PHYSICAL REVIEW C VOLUME 29, NUMBER 4 APRIL 1984

Electroexcitation of levels in ^{14}N between 12 and 21 MeV

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(Received 27 June 1983)

Four $M4$ transitions based on the stretched particle-hole excitation $(1d_{5/2}1p_{3/2}^{-1})_{4^-}$ have been observed in ^{14}N by inelastic electron scattering, and their form factors have been measured over the range $q=0.8$–2.8 fm^{-1}. The states are at 15.01 ($3^-,4^-$), 16.91 (5^-), 18.48 (5^-), and 20.11 MeV ($3^-,4^-$) and all have $T\approx1$. The 16.91 and 18.48 MeV states together exhaust 61–84 % of the possible $M4$ strength for $J^\pi=5^-$, the higher percentage obtaining for a phenomenological ground-state wave function which fits the static electromagnetic moments. The 15.01 and 20.11 MeV transitions exhaust over 60% of the $M4$ strength allowed for $J^\pi=3^-$, but the spins are still uncertain. Combined with recent ^{14}N(π,π') results, a triplet of 5^- states with $T=0,1,0$ is apparent at 14.66, 16.91, and 17.46 MeV and a three-state isospin-mixing scheme is invoked to describe the π^+/π^- asymmetries. Using the (e,e′) data as a calibration, and assuming a simplified ground-state wave function, the joint analysis indicates the 5^- triplet exhausts roughly 60% of the isovector 5^- strength but only about 35% of the isoscalar strength. Significant $C3$ strength is found in a triplet of levels at 12.82 (4^-), 11.24 (3^-), and 13.17 MeV (2^-?). The form factors are interpreted in terms of a weak coupling of the valence nucleons to a $K=3$ rotational excitation of a deformed core. Finally, an isovector $M2$ transition is found at 14.72 MeV (2^-?) and is analyzed in terms of a phenomenological shell-model configuration.

I. INTRODUCTION

So-called "stretched" nuclear states of unnatural parity offer great simplicity in their wave functions, making them good candidates for reaction studies where the structural details are presumed known. States of this type consist, to lowest order in $\hbar\omega$, of a single particle-hole configuration $(l'_{j'}l_j^{-1})_J$ where j', j, and J are the maximum values consistent with the orbital angular momenta l and l'. As a corollary, the corresponding electron scattering form factor contains no contribution from the convection current and is driven by a single spin (i.e., magnetization) current term. The transition density may therefore be deduced from the experimental form factor in a relatively model-independent manner and applied to other reactions which proceed by one-body operators from the ground state. The point to be emphasized is that this density carries all the required nuclear structure information.

The cross sections for excitation of a given stretched configuration by inelastic electron, proton, and pion scattering all depend on the same transition spin density,[1] although the expressions are complicated by distortion and other corrections. This connection between (e,e′) and (p,p′) was first utilized in a systematic study of the known stretched transitions to gain information on the high-momentum behavior of the nucleon-nucleon tensor force.[2] Included were the only known isovector M4 transitions in $1p$-shell nuclei, namely ^{12}C (19.5 MeV) and ^{16}O (18.98 MeV).

The link with (π,π') came following the inelastic pion scattering work of Holtkamp *et al.*[3] on the triplet of 4^- states in ^{16}O at 17.79, 18.98, and 19.80 MeV. From the observed π^+/π^- asymmetries in the cross sections, those authors concluded that the upper and lower $T=0$ states contained $T=1$ admixtures, probably from the middle ($T=1$) 18.98 MeV level, and estimates were made for the admixture amplitudes. Subsequently, these states were also observed in high-resolution electron scattering studies on ^{16}O over an extensive range of momentum transfers.[4] By utilizing the isospin-mixing amplitudes gained from the (π,π') work and the transition density from the 18.98 ($T=1$) form factor, predictions were made for the 17.79 and 19.80 MeV form factors which are in good agreement with the experimental data. Finally, the mixing amplitudes and transition densities were used to compute the corresponding inelastic (p,p′) cross sections, and these too are in reasonable accord with experiment.[4]

As the above example illustrates, a combination of data from various probes can reduce the ambiguity in some aspects of nuclear structure, and perhaps clarify details of the reaction mechanism itself. In keeping within this theme, we have instituted a search for $M4$ strength in ^{14}N using the electron scattering facility at Mainz.

The situation in ^{14}N is more complex than ^{16}O because the ground-state spin is 1^+, and the $(1d_{5/2}1p_{3/2}^{-1})_{4^-}$ strength can now be distributed between 3^-, 4^-, and 5^- states of the nucleus. Nevertheless, *we find a strong M4 transition to a state at 16.91 MeV, apparently 5^-, which exhausts about 59% of the M4 sum rule for $J^\pi=5^-$*. Significant $M4$ strength is also found at 20.1 and 15.0 MeV, probably corresponding to $J^\pi=3^-$ or 4^-. If both happen to be 3^- levels, together they exhaust about 62% of the

$M4$ sum rule for $J^\pi=3^-$. Finally, we have some evidence for a weak $M4$ transition at 18.5 MeV excitation, also reportedly a 5^- state.

The scattering of 162 MeV pions of both charges by ^{14}N has recently been studied by Geesaman *et al.*[5] who find evidence for 5^- states at 14.66, 16.86, and 17.46 MeV. The 16.86 MeV state probably corresponds to our 16.91 MeV level and therefore is predominantly $T=1$, but we see no indication of the other states which suggests they are mostly $T=0$. Thus, a pattern similar to ^{16}O emerges consisting of a triplet of stretched states in the sequence $T=0,1,0$. The analogy goes even further; we present arguments that the 18.5 MeV level has $T=1$ and a complex multiparticle-hole structure reminiscent of the 18.6 MeV ($4^-,T=1$) state of ^{16}O (Ref. 4). We have combined the (e,e′) and (π,π') data on the 5^- triplet in ^{14}N to obtain estimates of the isospin-mixing amplitudes, Coulomb matrix elements, etc., and comparison is made with the corresponding quantities deduced from the 4^- triplet in ^{16}O.

A search was made for 4^- states in ^{14}N which might have substantial $(1d_{5/2}1p_{3/2}^{-1})_{3^-}$ components and would manifest themselves as strong $C3$ transitions. The only reasonable candidate, at 12.82 MeV ($4^-,T=0$?) turns out to be more collective than the simple shell model predicts. We offer an interpretation of the form factor in terms of the weak coupling of valence nucleons to the rotational excitation of a deformed core similar to the 9.64 MeV ($3^-,T=0$) state of ^{12}C.

Finally, we observe a transition at 14.72 MeV excitation that appears to be $M2$ in nature and which we tentatively assign $J^\pi,T=2^-,1$. The form factor was analyzed within the framework of a simple $1p_{3/2}\rightarrow 2s\text{-}1d$ model and the phenomenological amplitudes are compared with those based on realistic particle-hole matrix elements. The structure is roughly analogous to the coupling of passive valence nucleons to the 16.58 MeV ($2^-,T=1$) level in ^{12}C.

II. EXPERIMENTAL DETAILS AND DATA ANALYSIS

This experiment was performed at the electron scattering facility of the Mainz 350 MeV linear accelerator. Details of the energy compression system, dispersion matching system, spectrometer, and beam-charge monitoring are given elsewhere (see, e.g., Ref. 6 and sources quoted therein). All measurements were made in the energy-loss mode permitting on-target currents of 20—30 μA within an energy spread of 0.1—0.2 %. The beam spot was about 8 mm square at the target position.

The targets consisted of cylindrical gas cells at room temperature filled with commerical nitrogen and oriented with their symmetry axes perpendicular to the scattering plane. The natural abundance of ^{15}N is less than 0.4% and presents no problem in the data analysis. Two different targets were used. For the low-energy runs, we utilized a target developed at the Institut für Kernphysik, Darmstadt, and kindly loaned to us. The cell body is 3 cm in diameter, 7 cm long and was machined from a single piece of aluminum alloy AlMgSi 0.5 to a wall thickness of 125 μm. A sectional view of the complete target is given in Ref. 7.

For the high-energy and extreme back-angle runs, a gas target developed at Mainz[8] was used. It consists of a cylinder 6 cm in diameter and 5 cm long made from AlMgSi 1 and machined to a thickness of 250 μm. The ends caps were fastened to the body with epoxy resin.

The Darmstadt and Mainz targets were charged with nitrogen to pressures of 8 and 11 bar, respectively, using the Darmstadt gas handling facility.

Electrons scattered by the target walls were prevented from entering the spectrometer by a collimator system similar to that described in Ref. 8. Straggling in the gas targets and kinematic broadening limited the overall resolution to 0.06—0.09 % as deduced from the elastic peak widths.

The incident beam energies varied from 112 to 300 MeV while the scattering angles ranged from 82° to 148.5°; the corresponding momentum transfers spanned $q=0.8$—2.8 fm^{-1}. The data at some excitation energies are actually a combination of two experiments, including one designed to explore the levels near 10 MeV and whose spectra extend into the region of current interest. The results for the 10 MeV region will be published elsewhere.

Figure 1 shows a spectrum extending to about 27 MeV excitation, obtained at a momentum transfer ($q\sim2.3$ fm^{-1}) where the 16.9 and 20.1 MeV peaks are especially pronounced relative to other transitions. Both are $M4$ in character as argued Sec. III. For future reference we also draw attention to the peaks at 12.82 MeV ($C3$), 14.72 MeV ($M2$), 15.01 MeV ($M4$), and 18.5 MeV ($M4$).

The inelastic spectra were fitted by the method of least squares using the line-shape program IPA, originally developed in Saskatoon and later modified at Mainz. This is a user-interactive code which is constructed on the premise that, in the absence of straggling, the "true" peak area as derived from the experimental area and the radiative correction factor, must be independent of the integra-

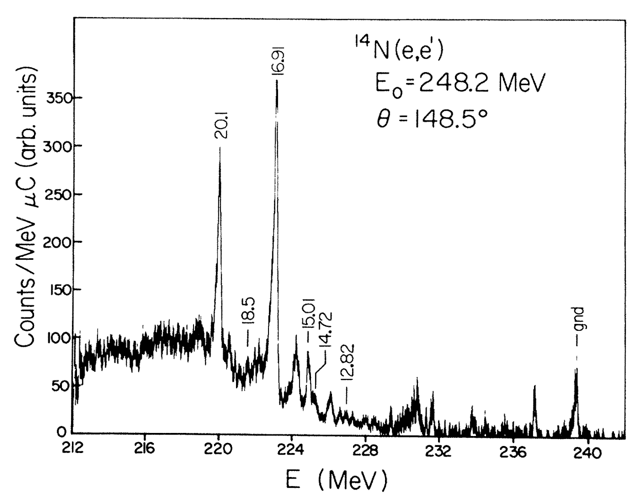

FIG. 1. Spectrum of electrons scattered by ^{14}N, extending from the ground state to about 27 MeV excitation, and indicating the peaks analyzed in the present work. The prominent structures at 16.91 and 20.1 MeV are $M4$ transitions, as are the weaker peaks at 15.01 and 18.5 MeV. The 12.82 and 14.72 MeV excitations are, respectively, $C3$ and $M2$ in character.

tion cutoff. The intrinsic response function of the scattering system is represented by a driving function of Gaussian shape with asymmetry options to accommodate the peculiarities of a given facility, or in the case of broad resonances, a Breit-Wigner or Lorentzian function. A phenomenological term in the code allows for peak distortion from Landau straggling.

