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Reply to "Comment on enhancement of forbidden nuclear beta decay by high-intensity radio-frequency fields"

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The negative conclusion in the Comment of Becker, Schlicher, and Scully about electromagnetic enhancement of beta decay is shown to be faulty. They have found an algebraic oversight in my paper, but correction of that oversight yields results strongly resembling the original. Becker, Schlicher, and Scully fail to find this. They then conduct an analysis which is highly implicit and incomplete. In attempting to analyze their very complicated expressions they claim not to find significant electromagnetic effects. Yet they also lose completely the electron retardation term of conventional forbidden beta decay. When they attempt to explain the difference between their results and mine, they misconstrue the momentum-translation technique and end up in a logical contradiction. They attempt also to apply a "no-go" theorem applicable only to plane-wave particles to my theory, which is built around the use of bound-state nuclear wave functions. This makes the no-go theorem inapplicable.

I. INTRODUCTION

Becker, Schlicher, and Scully[1] (hereafter referred to as BSS) dispute the recent analysis of electromagnetic enhancement of nuclear beta decay[2] (hereafter referred to as I), and claim that the effect is nonexistent. In their examination of I, BSS have found that a factor of i was omitted in the generating function of the Bessel function expansion. Insertion of the missing i does change the explicit analytical form of the results, but certainly does not make the result vanish. Predicted transition probabilities for electromagnetically enhanced beta decay remain very significant.

The BSS work can be separated into four parts: they insert the missing i factor and achieve a zero answer; they examine the procedures in I, and find fault with the nuclear wave functions employed; they attempt to repeat the analysis of I with a different technique, with the conclusion that the result should be too small to be of interest; and they attempt to equate the approach of I with other published work. Each of these four aspects of the BSS paper is defective.

The i factor is inserted uncritically by BSS. In I, only a very small portion of the generalized function $J_n(-\zeta, -\frac{1}{2}\eta)$ that occurs there was retained, because that portion appears to be dominant, and the rest of the amplitude was unnecessary. The obvious message from cancellation in phases when the i factor is inserted is that the entire $J_n(-\zeta, -\frac{1}{2}\eta)$ result explicitly given in I should be employed. This is, in fact, more straightforward than the partial approach adequate as an approximation in I. When the entire generalized Bessel function given in I is used, the results remain both analytically tractable and physically important.

When BSS examine the nuclear wave functions in I, they misunderstand the momentum-translation approximation[3] (MTA) that is employed there. They introduce needless and confusing gauge transformations which are never implied by the MTA nor employed in I. This leads then to a flagrant logical contradiction when they conclude that "the MTA wave function is nothing but the correct unperturbed state in the Coulomb gauge." The entire analysis of I is done in the Coulomb gauge, and the MTA wave function is transparently field dependent, since it consists of a field-dependent factor multiplied into the unperturbed wave function. The BSS conclusion is self-contradictory.

BSS attempt to repeat the calculation of I using a Green's function approach. There are a number of difficulties with the BSS calculation. Above all, their work is not finished. Their analytical forms are very complicated and highly implicit. They leave undone two crucial integrations which would lead to energy-conserving delta functions. Because they are never able to exhibit a delta function, all order of magnitude estimates are very risky. Not only do their residual expressions contain a singular function lurking within them, but because the delta function has not been extracted it is inadmissible to assign physical magnitudes to energies and momenta contained therein, as some of their own conclusions demonstrate. A point of major importance is that the BSS formalism intimately mixes allowed transition amplitudes with conventional retardation amplitudes and induced electromagnetic transition amplitudes. Although they have not actually achieved any final results, any attempt to achieve them by an approximation technique (such as the stationary phase method they imply) must cope with the dominance of allowed transitions. Great care must be taken that the approximation does not lose the electromagnetic contribution altogether, or critically distort it. A very important point here is that the BSS formalism has also lost track of the electron retardation term that is so vital in the conventional theory of forbidden beta decay. This retardation term exists initially, but it can no longer be found by the point of development where BSS also have trouble finding the electromagnetic term. By contrast, the allowed, conventional forbidden, and electromagnetic transition amplitudes are clearly identifiable and cleanly separable in I.

