quite good. It is possible to extract this information from the experiments quantitatively and reliably. Specific conclusions reached may be summarized as follows:

(i) Direct determination of the effective spin-isospin coupling interaction $V_{\sigma\tau}$(NN$\rightarrow$NΔ) is possible through the $p+A\rightarrow\Delta^{++}+B$ reaction.

(ii) The data on ^{6}Li(p,Δ^{++})^{6}He already suggest that this interaction is correctly described by the one-pion—and—one-rho—exchange interaction.

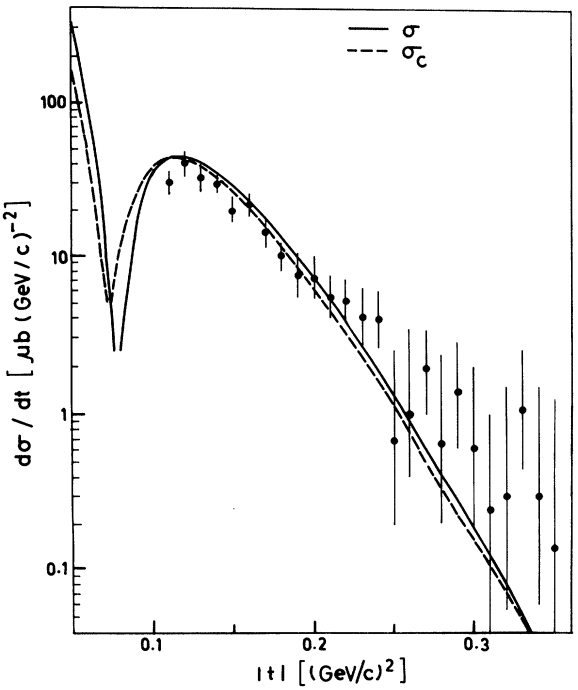

FIG. 7. Differential cross section for ^{6}Li(p,Δ^{++})^{6}He including (——) and excluding (— — —) the noncentral part of the interaction.

(iii) With this interaction, the distorted-wave Born approximation seems to be the correct theoretical framework in which the data on the $A(p,\Delta^{++})B$ reaction should be analyzed.

(iv) Sensitivity of the cross section to the Δ-nucleus optical potential is such that it requires better accuracy of the data than presently achieved. Considering that these kinds of experiments are difficult, it might be difficult to learn about the finer details of the Δ-nucleus optical potential from this reaction.

(v) Within the model used for the description of the reaction mechanism by us it seems that the value of the repulsive parameter g'_Δ at high momentum transfer is less than 0.4.

ACKNOWLEDGMENTS

It is a great pleasure to thank Jörg Hüfner, who suggested the problem of this paper and helped at every stage of the work. Thanks are also due to him, H. J. Pirner, and other members of the Institut for many useful discussions and the warm hospitality extended during my stay at Heidelberg. I also thank my colleague, Nataraj Sarma, at the Bhabha Atomic Research Centre, Bombay, for his useful comments in preparing this paper. This work was partly supported by the German Federal Ministry of Research and Technology (BMFT).

APPENDIX: TRANSITION DENSITY FOR ^{6}Li→^{6}He (g.s.)

Spin (isospin) quantum numbers for ^{6}Li and ^{6}He (g.s.) are 1^+ (0) and 0^+ (1), respectively. Using a $(1p)^2$ configuration, the most general wave functions for these states in the "intermediate coupling" scheme are given by

$$|A\rangle=\Psi_{^6\mathrm{Li}}(1^+,T_A=0)=[\alpha_A\,{}^3S_1+\beta_A\,{}^1P_1+\gamma_A\,{}^3D_1]\Phi^{M_{T_A}=0}_{T_A=0}\,, \tag{A1}$$

$$|B\rangle=\Psi_{^6\mathrm{He}}(0^+,T_B=1)=[\alpha_B\,{}^1S_0+\beta_B\,{}^3P_0]\Phi^{M_{T_B}=-1}_{T_B=1}\,, \tag{A2}$$