The basic shape parameters for each spectrum were determined, when feasible, from the elastic peak. The natural widths of states at high excitation are not negligible, therefore the corresponding width parameters in IPA were allowed to vary. In particular, the widths of very weak peaks were often locked to those of more prominent excitations and varied together, unless the resulting quality of fit dictated otherwise. The actual number of peaks and their approximate location were first established by scanning all the spectra and correlating them with the level scheme compiled by Ajzenberg-Selove.[9] The relative positions of peaks within a grouping were determined and locked together in subsequent fits. In this way a consistent pattern evolved for the spectra analyses. A typical fit in the 16 MeV region is illustrated in Fig. 2.

The excitation energies of the high-lying states as given by the line-shape fits are 14.72 ± 0.03, 15.01 ± 0.03, 16.91 ± 0.02, 18.48 ± 0.04, and 20.11 ± 0.02 MeV. Estimates for the natural widths of the most prominent peaks are

$$\Gamma(15.0)\approx 100 \text{ keV} ,$$

$$\Gamma(16.9)=170\pm20 \text{ keV} ,$$

$$\Gamma(20.1)=120\pm20 \text{ keV} ,$$

if we assume the intrinsic-response width and natural widths combine quadratically.

The peak areas obtained by integrating the line shapes were corrected in the usual manner for radiative and straggling losses and converted to cross sections by means of calibration constants deduced from elastic scattering. At angles where the elastic statistics are good, this is equivalent to normalizing directly against the elastic peak.

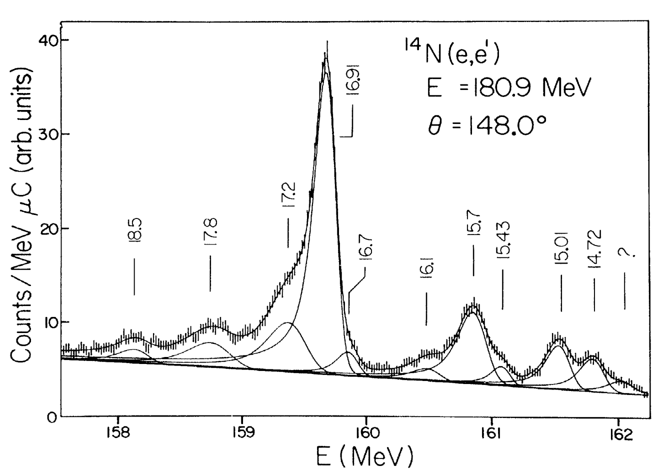

FIG. 2. A typical spectrum fit with the program IPA in the region of the 16.91 MeV (5^-) state. Most of the peaks correlate well with excitation energies tabulated in Ajzenberg-Selove (Ref. 9), although in several cases the J^π are unknown.

For the back-angle inelastic data we relied on elastic calibrations made at the same incident energies but at forward scattering angles. Since elastic and inelastic measurements were made with the same beam intensity, we were not sensitive to local density fluctuations in the target from beam heating. The elastic cross sections were calculated in distorted wave Born approximation (DWBA) with the parameters determined by Lahm[10] from elastic measurements recently made at Mainz.

The differential cross sections for the seven levels to be discussed are presented in Table I. The errors reflect the counting statistics and are obtained from the error matrix in the line-fitting program. Uncertainties in the calibration constants have also been included. The incident energies listed in Table I refer to the centers of the gas targets.

The form factors are shown in Figs. 3—5 plotted as a function of the effective momentum transfer $q_{\rm eff}$, given by

$$q_{\rm eff}=q\left[1+\frac{3\alpha Z}{2E_0R}\right] ,$$

where $R=(5/3)^{1/2}r_{\rm rms}$ is the radius of the equivalent uniformly charged sphere. The longitudinal and transverse form factors, F_L^2 and F_T^2, are related to the differential cross sections in plane wave Born approximation (PWBA) by

$$\frac{d\sigma}{d\Omega}=\sigma_M\left[\frac{q_\mu^4}{q^4}F_L^2+\left[\frac{q_\mu^2}{2q^2}+\tan^2\tfrac{1}{2}\theta\right]F_T^2\right] , \tag{1}$$

where the Mott cross section is

$$\sigma_M=\left[\frac{Ze^2}{2E_0}\right]^2\frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta}\left[1+\frac{2E_0}{Mc^2}\sin^2\tfrac{1}{2}\theta\right]^{-1} .$$

For the kinematic conditions of this experiment $q_\mu^2/q^2\approx1$.

III. INTERPRETATION OF THE RESULTS

A. Identification of the $M4$ transitions

The $M4$ character of the 15.0, 16.9, 18.5, and 20.1 MeV transitions is established by comparing their form factors with a model $M4$ form factor. In the simplest picture the ground state of ^{14}N consists of two nucleons in the $1p_{1/2}$ shell coupled to $J_0^\pi,T_0=1^+,0$, hence $M4$ excitations from the ground state, to lowest order in $\hbar\omega$, are necessarily of the form

$$[(1d_{5/2}1p_{3/2}^{-1})_{4^-,T}\otimes(1p_{1/2}^2)_{1^+,0}]_{J^\pi,T} ,$$

where $T=0$ or 1, and $J^\pi=3^-$, 4^-, or 5^-. Only the spin-dependent operator $\vec{Y}_{\lambda\lambda-1}\cdot\vec{\sigma}$ with $\lambda=4$ contributes to the stretched particle-hole transition, and the form factor may be written

$$F_{M4}^2(q)=(\tfrac{5}{4116})(2J+1)\left[\frac{q\mu_i}{M_{\rm n}}\right]^2$$

$$\times(1d_{5/2}\,|\,j_3(qr)\,|\,1p_{3/2})^2f_{\rm p}^2f_{\rm c.m.}^2 , \tag{2}$$

where $\mu_i=\mu_{\rm p}+\mu_{\rm n}$ for isoscalar transitions, $\mu_i=\mu_{\rm p}-\mu_{\rm n}$ for isovector transitions, $M_{\rm n}$ is the nucleon mass, and the

TABLE I. Differential cross sections for electroexcitation of ^{14}N levels considered in the present work. The cross sections are given in units of (10^n) μb/sr, where n is indicated in brackets. The transition multipolarities for each state were deduced from the form factors as discussed in the text. The J^π assignments for the 11.24, 14.72, 15.01, and 20.1 MeV states are tentative.

E_0 (MeV)	θ (deg)	Excitation energy (MeV) 11.24 (3^-?;$C3$)	12.82 (4^-;$C3$)	14.72 (2^-?;$M2$)	15.01 (3^-,4^-;$M4$)	16.91 (5^-;$M4$)	18.5 (5^-;$M4$)	20.1 (3^-,4^-;$M4$)
90.0	130.0	2.93±0.14(−3)						
112.5	110.0					1.89±0.47(−3)		
114.8	135.3	2.63±0.08(−3)	2.68±0.08(−3)					
124.0	82.0	1.22±0.05(−2)	1.20±0.04(−2)					
134.9	98.1					2.47±0.26(−3)		
	116.1					3.29±0.22(−3)		
135.0	128.1					3.67±0.16(−3)		
174.6	91.8	1.45±0.04(−2)	1.51±0.04(−2)			4.26±0.53(−3)		
	110.1	6.13±0.10(−3)	6.34±0.11(−3)	8.38±0.63(−4)	7.05±0.74(−4)	4.72±0.15(−3)		
	128.1	2.04±0.05(−3)	2.32±0.05(−3)	5.68±0.36(−4)	6.94±0.36(−4)			
217.0	91.9	8.85±0.15(−3)	9.52±0.15(−3)	7.37±0.58(−4)	8.77±0.52(−4)	5.44±0.15(−3)		
	116.2	1.43±0.03(−3)	1.64±0.03(−3)	4.07±0.25(−4)	5.43±0.27(−4)	3.44±0.08(−3)		
	128.1	5.47±0.20(−4)	6.85±0.19(−4)	3.00±0.19(−4)	4.18±0.19(−4)	2.73±0.06(−3)		
114.7	148.0			4.22±1.66(−4)	3.65±2.15(−4)	2.68±0.32(−3)		1.45±0.29(−3)
128.9	148.0			5.18±1.00(−4)	3.95±0.81(−4)	3.90±0.18(−3)		1.55±0.15(−3)
149.9	148.0			6.18±0.43(−4)	5.88±0.50(−4)	4.10±0.19(−3)		1.79±0.09(−3)
180.9	148.0			3.85±0.41(−4)	5.20±0.41(−4)	3.43±0.14(−3)	1.28±0.33(−4)	1.77±0.07(−3)
227.9	148.5		7.01±1.06(−5)	1.33±0.17(−4)	1.93±0.21(−4)	1.47±0.09(−3)	3.73±1.88(−5)	7.51±0.65(−4)
248.2	148.5		2.50±0.51(−5)	7.90±2.17(−5)	1.15±0.17(−4)	8.08±0.42(−4)	3.31±1.04(−5)	4.53±0.53(−4)
269.9	148.5		1.36±0.36(−5)	2.00±0.49(−5)	3.80±0.76(−5)	4.05±0.17(−4)	1.03±0.62(−5)	2.18±0.28(−4)
300.1	148.0					1.20±0.07(−4)	9.00±3.77(−6)	7.62±1.08(−5)

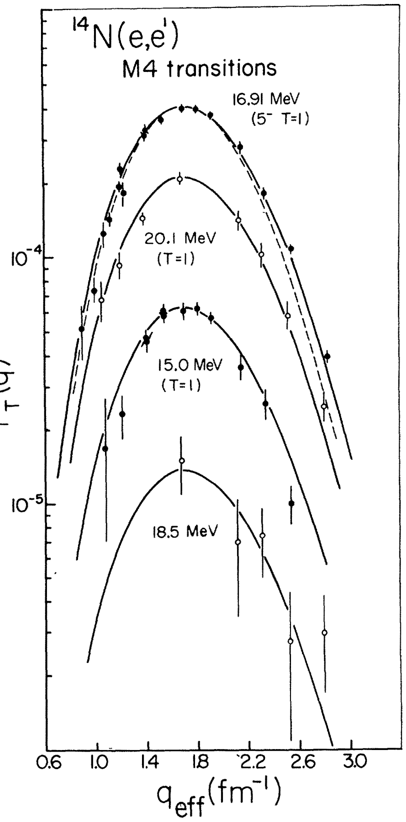

FIG. 3. Transverse form factors identified as $M4$ from their shape. Only the 16.91 MeV (5^-) state has a well-established spin, although there is good evidence the 18.5 MeV state is also 5^-. The 15.0 and 20.1 MeV states are either 3^- or 4^-, with a slight preference for 4^-. The isospin assignments follow from the strengths of the form factors. The 18.5 MeV state is also probably $T=1$ as discussed in the text. The solid curves are $M4$ form factors for a Woods-Saxon potential, normalized to the data. The dashed curve is a harmonic oscillator fit.

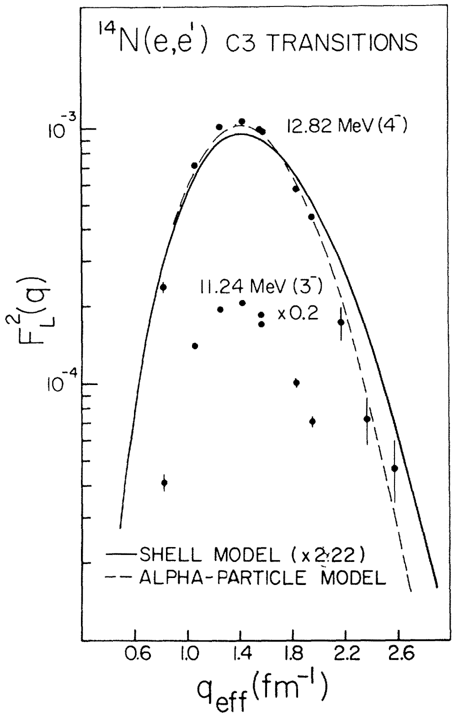

FIG. 4. $C3$ form factors for the 12.82 MeV (4^-) and 11.24 MeV (3^-?) states. The solid curve represents a renormalized fit to the shell model transition $(1d_{5/2}1p_{3/2}^{-1})_{3^-}$ with Woods-Saxon wave functions. The dashed curve is an unrenormalized fit with an alpha-particle model, as discussed in the text. The 11.24 MeV data apparently suffer from a small transverse contamination which has been removed only from the lowest-q point.

radial matrix element is evaluated between the indicated orbitals. The proton form factor f_p is taken from Ref. 11. For the "center-of-mass" form factor $f_{c.m.}$ we adopt the usual harmonic oscillator expression $f_{c.m.} = \exp(q^2b^2/4A)$.

The radial matrix element in Eq. (2) was evaluated using a Woods-Saxon potential whose parameters were determined from the proton binding energies and level schemes of ^{14}N and the adjacent nuclei ^{13}N and ^{15}N. The potential radius and diffuseness parameter were $R=2.95$ fm and $a=0.57$ fm, respectively. Proton binding energies, with the Coulomb interaction included, were taken as $E(1s_{1/2})=42$ MeV, $E(1p_{3/2})=12.5$ MeV, $E(1p_{1/2})=7.6$ MeV, and $E(1d_{5/2})=2.1$ MeV and differ from the values of Gambi *et al.*[12] mainly in the $1p_{3/2}$-$1p_{1/2}$ splitting. We have simulated the spin-orbit interaction by simply changing the depth of the central potential and have constrained the nuclear charge radius to the known value, $r_{rms}=2.56$ fm. The slight difference between proton and neutron wave functions due to the Coulomb interaction was taken into account in a crude fashion by simply averaging the respective $F_{M4}^2(q)$. The oscillator parameter for $f_{c.m.}$ was $b=1.68$ fm, similar to the values used by others[5,22] and consistent with the charge radius.

The model isovector form factor, normalized to each experimental form factor, is illustrated by the solid curves in Fig. 3. With the possible exception of the 18.5 MeV data which are rather sparse, the comparisons clearly indicate the $M4$ nature of each transition, with no suggestion

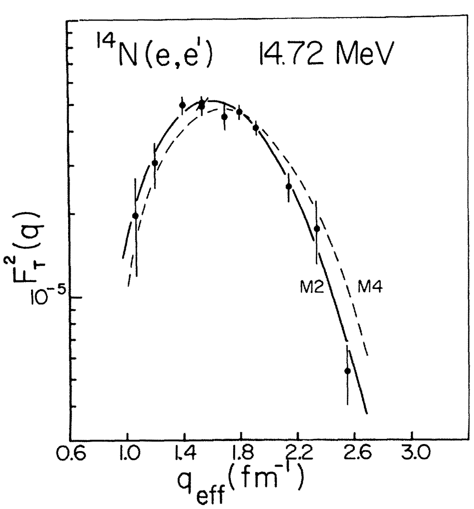

FIG. 5. Transverse form factor for the 14.72 MeV state. The dashed curve is a fit to the data using the same $M4$ shape as in Fig. 3. The solid curve is a fit to the $M2$ form factor within the shell-model basis $(2s\,1d)(1p_{3/2}^{-1})$ using harmonic oscillator functions. The assignment $J^\pi,T=2^-,1$ is favored for this state.

of competing multipolarities. The normalization factors will be discussed in Sec. III B.

The 16.91 MeV data were also fitted using harmonic oscillator wave functions in Eq. (2), treating the normalization and oscillator parameter as variables. (The same parameter b was used in $f_{\text{c.m.}}$.) Since it is generally believed that oscillator wave functions are inadequate at high momentum transfers, the fitting region was arbitrarily restricted to $q<2.2$ fm^{-1}. The result is given by the dashed curve in Fig. 3, corresponding to $b=1.59$ fm and a normalization factor about 9% less than the Woods-Saxon value.

While some discrepancy is apparent at large momentum transfers, it is interesting that the oscillator functions fit as well as they do. Actually, examination of the integrand of the radial matrix elements reveals that even at $q \approx 3$ fm^{-1} the major contribution to the integral comes from $r<4$ fm, and in this region the Woods-Saxon density $R(1d_{5/2})R(1p_{3/2})$ differs from the oscillator density by less than 10%. The difference between the form factors in Fig. 3 beyond the maximum is largely due to subtle cancellations within this region, magnified by the slightly different $f_{\text{c.m.}}$ factors used for each model. The diverging asymptotic behaviors of the Woods-Saxon and oscillator functions is more evident at low momentum transfers, where the transition matrix element becomes roughly proportional to the r^3 moment of the respective densities, hence emphasizing the outer regions.

B. Quenching of the $M4$ strength

A "sum rule" for $M4$ transitions in p-shell nuclei can be constructed that does not depend on the isospin character of the excited states, but does assume the ground states are based on a closed $1p_{3/2}$ shell, at least those parts which can connect to the stretched excited states. We do not pretend the actual ^{14}N ground state is this elementary; the function of the sum rule is simply to provide a convenient calibration for the observed strengths. We first derive the expression, then discuss the excited-state spins, and finally compare with the experimental $M4$ strengths. The total 5^- ($M4$) isovector strength expected for a more realistic ^{14}N ground state is also considered.

The nuclear ground state with spin J_0 and isospin $T_0=0$ will be expressed as

$$\Psi_0(J_0)=A\varphi_0(J_0)+\sum_i B_i\tilde{\varphi}_i(J_0) , \tag{3}$$

where φ_0 represents the closed $p_{3/2}$ shell-model configuration, i.e., $\varphi_0=1p_{1/2}^2\,(1^+)$ in the case of ^{14}N. The second term in Eq. (3) represents possible components in the ground state that, for whatever reason, cannot connect to the excited states by one-body operators.

The excited states of spin J and of generally mixed isospin are expanded as

$$\Psi_n(J)=\alpha_n[(\lambda;1)_{\text{ph}}\otimes\varphi_0(J_0)]_J + \beta_n[(\lambda;0)_{\text{ph}}\otimes\varphi_0(J_0)]_J+\sum_k \gamma_{nk}(\text{complex})_J , \tag{4}$$

where α_n and β_n are, respectively, the $T=1$ and $T=0$ amplitudes, $(\lambda;T)_{\text{ph}}$ is the stretched particle-hole excitation coupled to maximum angular momentum $\lambda=4$,

$$(\lambda;T)_{\text{ph}}=(1d_{5/2}\otimes 1p_{3/2}^{-1})_{4^-,T} ,$$

and the "complex" configurations are any that cannot be accessed from the ground state by one-body operators.

We introduce the form factor $F_{T=1}^2(q)$ for a pure isovector transition between hypothetical states containing only one configuration, that is $A=1$, $\alpha_n=1$ for $n=1$, and all other coefficients in Eqs. (3) and (4) vanish. *Thus $F_{T=1}^2(q)$ saturates the isovector $M4$ strength for a given excited state spin J.* If we define the normalization factor $N_n(J)$ by

$$N_n(J)=\frac{F_n^2(q)}{F_{T=1}^2(q)} , \tag{5}$$

where $F_n^2(q)$ is the $M4$ form factor between the states given by Eqs. (3) and (4), then it follows (see the Appendix)

$$N_n(J)=(A\alpha_n+\mu A\beta_n)^2 , \tag{6}$$

where

$$\mu=\left|\frac{\mu_{\text{p}}+\mu_{\text{n}}}{\mu_{\text{p}}-\mu_{\text{n}}}\right|=0.187 . \tag{7}$$

The above derivation relies on the fact that the convection current does not contribute to the excitation of $(\lambda,T)_{\text{ph}}$.

Finally, we sum $N_n(J)$ over the complete set of states n using the orthonormal properties of the amplitudes,

$$\sum_n \alpha_n^2=1, \quad \sum_n \alpha_n\beta_n=0, \text{ etc. } ,$$

and obtain the completeness condition or sum rule

$$\sum_n N_n(J)=(1+\mu^2)\left[1-\sum_i B_i^2\right] . \tag{8}$$

This result is valid insofar as particle-hole excitations higher than $(\lambda;T)_{\text{ph}}$ are not important and the $1p_{3/2}$ shell is closed in the $0\hbar\omega$ part of the ground state.

In the simplest shell model schemes the amplitudes B_i are negligible and Eq. (8) becomes

$$\sum_n N_n(J)\approx 1+\mu^2 . \tag{9}$$

Note that if all the states $\Psi_n(J)$ have been found experimentally and Eq. (9) is not satisfied, one cannot argue that complex configurations in the excited states are responsible for the quenching since in fact the sum rule is independent of the amplitudes γ_{nk}. On the other hand, complex terms in the ground state can suppress the total $M4$ strength, as indicated by Eq. (8).