where Φ denotes the isospin part of the wave function and the rest stands for the spin-spatial part. The latter has been written in the spectroscopic notation $^{2S+1}L_J$ to denote the various quantum numbers. (α,β,γ) are the configuration mixing coefficients, to be determined from the considerations of the various properties of the state concerned. With these wave functions, the spin-isospin transition density becomes

$$\begin{aligned}\rho^{BA}_{\mu,-1}(\vec{r})&=\langle A\mid\sum_i\delta(\vec{r}-\vec{r}_i)\sigma_\mu(i)\tau_{-1}(i)\mid B\rangle\\&=2\sum_{\substack{L_iS_i\\L_fS_f\\M_{L_i}M_{S_i}\\M_{L_f}M_{S_f}}}C_{L_fS_f}C_{L_iS_i}(L_iS_iM_{L_i}M_{S_i}\mid 1M_i)(L_fS_fM_{L_f}M_{S_f}\mid 00)\\&\quad\times\langle\Psi^{M_{L_f}M_{S_f}}_{L_fS_f}(1,2)\mid\sigma_\mu(1)\mid\Psi^{M_{L_i}M_{S_i}}_{L_iS_i}(1,2)\rangle\langle\Psi^{M_{T_B}=-1}_{T_B=1}(1,2)\mid\tau_{-1}(1)\mid\Psi^{M_{T_A}=0}_{T_A=0}(1,2)\rangle\,.\end{aligned} \tag{A3}$$

Here, instead of writing the wave functions in detailed form, like Eqs. (A1) and (A2), we have used the compact form. Each LS coupled wave function is denoted by $\Psi^{M_LM_S}_{LS}$, and the corresponding mixing coefficient by C_{LS}.

The isospin part of the matrix element in Eq. (A3) is simple to evaluate. It yields

$$\langle\Psi^{M_{T_B}}_{T_B}(1,2)\mid\tau_{-1}(1)\mid\Psi^{M_{T_A}}_{T_A}(1,2)\rangle=\sum_\nu\frac{(-1)^{(1/2)-\nu}}{\sqrt{2}}\langle\phi_{-1/2}(1)\phi_{-1/2}(2)\mid\tau_{-1}(1)\mid\phi_\nu(1)\phi_{-\nu}(2)\rangle=1\,. \tag{A4}$$

In general, this matrix element would give a delta function corresponding to isospin transfer (ΔT) equal to one.

The spin part of the matrix element yields

$$\langle \Phi_{S_f}^{M_{S_f}}(1,2)\,|\,\sigma_\mu(1)\,|\,\Phi_{S_i}^{M_{S_i}}(1,2)\rangle = \sum_{\substack{m_{S_1} m_{S_2} \\ m'_{S_1} m'_{S_2}}} (\tfrac{1}{2}\tfrac{1}{2} m'_{S_1} m'_{S_2}\,|\,S_f M_{S_f})(\tfrac{1}{2}\tfrac{1}{2} m_{S_1} m_{S_2}\,|\,S_i M_{S_i})$$

$$\times \langle \phi'_{m_{S_1}}(1)\phi'_{m_{S_2}}(2)\,|\,\sigma_\mu(1)\,|\,\phi_{m_{S_1}}(1)\phi_{m_{S_2}}(2)\rangle$$

$$= (-1)^{S_i - S_f}[6(2S_i+1)]^{1/2} W(S_f \tfrac{1}{2} 1 \tfrac{1}{2}; \tfrac{1}{2} S_i)(S_i \tfrac{1}{2} M_{S_i}\mu\,|\,S_f M_{S_f}) , \tag{A5}$$

where $W(abcd;ef)$ is the Racah coefficient. This expression clearly shows the spin selection rule that the spin transfer (ΔS) in the reaction under consideration, like isospin transfer, is restricted to unity.