We wish to apply Eqs. (8) or (9) to the present data but unfortunately there is some ambiguity in the spins J. Therefore, the available information on each state will be reviewed.

(i) 16.91 MeV. This state is seen in the isospin-forbidden reaction ^{12}C(d,α_2)^{10}B(1.74) and the angular distribution suggests $J^\pi=5^-$ (Ref. 13). The recent ^{14}N(π,π') measurements of Geesaman *et al.*[5] also favor this assign-

ment. The magnitude of the 16.91 MeV form factor clearly indicates $T=1$, and if $J^{\pi}=5^{-}$ then the analog state in ^{14}C (14.9 MeV) should not be accessible via a one-body operator from the ^{14}C ground state; indeed, no evidence for the analog is seen in inelastic pion scattering from ^{14}C (Ref. 14). Finally, the ^{14}N shell model calculations of Glaudemans *et al.*[15] place the lowest $5^{-},T=1$ state at 17.2 MeV, close to the observed state. From these considerations we adopt $J^{\pi},T=5^{-},1$ for the 16.91 MeV state.

(ii) 18.5 MeV. This state is also populated by ^{12}C(d,α_2)^{10}B(1.74); actually it is more strongly excited than the 16.91 MeV level, in contrast to the electroexcitation cross sections (Fig. 3). The α_2 angular distribution is consistent with $J^{\pi}=5^{-}$. Clark and Kemper[16] have recently investigated the three-particle transfer reaction ^{11}B(^{6}Li,^{3}H)^{14}N and report a state at 18.40±0.06 MeV whose spin parity could be 3^{-} or 5^{-}. The peak we observe falls at 18.48±0.04 MeV so the two may or may not correspond to the same level. Nevertheless we will assume $J^{\pi},T=5^{-},1$ for the 18.5 MeV state, where the isospin assignment will be argued later.

(iii) 15.0 MeV. The reduced strength of this form factor (Fig. 3) compared to the 16.9 and 20.1 MeV data is characteristic of a $T=0$ state having some $T=1$ admixture, in which case it should be readily excited by inelastic pion scattering. The ^{14}N(π,π') measurements, however, show little, if any, evidence for a level at 15.0 MeV. It seems more reasonable to assume $T\approx1$ since then the (π,π') and (e,e′) cross sections would be suppressed to the same degree, possibly below the level of detectability in the pion spectra.

Of the allowed spin-parities, 3^{-}, 4^{-}, or 5^{-}, we reject the last from theoretical considerations. The lowest $5^{-},T=1$ state in the calculations of Glaudemans *et al.*[15] was previously identified with the 16.91 MeV state and the next theoretical candidate lies near 22 MeV, too high to be associated with the 15.0 MeV state. Of the remaining options, we have a *slight* preference for $J^{\pi}=4^{-}$ for two reasons. First, Glaudemans *et al.* predict the second $4^{-},T=1$ state at 15.5 MeV which is compatible in energy with the observed state. Second, considering the orbital energetics, one might expect a $J^{\pi}=3^{-}$ form factor to have a substantial $M2$ component arising from $1p_{1/2}\rightarrow1d_{5/2}$ transitions, but we see no indication of the characteristic $M2$ q dependence in the present data.

Thus, without ruling out $J^{\pi}=3^{-}$, we tentatively assign $J^{\pi},T=4^{-},1$ to the 15.0 MeV level. [To further confuse the issue, Weller[17] reports a state at 15.0 MeV from the elastic reaction ^{10}B(α,α)^{10}B whose J^{π} could be 2^{-}, 3^{-}, or 4^{-}, with 4^{-} preferred. Since the total and α widths are virtually identical, the isospin must be predominantly $T=0$. However, the total width is 250±20 keV, somewhat larger than the upper limit of ~100 keV found in the present work.]

(iv) 20.1 MeV. A state at 20.1 MeV is excited by inelastic pion scattering,[5] and from the π^{+}/π^{-} cross section ratios it appears the isospin is quite pure, which could explain why it is not seen in the isospin-forbidden reaction ^{12}C(d,α_2)^{10}B(1.74) (Ref. 13). Assuming this state is also the one detected in the present work, the isospin must be $T=1$ from the intensity of the experimental form factor (Fig. 3).

The spin-parity is likely $J^{\pi}=4^{-}$ for reasons similar to those given in (iii). Furthermore, theory predicts $4^{-},T=1$ states at 18.9 and 19.9 MeV and the latter is close to the observed energy.

An unresolved discrepancy exists in the multipole character of the 20.1 MeV transitions as deduced from the electron and pion scattering data. The pion angular distribution has been interpreted as (perhaps) being a mixture of $M4$ and $M2$ contributions, implying $J^{\pi}=3^{-}$, while the form factor is consistent with pure $M4$. The isovector $M2$ form factors for the single-particle transitions $1p_{1/2}\rightarrow 1d_{5/2}$ and $1p_{3/2}\rightarrow1d_{5/2}$ peak at around $q=1$ fm^{-1}, so if these components contribute to the present data, their amplitudes must be very small. Again, without ruling out $J^{\pi}=3^{-}$, we tentatively assign $J^{\pi},T=4^{-},1$ to the 20.1 MeV state.

The normalization factors $N_n(J)$ as defined by Eq. (5), where $F_n^2(q)$ are the experimental form factors and $F_{T=1}^2(q)$ is given by Eq. (2), are presented in Table II for all possible excited-state spins. The extent to which the $M4$ sum rule, Eq. (9), is exhausted by these levels is included and is independent of our assumptions concerning the isospins. From these results we may draw some conclusions and make a few conjectures.

About 59% of the $M4$ strength belonging to the "superstretched" configuration

$$[(1d_{5/2}1p_{3/2}^{-1})_{4^-}\otimes(1p_{1/2}^{2})_{1^+,0}]_{5^-}$$

is concentrated in a single state at 16.91 MeV, and altogether we can account for about 61% of the $J^{\pi}=5^{-}$ sum rule. By way of comparison, the 18.98 MeV ($4^{-},T=1$) level in ^{16}O contains about 47% of the $(1d_{5/2}1p_{3/2}^{-1})_{4^-}$ $M4$ strength, while in total about 55% of the sum rule is accounted for (51% if the 20.5 MeV state is not 4^{-}). These estimates were made using the Woods-Saxon parameters of Gambi *et al.*[12] for ^{16}O, and the MIT electron scattering data.[4] Similar reductions in stretched-transition strengths

TABLE II. (a) Experimental normalization factors defined by Eq. (5), for various excited-state spin assignments. The 16.9 and 18.5 MeV states are probably 5^{-}, while the 15.0 and 20.1 MeV states are more likely 3^{-} or 4^{-}. The errors on the N_n, of purely statistical origin, are 1% (16.91), 3% (20.1), 2% (15.0), and 16% (18.5). (b) Fraction of the stretched-configuration $M4$ sum rule, Eq. (9), exhausted by the observed levels.

(a)	E_x (MeV)	N_n (5^{-})	N_n (4^{-})	N_n (3^{-})
	16.9	0.61	(0.75)	(0.96)
	20.1	(0.31)	0.38	0.49
	15.0	(0.093)	0.11	0.15
	18.5	0.021	(0.025)	(0.032)

(b)	E_x (MeV)	J^{π}	$\sum N_n(J)$	Fraction of $M4$ sum rule
	16.9,18.5	5^{-}	0.63	61%
	15.0,20.1	4^{-}	0.49	47%
		3^{-}	0.64	62%

have been noted all the way up to the lead region.[2]

The 15.0 and 20.1 MeV states of ^{14}N together carry about 47% of the $M4$ sum rule for $J^{\pi}=4^-$, hence the $M4$ quenching seems to be spin dependent. Fragmentation of the 4^- states is expected since other configurations may now contribute to the wave function, for example,

$$[(1d_{5/2}1p_{3/2}^{-1})_{4^-}\otimes(1p_{1/2}^2)_{0^+,1}]_{4^-} .$$

This term should play an important role in any 4^- states of ^{14}C which can be excited by inelastic scattering from the ^{14}C ground state, although the analog term in ^{14}N cannot be reached from the ^{14}N ground state by a one-body operator. Since the 4^- levels of ^{14}C at 11.67 and 17.26 MeV as observed in pion scattering[14] have no visible analogs in the present ^{14}N spectra, and conversely the analogs of the 15.0 and 20.1 MeV states of ^{14}N are not evident in the ^{14}C(π,π') spectra, all these states must be fairly pure with respect to the valence configurations $(1p_{1/2}^2)_{1^+,0}$ and $(1p_{1/2}^2)_{0^+,1}$. Therefore, it is unlikely the 15.0 and 20.1 MeV states are split because of large $(1p_{1/2}^2)_{0^+,1}$ admixtures in the wave functions. Anyway, the presence of such terms or any other complex pieces has no influence on the sum rule.

An interesting situation develops if both the 15.0 and 20.1 MeV states happen to have $J^{\pi}=3^-$ instead of 4^-. As Table II shows, 62% of the corresponding $M4$ sum rule would be exhausted. This is virtually the same as we find for $J^{\pi}=5^-$ and would imply the $M4$ quenching is somehow intrinsic to the stretched particle-hole excitation and not a function of J. Each of these conjectures has important implications for the source of the quenching, but until the spins have been determined, little more can be said.

According to the more general sum rule, Eq. (8), we should expect some quenching due to ground-state components that cannot link to the excited states by one-body operators. A classic example is ^{16}O whose ground state is believed to have large 2p-2h and 4p-4h admixtures. As we have seen, the $M4$ sum rule is only 51—55 % satisfied, and Eq. (8) gives

$$\left[\sum_i B_i^2\right]^{1/2}\approx 0.67-0.70 ,$$

which compares nicely with the value 0.70 deduced from the shell-model wave function of Zuker *et al.*[18] if we ignore possible 3p-3h admixtures in the 4^- states.

Similarly, from the observed 5^- ($M4$) strength in ^{14}N and Eq. (8) we obtain

$$\left[\sum_i B_i^2\right]^{1/2}\approx 0.62 , \qquad (10)$$

however, in this case there is no theoretical or experimental evidence for such large multiparticle-hole admixtures in the ground state. The ^{14}N and ^{14}C ground states in the shell model calculations of Lie,[19] for example, have 2p-4h intensities of only 4%, while the recent ^{12}C(t,p) work by Fortune and Stephans[20] suggests the ^{14}C ground state has a 2p-4h admixture of about 12%, still much less than required by Eq. (10).

Some reduction in the $M4$ strength relative to Eqs. (8) and (9) can be expected when allowance is made for the open $p_{3/2}$ shell terms that, unlike the ^{16}O case, can occur in the ^{14}N ground state in lowest order. We will estimate the effect on isovector transitions to the 5^- states in the absence of isospin mixing.

The general ^{14}N ground state is similar to Eq. (3), except the $0\hbar\omega$ part becomes

$$\varphi_0(1^+)=A_1(p_{1/2}^{-2})+A_2(p_{1/2}^{-1}p_{3/2}^{-1})+A_3(p_{3/2}^{-2}) . \qquad (11)$$

The 5^- ($T=1$) basis includes all 1p-3h terms between the $1d_{5/2}$ orbital and the whole $1p$ shell, plus possible complex terms as in Eq. (4). The normalizations $N_n(J)$ are defined as in Eq. (5), using the same model form factor $F_{T=1}^2(q)$ as before, but now $F_n^2(q)$ incorporates the full ground state. With the help of fractional parentage expansions and the orthonormal properties of the excited state amplitudes, the $M4$ sum rule may be written

$$\sum_n N_n(5^-,T=1)=A_1^2+0.54A_2^2+0.70A_3^2 . \qquad (12)$$

This is equivalent to the isovector part of Eq. (8) in the closed-shell limit $A_2=A_3=0$.

Two sets of amplitudes will be used in the calculation, those based on the Cohen-Kurath matrix elements and designated CK-I,[21] and the phenomenological functions of Ensslin *et al.*[22] They differ markedly in the intensity of the $p_{1/2}^2$ configuration, constituting 87% of the CK-I ground state but only 46% of the Ensslin ground state. Neither one contains any 2p-4h admixtures, i.e., $B_i=0$.

The sum rules Eq. (12) for each of these wave functions, and the degree to which each is saturated by the 16.9 and 18.5 MeV transitions, are

$$\sum_n N_n(5^-,T=1)=0.94 \quad \text{(Cohen-Kurath)}$$

$$\rightarrow 67\% \text{ exhausted}$$

and

$$\sum_n N_n(5^-,T=1)=0.75 \quad \text{(Ensslin)}$$

$$\rightarrow 84\% \text{ exhausted} .$$

Thus we reach about the same conclusion with the CK-I wave function as with the elementary sum rule, Eq. (9), namely that the isovector $M4$ strength is far from being saturated by the 16.9 and 18.5 MeV transitions. On the other hand, we come much closer to exhausting the sum rule calculated with the Ensslin wave function. Although this wave function is consistent with several electromagnetic properties of ^{14}N, it has yet to be explained in terms of the fundamental two-body interaction.

C. Isospin mixing of the 5^- states

Three 5^- states have been identified at 14.66, 16.86, and 17.46 MeV in the pion scattering work of Geesaman *et al.*,[5] and three-state isospin mixing has been proposed to explain the observed π^+/π^- cross section ratios. Such a scheme was earlier used by Holtkamp *et al.*[3] and Barker *et al.*[23] to describe the π^+/π^- ratios for the 4^- triplet in ^{16}O near 19 MeV excitation. In ^{14}N, the mutually comparable (π,π') cross sections contrast sharply with the overwhelming predominance of the 16.91 MeV state in the (e,e′) spectra (we assume the 16.91 and 16.86 MeV excitations are the same state), indicative of the same isospin sequence $T=0,1,0$ as in ^{16}O. In this section we pursue in a qualitative fashion the three-state mixing hypothesis and show that it is in accord with the electron scattering results.

Following Holtkamp *et al.*,[3] the physical 5^- states are expanded in terms of unperturbed eigenstates of isospin as

$$\begin{aligned}\Psi_1(14.66)&=|0\rangle+\epsilon_1|1\rangle+\epsilon_3|0'\rangle\ ,\\ \Psi_2(16.91)&=-\epsilon_1|0\rangle+|1\rangle+\epsilon_2|0'\rangle\ ,\\ \Psi_3(17.46)&=-\epsilon_3|0\rangle-\epsilon_2|1\rangle+|0'\rangle\ ,\end{aligned}\tag{13}$$

to first order in the Coulomb-mixing amplitudes ϵ_i. The eigenstates $|T\rangle$ are built upon the stretched particle-hole configuration and in general also contain complex multiparticle-hole admixtures not accessible from the ground state. We express them as

$$\begin{aligned}|0\rangle&=a\,|\text{1p-3h:0}\rangle+\Sigma a_i\,|\text{complex}\rangle\ ,\\ |1\rangle&=b\,|\text{1p-3h:1}\rangle+\Sigma b_i\,|\text{complex}\rangle\ ,\\ |0'\rangle&=c\,|\text{1p-3h:0}\rangle+\Sigma c_i\,|\text{complex}\rangle\ .\end{aligned}\tag{14}$$

It will be assumed the 1p-3h terms have a common structure differing only in their isospin coupling. Thus, to the extent that Eq. (3) is an adequate representation of the ground state, Eqs. (13) and (14) together are equivalent to the isospin-mixed states defined by Eq. (4). In view of the results of the preceding section, we do not impose the condition $a^2+c^2=1$ or $b^2=1$ on the eigenstates, i.e., we will allow for unobserved or otherwise quenched $M4$ strength. This differs from the approach used by Barker *et al.*[23] in their treatment of ^{16}O. There, the physical states were expanded in terms of pure 1p-1h stretched configurations with $T=0$ and 1, plus a single complex term, consequently only three 4^- states could be accommodated and these were assumed to saturate the $M4$ strength. The ^{16}O(p,p′) cross sections predicted by this model are in good agreement with experiment, but the inelastic electron scattering form factors $F^2_{M4}(q)$ are overestimated by roughly a factor of 2.

As in Refs. 3 and 23, the inelastic pion scattering cross section for a given state is given to sufficient approximation by

$$\sigma_\pi^{\pm}(n)=f^2(\alpha_n\pm c_\pi\beta_n)^2\ ,\tag{15}$$

where α_n and β_n are the excited-state $T=1$ and 0 amplitudes, f^2 is a factor common to those levels based on the *same* 1p-3h stretched configuration under the same pion kinematic conditions, and $c_\pi=2$ if the pion-nucleon interaction is dominated by the (3,3) resonance.

The mixing amplitudes ϵ_i can be deduced from the ratios of the experimental cross sections without depending on an explicit knowledge of c_π. Specifically, from Eqs. (13)–(15) we obtain

$$\epsilon_1^2=\left[\frac{1+\sqrt{R_1}}{1+\sqrt{R_2}}\right]^2\left[\frac{\sigma_\pi^+(1)}{\sigma_\pi^+(2)}\right],$$

$$\epsilon_2^2=\left[\frac{1+\sqrt{R_3}}{1+\sqrt{R_2}}\right]^2\left[\frac{\sigma_\pi^+(3)}{\sigma_\pi^+(2)}\right],$$

$$\epsilon_3^2r_2+\epsilon_1\epsilon_2\epsilon_3(r_3-r_1)+r_2+\epsilon_1^2r_1+\epsilon_2^2r_3=0\ ,$$

where the state labels are as in Eq. (13). In these expressions R_n is the π^-/π^+ ratio for the state n,

$$R_n=\frac{\sigma_\pi^-(n)}{\sigma_\pi^+(n)}$$

and

$$r_n=\frac{1-\sqrt{R_n}}{1+\sqrt{R_n}}\ .$$

The Coulomb matrix elements may in turn be derived from the mixing amplitudes and excitation energies through the relations

$$\begin{aligned}\langle 0|H_c|1\rangle&=\epsilon_1(E_1-E_2)\ ,\\ \langle 0'|H_c|1\rangle&=\epsilon_2(E_2-E_3)\ ,\\ \langle 0'|H_c|0\rangle&=\epsilon_3(E_1-E_3)\ .\end{aligned}$$

It should be noted that the above expressions for the ϵ_i are a bit ambiguous since in principle the roots $\sqrt{R_n}$ may be of either sign. We find that reasonable values for $\langle 0|H_c|1\rangle$ and $\langle 0'|H_c|1\rangle$ are obtained only if the positive root of R_2 and the negative roots of R_1 and R_3 are used.

From the ^{14}N pion scattering data[5] and the above relations one gets

$$\left.\begin{aligned}&\epsilon_1\approx 0.08\quad \langle 0|H_c|1\rangle\approx -170\text{ keV}\\ &\epsilon_2\approx 0.19\quad \langle 0'|H_c|1\rangle\approx -100\text{ keV}\\ &\epsilon_3\approx 0\qquad\ \langle 0|H_c|0'\rangle\approx 0\text{ keV}\ .\end{aligned}\right\}(^{14}\text{N})\tag{16}$$

By way of comparison, the corresponding quantities extracted from the ^{16}O(π,π') measurements are[3]

$$\left.\begin{aligned}&\epsilon_1\approx 0.12\quad \langle 0|H_c|1\rangle\approx -150\text{ keV}\\ &\epsilon_2\approx 0.12\quad \langle 0'|H_c|1\rangle\approx -100\text{ keV}\\ &\epsilon_3\approx -0.01\quad \langle 0|H_c|0'\rangle\approx 17\text{ keV}\ .\end{aligned}\right\}(^{16}\text{O})\tag{17}$$

Although the agreement between the two sets of Coulomb matrix elements seems favorable, it is less impressive when careful consideration is given to the errors. We conservatively estimate the uncertainties in $\langle 0|H_c|1\rangle$ and $\langle 0'|H_c|1\rangle$ in Eq. (16) at about 80% and 35%, respectively, while the best that can be said for $\langle 0|H_c|0'\rangle$ is that it is consistent with zero.

The relative magnitudes of the 1p-3h amplitudes in Eq. (14) are deduced from the pion cross section ratios in a particularly straightforward manner when ϵ_3 is negligible, and because the 16.91 MeV peak is strong in the (e,e′) spectra we choose to normalize the amplitudes against b. For $c_\pi=2$ we obtain

$$a/b=0.53\quad c/b=0.55\ ,\tag{18}$$

surprisingly close to the analogous quantities in ^{16}O as found by Holtkamp *et al*;[3] in their notation $\hat{\alpha}=0.51$ and

$\hat{\gamma}=0.53$, respectively.