After integrating over the particle numbered "2," the spatial part of Eq. (A3) yields

$$\int d\vec{r}_2 \Phi_{L_f}^{M^*_{L_f}}(1,2)\Phi_{L_i}^{M_{L_i}}(1,2) = \sum_{m_1 m_2 m'_1 m'_2} (llm'_1 m'_2\,|\,L_f M_{L_f})(llm_1 m_2\,|\,L_i M_{L_i})\phi^*_{lm'_1}(1)\phi_{lm_1}(1)\delta m'_2 m_2 , \tag{A6}$$

where l is the single particle orbital angular momentum. In the present case of ^{6}Li$\rightarrow^6$He (g.s.) it corresponds to a $1p$ orbit. Combining the angular part of the wave functions in Eq. (A6) we finally obtain

$$\int d\vec{r}_2 \Phi_{L_f}^{M^*_{L_f}}(1,2)\Phi_{L_i}^{M_{L_i}}(1,2) = (-1)^{l+M_{L_i}+M_{L_f}}(2l+1)[(2L_i+1)/4\pi]^{1/2}$$

$$\times \sum_L W(L_f l L l; l L_i)(L_i L - M_{L_i} M_L\,|\,L_f - M_{L_f})(ll00\,|\,L0)\rho_l(r_1)Y_{LM_L}(\hat{r}_1) . \tag{A7}$$

Here, $\rho_l(r)$ is the radial transition density and is normalized to unity.

Combining Eqs. (A4)–(A7) and setting $l=1$, we finally get the following expression for the spin-isospin transition density:

$$\rho^{BA}_{\mu,-1}(\vec{r}) = (-1)^{M_i+1}6(6/4\pi)^{1/2} \sum_{\substack{L_i S_i \\ L_f = S_f \\ L=0,2}} C_{L_i S_i} C_{L_f S_f}(2S_f+1)[(2S_i+1)(2L_i+1)/(2L_f+1)]^{1/2}$$

$$\times W(S_f \tfrac{1}{2} 1 \tfrac{1}{2}; \tfrac{1}{2} S_i) W(L_f 1 L 1; l L_i) W(1 L_i 1 S_f; S_i L)$$

$$\times (1 L M_i - M_L\,|\,1-\mu)(1100\,|\,L0)\rho_l(r)Y_{LM_L}(\hat{r}) . \tag{A8}$$

Considering the appropriate combination of quantum numbers summed in this expression, we get

$$\rho^{BA}_{\mu,-1}(\vec{r}) = \rho^{L=0}_{\mu,-1}(\vec{r}) + \rho^{L=2}_{\mu,-1}(\vec{r}) , \tag{A9}$$

where L, as seen in Eq. (26), is the transferred orbital angular momentum. The expressions for ρ are the following:

$$\begin{aligned} \rho^{L=0}_{\mu,-1}(\vec{r}) &= C_0 \delta_{M_i,-\mu}(4\pi)^{-1/2}\rho_l(r)Y_{00}(\hat{r}) , \\ \rho^{L=2}_{\mu,-1}(\vec{r}) &= C_2(12 M_i - M_L\,|\,1-\mu)(4\pi)^{-1/2}\rho_l(r)Y_{2M_L}(\hat{r}) . \end{aligned} \tag{A10}$$

Here, C_0 and C_2 are the weights for $L=0$ and $L=2$ transferred orbital angular momenta, respectively. In terms of the configuration mixing coefficients of Eqs. (A1) and (A2), they are given by

$$\begin{aligned} C_0 &= 2[-C_i(^3S_1)C_f(^1S_0) + 3^{-1/2}C_i(^1P_1)C_f(^3P_0)] \\ &= 2(-\alpha_A\alpha_B + 3^{-1/2}\beta_A\beta_B) , \end{aligned} \tag{A11}$$

$$\begin{aligned} C_2 &= 4(10)^{-1/2}C_i(^3D_1)C_f(^1S_0) + 2(6)^{-1/2}C_i(^1P_1)C_f(^3P_0) + 3(5)^{-1/2}C_i(^3D_1)C_f(^3P_0) \\ &= 4(10)^{-1/2}\gamma_A\alpha_B + 2(6)^{-1/2}\beta_A\beta_B + 3(5)^{-1/2}\gamma_A\beta_B . \end{aligned} \tag{A12}$$

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