At this point we turn to the electron scattering data for information on the 1p-3h amplitude b. The normalization factor for the 16.91 MeV transition, defined by Eq. (5), becomes

$$N_2(5^-)=b^2\left[1+\mu\left[-\epsilon_1\frac{a}{b}+\epsilon_2\frac{c}{b}\right]\right]^2 \tag{19}$$

for the ground state given by Eq. (3) with $B_i=0$. Combining Eqs. (16), (18), (19), and the experimental normalization in Table II, one finds

$$\begin{aligned} a&=0.41\ ,\\ b&=0.77\ ,\\ c&=0.43\ . \end{aligned} \tag{20}$$

Thus, within the triplet of 5^- *states at 14.66, 16.91, and 17.46 MeV we can account for about 60% of the isovector* 5^- *strength but only about 35% of the isoscalar strength.* These should be treated as lower limits since, as we have seen, the simplified ground state tends to overestimate the expected $M4$ strength. At least some of the missing $T=0$ strength may be dispersed at higher energies. For example, four rather broad $J^\pi=5^-$ resonances are reported above 20 MeV excitation[9] and their absence in the present (e,e′) spectra may indicate $T\approx 0$. We observe two narrow states at around 21.1 and 21.4 MeV whose form factors are similar in magnitude to the 18.5 MeV data shown in Fig. 3, and whose shapes are representative of $M4$ transitions, but otherwise nothing is known of their characteristics.

Finally, it is of interest to see how the above amplitudes fare with respect to the 14.66 and 17.46 MeV $(5^-,T\approx 0)$ form factors. Unfortunately neither of these transitions was clearly discernible in the (e,e′) spectra (see Fig. 2) so we are restricted to upper limit considerations. The $M4$ normalization factors which follow from Eqs. (16) and (20) are

$$N_1\ (14.66)=0.018\ ,$$

$$N_3\ (17.46)=0.004\ .$$

Translated into the current electron scattering spectra, N_1 (14.66) would correspond to a peak barely at the threshold of detection (the adjacent 14.72 MeV peak is about five times more intense, making their resolution difficult) while N_3 (17.46) is well below threshold. While this is not a quantitative test, at least both predictions are not inconsistent with experiment. To conclude, the three-state mixing scheme seems to provide a valid description of the 5^- triplet in ^{14}N with implications quite similar to those which derive from a similar analysis of the 4^- triplet in ^{16}O.

D. The 18.5 MeV (5^-) state

This state is weakly excited by electron scattering and would otherwise be of no particular interest here were it not for a few exceptional properties, some of which we have already alluded to. It appears as a narrow resonance in the isospin-forbidden reaction ^{12}C(d,α_2)^{10}B(1.74) with about twice the intensity of the 16.91 MeV (5^-) peak,[13] while in the (e,e′) spectra the latter is the stronger by a factor of 30. [This is a bit misleading since if the differences in the $L=5$ α-penetrability factors are taken into account, the ratio of the (d,α_2) cross sections becomes comparable to the (e,e′) ratio.] Within statistics, the 18.5 MeV state is not seen in inelastic pion scattering from ^{14}N.

The relative magnitudes of the 18.5 and 16.91 MeV $M4$ form factors are very close to what one expects for pure isoscalar and isovector excitations, respectively, but in that event the 18.5 MeV state should be strongly excited by inelastic pion scattering, contrary to experiment.[5] An alternative explanation consistent with the (e,e′) and (π,π') data is that the 18.5 MeV state consists predominantly of 3p-5h and other complex configurations not reached by one-body operators from the ^{14}N ground state, therefore both reactions would be suppressed. The small 1p-3h admixture must then be mostly $T=1$ to account for the observed (e,e′) strength, and accordingly we find for the stretched particle-hole amplitude $\alpha(T=1)\approx 0.14$.

Some support for this interpretation comes from the three-particle transfer reactions ^{11}B(^{6}Li,^{3}He)^{14}C and ^{11}B(^{6}Li,^{3}H)^{14}N, recently studied by Clark and Kemper.[16] If one views ^{11}B as a five-hole system, these reactions should preferentially populate 3p-5h states in the daughter nuclei. Indeed, a state is observed in ^{14}N at 18.4 MeV, less strongly excited than the 16.91 MeV state, which we are tempted to identify with the 18.5 MeV level of the present work. On the other hand, the analog peak in the ^{11}B(^{6}Li,^{3}He)^{14}C spectra at 16.4 MeV is very strong, in fact it is the most prominent feature in the data. It is interesting that no evidence for a state at this energy occurs in the inelastic pion scattering work on ^{14}C (Ref. 14). This does not necessarily support the 3p-5h model though, since even the 1p-3h configuration

$$[(1d_{5/2}1p_{3/2}^{-1})_{4^-}\otimes(1p_{1/2}^{2})_{1^+}]_{5\_}$$

cannot connect to the ^{14}C ground state.

The much greater strength of the 16.4 MeV peak in ^{14}C compared to the analog in ^{14}N at 18.4 MeV seems at first glance to preclude a common multiparticle-hole structure for these states. Such an asymmetry might obtain, however, if we allow for isospin mixing in the ^{14}N state. The (^{6}Li,^{3}He) reaction can only populate $T=1$ levels in ^{14}C while the (^{6}Li,^{3}H) reaction can excite both $T=0$ and $T=1$ levels in ^{14}N, where the incident-channel isospins of ^{11}B and the transferred ^{3}He occur with opposite phases, i.e.,

$$|\tfrac{1}{2},-\tfrac{1}{2}\rangle\otimes|\tfrac{1}{2},\tfrac{1}{2}\rangle=\frac{1}{\sqrt{2}}(|1,0\rangle-|0,0\rangle)\ .$$

Thus, if the 3p-5h components of the 18.4 MeV state are strongly isospin mixed, a partial cancellation may occur between the isoscalar and isovector matrix elements yielding a somewhat reduced cross section.

In ^{16}O a 4^- state of complex structure has been detected at 18.6 MeV whose properties were summarized in the review by Bertozzi.[4] It is observed by inelastic electron and proton scattering, by the (^{6}Li,^{3}H) three-particle transfer reaction, but not by inelastic pion scattering, from which it has been concluded that $T=1$ and the 1p-1h component is small. Thus, it seems an overall pattern of similarity exists between the high-lying quartet of 4^- states in ^{16}O and the quartet of 5^- states in ^{14}N.

IV. OTHER TRANSITIONS

A. The 12.82 MeV (4^-) and 11.24 MeV (3^-?) states

The discussion so far has centered on the $M4$ transitions to 5^-, 4^-, or 3^- states presumably based on stretched particle-hole configurations. Of course, the structures of the 4^- and 3^- states are not necessarily unique, for example, one could imagine contributions from the octupole particle-hole excitation

$$[(1d_{5/2}1p_{3/2}^{-1})_{3^-}\otimes(1p_{1/2}^{2})_{1^+}]_J \tag{21}$$

which would manifest itself in the form of a longitudinal $C3$ form factor. Since no appreciable longitudinal strength was detected in the 15—20 MeV region, most of the above octupole strength must lie elsewhere.

Examination of the (e,e′) spectra in the 10—15 MeV region reveals two peaks that carry significant $C3$ strength and we identify these with the states at 12.82 MeV (4^-) and 11.24 (3^-?). The experimental form factors are presented in Fig. 4. A Rosenbluth plot for matching-q points near the peak of the 12.82 MeV form factor indicates a negligible transverse contribution, so these data have been treated as longitudinal. The 11.24 MeV data will also be treated as longitudinal, although a similar Rosenbluth separation at $q=0.8$ fm^{-1} hints at a small transverse contribution.

First, we consider the 12.82 MeV results. The $C3$ form factor based on the shell model [i.e., Eq. (21)] and the Woods-Saxon parameters of Sec. III A, is shown as the solid curve in Fig. 4. To compare with experiment, it was necessary to renormalize the theoretical $F_{C3}^2(q)$ upward by a factor of 2.22, thus implying the 12.82 MeV state is collective to some degree. Assuming the isospin is $T=0$, this enhancement corresponds to the nucleon total effective charges

$$e_{\text{eff}}(\text{p})=1.25\,|\,\text{e}\,|\ ,$$

$$e_{\text{eff}}(\text{n})=0.25\,|\,\text{e}\,|\ .$$

If the basis of Eq. (21) is expanded to include octupole excitations to the $1d_{3/2}$ shell, one gains a little in the collectivity but not sufficiently. Instead, we will reinterpret the ^{14}N state in terms of a weak coupling of the valence nucleons to a $J^\pi=3^-$ excitation of the ^{12}C core. A natural candidate here would be the 9.64 MeV ($3^-,T=0$) level which is also more collective than the simple shell model description.

Many of the levels in ^{12}C are known to be highly deformed and early attempts to unify their description were often based on the α-particle model. According to this model, the 9.64 MeV (3^-) state is a $K=3$ rotational excitation of a triangular arrangment of three α particles, about an axis perpendicular to their plane. In its most primitive form, the clusters are rigidly fixed relative to each other and nucleon antisymmetrization between clusters is ignored. Our purpose for considering this rather naive version is simply to see, in a qualitative fashion, whether the weak coupling picture can be improved with a deformed core excitation.

The $C3$ form factor for ^{12}C is given in the α-particle model by[24]

$$F_{C3}^2(q)\,|_{^{12}\text{C}}=(\tfrac{35}{16})[j_3(x)f_\alpha(q)f_{\text{c.m.}}]^2\ , \tag{22}$$

where

$$x=2qd/\sqrt{3}$$

and $2d$ is the separation between the clusters. In the harmonic oscillator scheme, the individual cluster form factors are

$$f_\alpha(q)=e^{-y}f_{\text{p}}\ ,$$

where $y=(qb/2)^2$ and f_{p} is the proton form factor.[11] The term $f_{\text{c.m.}}=\exp(y/A)$ has been included in Eq. (22) to at least partly correct for the lack of translational invariance of the internal nuclear wave function, although this procedure is strictly valid only at the shell model limit.

The ^{14}N form factor which derives from a weak coupling of the valence particles to the rotational excitation of the α-particle core is

$$F_{C3}^2(q)\,|_{^{14}\text{N}}=(\tfrac{6}{7})^2\frac{2J+1}{21}F_{C3}^2(q)\,|_{^{12}\text{C}}\ , \tag{23}$$

where the possible final state spins are $J=2$, 3, or 4.

The experimental form factors for the 9.64 MeV (^{12}C) and 12.82 MeV (^{14}N) states were fitted by Eqs. (22) and (23), treating b and d as the only free parameters. The ^{12}C data were selected points from Ref. 25 in the range $q=0.9$—2.5 fm^{-1}. The best-fit values are

$$^{12}\text{C}:\ b=1.46\text{ fm},\quad d=1.85\text{ fm}\ ;$$
$$^{14}\text{N}:\ b=1.38\text{ fm},\quad d=1.61\text{ fm}\ . \tag{24}$$

The fit to the ^{14}N data is illustrated by the dashed curve in Fig. 4. We see that the required degree of collectivity is readily provided by the model and the q dependence of the data is closely duplicated, more so than by the shell model. It is encouraging that the oscillator parameter b does not deviate substantially from the free α-particle value $b=1.31$—1.42 (depending on the rms radii one accepts for ^{4}He and the proton). The smaller size of the cluster triangle in ^{14}N as compared to ^{12}C, evident from Eq. (24), could be attributed to the binding effect of the two valence nucleons. The classical moment of inertia of the α core is proportional to d^2, therefore the excitation energy should be larger in ^{14}N than ^{12}C, which is consistent with experiment.

The foregoing phenomenological exercise lends some credence to our hypothesis that the 12.82 MeV (4^-) state is built on a deformed core excitation. The model predicts a triplet of states with $J^\pi=2^-$, 3^-, and 4^- and we now speculate on the other two members. From angular momentum algebra the form factors should be in the ratio

$$\frac{F_{C3}^2(3^-)}{F_{C3}^2(4^-)}=\tfrac{7}{9}\approx 0.8 \tag{25}$$

and

$$\frac{F_{C3}^2(2^-)}{F_{C3}^2(4^-)}=\tfrac{5}{9}\approx 0.6 \ . \tag{26}$$

Possible candidates include the peak at 11.24 MeV, mentioned earlier, and a weaker excitation at around 13.17 MeV.

The narrow state at 11.24 MeV, tentatively assigned $J^\pi=3^-$ in the compilation,[9] has a form factor very similar in shape to the 12.82 MeV $C3$ form factor as shown in Fig. 4 and therefore is probably also $C3$ in nature, although as already noted, there are some transverse contributions. From the measurements we find

$$\frac{F_{C3}^2(11.24;3^-)}{F_{C3}^2(12.82;4^-)}\approx 0.9 \ .$$

The inelastic pion scattering measurements of Geesaman *et al.*[5] give nearly identical $L=3$ angular distributions for these two levels, except for a scale factor. The cross sections are in the ratio

$$\frac{\sigma_\pi^+(11.24;3^-)}{\sigma_\pi^+(12.82;4^-)}\approx 0.8$$

in good agreement with Eq. (25). Thus, the 11.24 MeV state could well be the 3^- member of the multiplet.

The only reasonable candidate for the 2^- state in the (e,e′) spectra is a narrow peak at about 13.17 MeV excitation of unknown spin and parity. The form factor resembles the others in shape although there appears to be a substantial transverse component to the cross section, possibly from unresolved neighboring levels; unfortunately we have only a single pair of matching-q points. Ignoring any transverse contamination, the form factor ratio is

$$\frac{F_{C3}^2(13.17;2^-?)}{F_{C3}^2(12.82;4^-)}\approx 0.3\text{—}0.5 \ ,$$

being somewhat q dependent.

In the pion scattering work a state is reported at 13.14 MeV that probably coincides with the 13.17 MeV peak in the (e,e′) spectra. The pion angular distribution bears a strong resemblance to the $L=3$ distribution of the 12.82 and 11.24 MeV states and further substantiates the $C3$ character of the form factor. The relative cross sections are

$$\frac{\sigma_\pi^+(13.14;2^-?)}{\sigma_\pi^+(12.82;4^-)}\approx 0.4 \ ,$$

which is compatible with the (e,e′) ratio but lower than expected from Eq. (26). Of the J^π values allowed for $C3$ transitions from the 1^+ ground state, a 2^- assignment is certainly the most consistent with these ratios, and it could be argued that the deviation from Eq. (26) is caused by the larger configuration space available to a 2^- state compared to a 4^- state.

As a final remark, we note that the centroid of the ^{14}N multiplet as given by the d parameters in Eq. (24), using the 9.64 MeV (3^-) state of ^{12}C as a calibration, is 12.7 MeV, while the centroid of the candidates we have proposed is 12.4 MeV.

B. The 14.72 MeV state

The (e,e′) spectra reveal a peak at 14.72 ± 0.03 MeV which is partially resolved from the 15.0 MeV peak (see Fig. 6) and whose natural width we estimate to be $\Gamma\approx 100$ keV. The nearest state of known J^π is reported[9] at 14.66 MeV (2^-), although we see little evidence for it in the present work. Furthermore, we see no indication of the 14.66 MeV state found in the pion scattering measurements[5] and identified as the lowest member of the isospin-mixed 5^- triplet. Since the nearest visible candidate for either of these states is the 14.72 MeV peak, we will consider whether the experimental form factor is consistent with either a high-multipolarity ($M4$) transition or a $J^\pi=2^-$ level.

The 14.72 MeV form factor (Fig. 5 and Table I) shares two features with the $M4$ transitions: There is no diffraction minimum, and fixed-q measurements at different angles establish that it is transverse in nature. However, comparison with the 15.01 MeV $M4$ form factor seems to indicate a different multipolarity is involved. In the raw spectra shown in Fig. 6 the relative strengths of the two peaks are clearly changing with momentum transfer, in

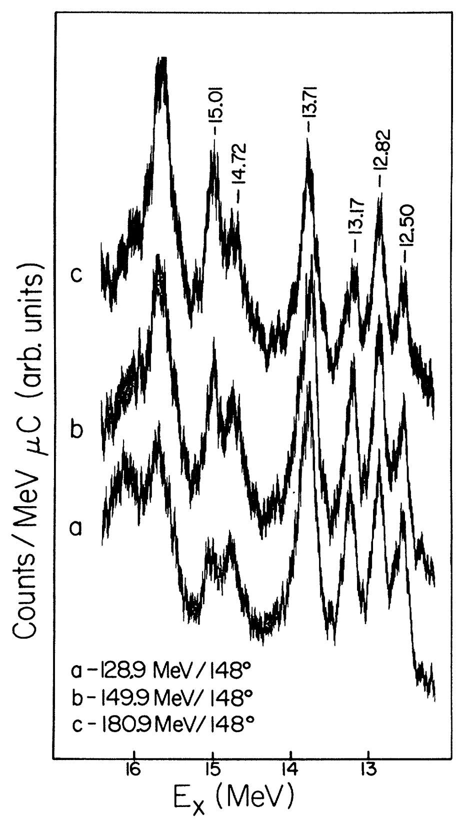

FIG. 6. Several spectra in the region of the 14.72 and 15.01 MeV peaks, showing a variation in their relative strengths with increasing momentum transfer. The 14.72 MeV transition is therefore distinct from the 14.66 MeV $M4$ transition observed in ^{14}N(π,π').

contrast to the behavior expected if both excitations were pure $M4$. Figure 5 shows a fit to the form factor using the $M4$ shape described earlier, and a systematic trend in the ratio fit/data with increasing q is quite apparent.

A more consistent picture emerges if, as we now argue, the 14.72 MeV form factor represents a pure $M2$ transition to a 1^-, 2^-, or 3^- $T=1$ state. For the present we will assume $J^\pi=2^-$. To ensure a unique transition multipolarity, the wave function is restricted to a linear combination of the terms

$$[(l_j 1p_{3/2}^{-1})_{2^-} \otimes (1p_{1/2}^2)_{1^+}]_{2^-} ,$$

where l_j represents the complete $2s$-$1d$ shell, and the two valence p-shell nucleons are assumed to be passive. Thus, the particle-hole excitation occupies the same basis as the 16.58 MeV $(2^-,T=1)$ state of ^{12}C to which we will refer shortly. The $M2$ form factor was evaluated using harmonic oscillator wave functions in which the length parameter, the three configuration amplitudes, as well as an overall normalization factor for $F_{M2}^2(q)$ were determined by a least-squares fit to the data. As expected, the normalization was somewhat greater than unity when the excited state was assumed to be $T=0$, so subsequent analyses were made for $T=1$. The optimum chi-square ($x_\nu^2 \approx 0.5$ per degree of freedom) achieved with this model represents a significant improvement over the previous $M4$ fit, but the freedom is too great to permit a unique determination of the amplitudes and normalization.

At this point we turn for guidance to the 16.58 MeV $(2^-,T=1)$ state of ^{12}C. Donnelly[26] has calculated the amplitudes for this level within the particle-hole basis $(2s\,1d)(1p_{3/2}^{-1})$, however, the resulting form factor $F_{M2}^2(q)$ must be renormalized by about 0.4 to give agreement with the recent measurements of Deutschmann.[27] If we assume the same factor applies to the hypothetical $M2$ form factor in ^{14}N, our fitting procedure yields the phenomenological amplitudes listed in Table III and the solid curve in Fig. 5. As it happens, the resulting $(1d_{5/2}1p_{3/2}^{-1})$ amplitude is nearly independent of the actual normalization chosen, while the $(1d_{3/2}1p_{3/2}^{-1})$ amplitude is quite sensitive. The oscillator length parameter yielding the best chi-square is $b=1.66$ fm^{-1}, close to the value $b=1.68$ fm^{-1} deduced from the ground-state rms radius (Sec. III A).

It is instructive to compare with the ^{12}C wave function calculated by Donnelly, also included in Table III. The relative phases of the large amplitudes are the same, and the increase in the $(1d_{5/2}1p_{3/2}^{-1})$ strength in ^{14}N relative to ^{12}C seems to be consistent with a reduction in the $1d_{5/2}$-$2s_{1/2}$ splitting as one moves through this region of the p shell. Also included in Table III are wave functions for ^{12}C and ^{14}N derived from a revised set of the Kuo-Brown two-body matrix elements. For ^{12}C the single-particle energies employed by Donnelly[26] were used, while for ^{14}N unperturbed particle-hole energies relative to the $(2s_{1/2}1p_{3/2}^{-1})$ state were taken to be

$$E_0(1d_{5/2}1p_{3/2}^{-1}) = -0.03 \text{ MeV} ,$$

$$E_0(1d_{3/2}1p_{3/2}^{-1}) = 4.77 \text{ MeV} ,$$

from considerations of the level scheme of ^{15}N. Aside from the $(1d_{3/2}1p_{3/2}^{-1})$ amplitude, which in any case is rather uncertain, the phenomenological ^{14}N results are in reasonable accord with the theoretical values and lend support to our model. Roughly speaking, the 14.72 MeV state is analogous to the coupling of two valence nucleons to the 16.58 MeV $(2^-,T=1)$ state of ^{12}C.

Although we have assumed $J^\pi=2^-$ for the 14.72 MeV state, the present model also accommodates pure $M2$ transitions to hypothetical 1^- or 3^- states, and aside from different overall normalizations, the form factors are identical. Our preference for the 2^- assignment stems from consideration of the phenomenological $(1d_{3/2}1p_{3/2}^{-1})$ amplitude if $J^\pi=1^-$ or 3^-. The least-squares fits to the 14.72 MeV data with these form factors, renormalized by 0.4 as before, give -0.31 $(J^\pi=1^-)$ and 0.34 $(J^\pi=3^-)$ for the amplitude, both values being much larger than can be accounted for in the present scheme.

We now return to the question of whether the level we observe at 14.72 MeV could in fact be the 14.66 MeV (2^-) state mentioned at the beginning of this section. Although the spins are consistent, the 14.66 MeV state is strongly excited by the elastic reaction $^{10}B(\alpha,\alpha)^{10}B$ (Ref. 28) and the total width seems to consist almost entirely of the α-decay width, suggesting $T=0$. On the other hand, the present phenomenological analysis of the 14.72 MeV form factor favors $T=1$. Thus, unless there is considerable isospin mixing, it is unlikely that these states are one and the same.

TABLE III. Particle-hole amplitudes for the 14.72 MeV $(2^-,T=1)$ state of ^{14}N. The phenomenological amplitudes and uncertainties are based on a fit to the data using a fixed normalization factor of 0.40 for $F_{M2}^2(q)$. For comparison, the amplitudes for the 16.58 MeV $(2^-,T=1)$ state of ^{12}C are included.

$(2s_{1/2}1p_{3/2}^{-1})$	$(1d_{5/2}1p_{3/2}^{-1})$	$(1d_{3/2}1p_{3/2}^{-1})$
	^{14}N	
0.80±0.03	−0.58±0.04	0.18±0.03[a]
0.84	−0.54	0.04[b]
	^{12}C	
0.94	−0.34	−0.02[c]
0.92	−0.39	0.02[d]

[a]Phenomenological (present work).
[b]Theoretical (present work).
[c]Theoretical (Donnelly, Ref. 26).
[d]Theoretical (present work).

V. CONCLUSION

Four $M4$ transitions based on the stretched particle-hole excitation $(1d_{5/2}1p_{3/2}^{-1})_{4^-}$ have been observed in the 15—20 MeV region of ^{14}N by inelastic electron scattering. They correspond to states at 15.0 MeV $(3^-,4^-)$, 16.91 MeV (5^-), 18.5 MeV (5^-), and 20.1 MeV $(3^-,4^-)$, and all have isospin $T\approx 1$. No other significant concentration of isovector $M4$ strength is seen up to $\sim$28 MeV excitation.

Using a hypothetical isovector $M4$ transition between a closed $1p_{3/2}$ shell and the $1d_{5/2}$ shell as a reference, we account for 61% of the $M4$ strength for 5^- states, independent of possible isospin mixing, with the majority concentrated in the 16.91 MeV state. The 15.0 and 20.1 MeV transitions exhaust 62% of the $M4$ sum rule for $J^{\pi}=3^{-}$, comparable to our finding for the 5^{-} states, but the spin assignments for these states are still uncertain.

These results are modified somewhat when more realistic ground state wave functions are employed in the sum rule. Thus, we find 67% of the isovector $M4$ strength for 5^{-} states is exhausted when a Cohen-Kurath wave function is used, and this increases to 84% with the phenomenological ground state of Ensslin *et al.* The phenomenological amplitudes have proved successful in other areas, for example, Singham and Tabakin[29] showed that they removed the discrepancy between theory and experiment for the reaction $^{14}\mathrm{N}(\gamma,\pi^{-})^{14}\mathrm{O}_{\mathrm{g.s.}}$, which exists for other wave functions including CK. Nevertheless, considering the history of stretched transitions in other nuclei, we remain skeptical that nearly all the theoretical 5^{-} isovector strength in ^{14}N is accounted for by the 16.91 MeV level.

Besides the 16.91 MeV state, two other 5^{-} states at 14.66 and 17.46 MeV are strongly excited by inelastic pion scattering, and from their absence in the (e,e′) spectra it is safe to conclude that $T\approx 0$. The π^{-}/π^{+} asymmetries suggest some isospin mixing is occurring in the triplet, and by invoking a three-state mixing scheme we arrive at a set of Coulomb matrix elements consistent with those derived from (π,π') measurements on the triplet of 4^{-} levels in ^{16}O.

From a joint analysis of the (e,e′) and (π,π') data, we conclude the 5^{-} triplet accounts for roughly 60% of the possible isovector 5^{-} strength in ^{14}N but only about 35% of the isoscalar strength. This is in marked contrast with the findings of Geesaman *et al.*[5] who claim their (π,π') cross sections virtually exhaust all the 5^{-} strength predicted with Cohen-Kurath and related wave functions. It is possible to increase the ratios in Eq. (18) so that the isoscalar strength becomes comparable to the isovector, by decreasing the value of c_{π} from 2 as used here to 1.4 as advocated by Barker *et al.*[23] This would mean the pion-nucleus interaction at 162 MeV is not completely dominated by the $\Delta(1232)$ resonance, but also has some $T=\frac{1}{2}$ contribution. However, to approach saturation of the isovector 5^{-} strength would require a severe reduction in the magnitude of our reference form factor $F_{T=1}^{2}(q)$, and this is not accomplished by using the CK ground state wave function. Since it is unlikely some mechanism is quenching the electromagnetic strength without influencing the inelastic pion scattering cross sections, we cannot explain the apparent discrepancy with Geesaman *et al.*

The (e,e′) and (π,π') experiments together detect four 5^{-} states (14.66, 16.91, 17.46, and 18.5 MeV), the first three are in the sequence $T=0,1,0$ and the final one, apparently $T=1$, contains large multiparticle-hole admixtures. This is very similar to the pattern of stretched 4^{-} states in ^{16}O, where the $T=0,1,0$ triplet occurs at 17.79, 18.98, and 19.80 MeV, and the complex $T=1$ level lies at 18.6 MeV.

We observe a concentration of longitudinal $C3$ strength in the triplet of levels at 12.82 MeV (4^{-}), 11.24 MeV (3^{-}), and 13.17 MeV (2^{-}?), whose form factors are in the same ratio as the corresponding (π,π') cross sections. The form factors were interpreted in terms of a weak coupling of the $(1p_{1/2}^{2})_{1^{+}}$ valence nucleons to the $K=3$ rotational excitation of a deformed core. The latter was based on a simplified version of the triangular α-particle model used to describe the 9.64 MeV (3^{-}) level of ^{12}C. The shape of the $C3$ form factor for the 12.82 MeV level and the collectivity are reproduced, and the predicted excitation energy of the triplet centroid is close to the experimental value.

Finally, we observe an $M2$ transition at 14.72 MeV, close in energy to the 5^{-} level detected in the pion scattering work. When analyzed within the same basis as the 16.58 MeV ($2^{-},T=1$) level of ^{12}C, and with the same quenching factor, we derive a set of phenomenological amplitudes that are very similar to those generated from two-body matrix elements. We tentatively assign $J^{\pi},T=2^{-},1$ to this state.

ACKNOWLEDGMENTS

The experimental aspect of this work was completed while one of us (J.C.B.) was an Alexander-von-Humboldt Fellow (1981) at the University of Mainz, and the assistance of the AvH is gratefully acknowledged. We wish to thank P.W.M. Glaudemans, K. W. Kemper, and D. F. Geesaman for communicating results of their works to us prior to publication. Helpful discussions with B. Zeidman and H. Weller are appreciated. We are grateful to D. Bender for preparing the gas targets.

APPENDIX

Here we give a brief derivation of the normalization factor $N_n(J)$ as expressed by Eq. (6). The underlying assumptions have been previously described in Sec. III B.

The form factor for the magnetic λ-pole transition to the stretched state n is defined by

$$F_n^2(q)=\frac{4\pi}{Z^2}\frac{1}{2J_0+1}\,|\langle J||Q_\lambda||J_0\rangle|^2\,, \tag{A1}$$

where

$$Q_\lambda=\frac{iq}{M_{\mathrm{n}}}\left[\frac{\lambda+1}{2\lambda+1}\right]^{1/2}\frac{\mu_i}{2}j_{\lambda-1}(qr)\vec{\mathrm{Y}}_{\lambda\lambda-1}\cdot\vec{\sigma} \tag{A2}$$

and

$$\mu_i=\left[\frac{\mu_{\mathrm{p}}+\mu_{\mathrm{n}}}{2}\right]+\left[\frac{\mu_{\mathrm{p}}-\mu_{\mathrm{n}}}{2}\right]\tau_3\,, \tag{A3}$$

the other symbols having their usual meaning. The convection current and $\vec{\mathrm{Y}}_{\lambda\lambda+1}\cdot\vec{\sigma}$ terms, present in the general form of Q_λ, do not contribute for reasons of angular momentum. Inserting the initial and final states defined by Eqs. (3) and (4), and using the fact that the angular momentum coupling is the same for both the $T=0$ and $T=1$ excited-state components, one obtains

$$F_n^2(q)=f(J_0,J,\lambda)[\alpha_n A\langle(\lambda;1)_{\mathrm{ph}}||Q_\lambda||0\rangle+\beta_n A\langle(\lambda;0)_{\mathrm{ph}}||Q_\lambda||0\rangle]^2\,, \tag{A4}$$

where $f(J_0,J,\lambda)$ contains the coefficients from the angular momentum algebra and other common factors. Next, we apply the Wigner-Eckart theorem to the isospin dependence and write

$$\langle(\lambda;T)_{\rm ph}||Q_\lambda||0\rangle=(-1)^T\begin{bmatrix}T & T & 0\\ 0 & 0 & 0\end{bmatrix}\times\langle(\lambda;T)_{\rm ph}|||Q_\lambda|||0\rangle\ . \quad \text{(A5)}$$

The particle-hole excitation $(\lambda;T)_{\rm ph}$ has the same orbital structure for both isospins, so it follows from Eqs. (A2) and (A3) that

$$\frac{\langle(\lambda;0)_{\rm ph}|||Q_\lambda|||0\rangle}{\langle(\lambda;1)_{\rm ph}|||Q_\lambda|||0\rangle}=\frac{\langle\frac{1}{2}||\frac{1}{2}||\frac{1}{2}\rangle}{\langle\frac{1}{2}||\frac{\tau_3}{2}||\frac{1}{2}\rangle}\left[\frac{u_{\rm p}+\mu_{\rm n}}{\mu_{\rm p}-\mu_{\rm n}}\right]=\frac{1}{\sqrt{3}}\left[\frac{\mu_{\rm p}+\mu_{\rm n}}{\mu_{\rm p}-\mu_{\rm n}}\right]\ . \quad \text{(A6)}$$

Finally, we introduce the hypothetical initial and final states with amplitudes $A=1$ $(T=0)$ and $\alpha_1=1$ $(T=1)$, connected by the form factor $F_{T=1}^2(q)$, and from Eqs. (A4)—(A6) obtain

$$N_n(J)\equiv\frac{F_n^2(q)}{F_{T=1}^2(q)}=\left[A\alpha_n+A\beta_n\left[\frac{\mu_{\rm p}+\mu_{\rm n}}{\mu_{\rm p}-\mu_{\rm n}}\right]\right]^2\ . \quad \text{(A7)}$$

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