BSS attempt to equate the calculation of I with other work[4,5] on electromagnetic effects in beta decay. That other work considers electromagnetic influences solely through the effect of the field on the beta particle, treated purely as a Volkov plane-wave electron. The "no-go" theorem of Ref. 6 demonstrates that the seemingly nontrivial results of Refs. 4 and 5 were simply an outcome of inadequate mathematical approximations. In contrast to this, the method of I is to treat the nucleon undergoing beta decay as an integral part of an atomic nucleus. Not only does this imply that the beta-decay nucleon itself is described by a finite-range nuclear wave function, but because of the four-fermion character of beta decay, the coordinates of the decay electron are similarly confined in space. That is, their plane-wave character is modified, and the strictures of Ref. 6 do not apply. This statement holds true whether or not the nuclear wave function includes interaction with the field—as it does in I.

In Sec. II below, the general form is exhibited of the results in I after introduction of the i factor. The effects on angular momentum and/or parity forbiddenness are shown explicitly, and a general order of magnitude is found for electromagnetic enhancement of forbidden beta decay. A brief review of the MTA is given in Sec. III in order to stress those aspects of the MTA which are misunderstood by BSS. It is also pointed out that the calculation of I can be done without using the MTA, and with the same results, albeit with more difficulty and less generality. Finally, in Sec. IV, a critique of the BSS calculation is presented. Some further remarks are included on the crucial distinctions between I and other attempts to treat external electromagnetic influences on beta decay.

II. SURVEY OF RESULTS

In I, the transition amplitude for beta decay in the presence of a plane wave electromagnetic field is treated in three parts, corresponding to a natural subdivision following from evaluation of traces of Dirac matrices. These three parts are designated as the direct term, the spin term, and the interference term. They are treated in parallel in I. For present purposes, it is adequate to consider the direct term only. Extension of the same procedures to the other two terms can be carried out in an obvious fashion.

As pointed out by BSS, a factor i^{-j} is missing on the right-hand side of Eq. (56) of I, and a factor of i^m is needed in Eq. (57). Insertion of those factors leads to an extra factor i^{m-j} in Eqs. (58), (60), and (64), which are otherwise unchanged. The next place in I in which these factors matter is in Sec. IV C, after evaluation of the asymptotic form of the generalized Bessel function $J_n(-\zeta,-\frac{1}{2}\eta)$ in Sec. IV B.

The procedure is now to employ the asymptotic $J_n(-\zeta,-\frac{1}{2}\eta)$ in a completely straightforward fashion. The sums over j and m in Eq. (64) of I are to be replaced by sums over m and l, where

$$l=m-j\ . \tag{1}$$

These two sums are, explicitly,

$$\sum_j\sum_m i^{m-j}J_n(-\zeta,-\tfrac{1}{2}\eta)J_{n-j+m}(-\zeta,-\tfrac{1}{2}\eta)J_j(e\vec{a}\cdot\vec{r})J_m(e\vec{a}\cdot\vec{r}\,')$$
$$=\sum_l i^lJ_n(-\zeta,-\tfrac{1}{2}\eta)J_{n+l}(-\zeta,-\tfrac{1}{2}\eta)\sum_m J_{m-l}(e\vec{a}\cdot\vec{r})J_m(e\vec{a}\cdot\vec{r}\,')\ . \tag{2}$$

The sum over m gives

$$\sum_m J_{m-l}(e\vec{a}\cdot\vec{r})J_m(e\vec{a}\cdot\vec{r}\,')=(-)^lJ_l(e\vec{a}\cdot\vec{r}-e\vec{a}\cdot\vec{r}\,')\ . \tag{3}$$

The generalized Bessel functions in Eq. (2) are to be replaced by the asymptotic forms stated in Eq. (94), given the definitions of terms as stated in Table I and in Eq. (95) of I. Equation (2) above then gives

$$\sum_j\sum_m i^{m-j}J_n(-\zeta,-\tfrac{1}{2}\eta)J_{n-j+m}(-\zeta,-\tfrac{1}{2}\eta)J_j(e\vec{a}\cdot\vec{r})J_m(e\vec{a}\cdot\vec{r}\,')$$
$$\approx\frac{1}{2\pi\eta c}\sum_l\left[\frac{\cos\phi_1}{[1-(c-b)^2]^{1/4}}+\frac{\cos\phi_2}{[1-(c+b)^2]^{1/4}}\right]\left[\frac{\cos(\phi_1+l\theta_1)}{[1-(c-b)^2]^{1/4}}+\frac{\cos(\phi_2+l\theta_2)}{[1-(c+b)^2]^{1/4}}\right](-i)^lJ_l(e\vec{a}\cdot\vec{r}-e\vec{a}\cdot\vec{r}\,')$$
$$\approx\frac{1}{4\pi\eta c}\sum_l\left[\frac{\cos l\theta_1}{[1-(c-b)^2]^{1/2}}+\frac{\cos l\theta_2}{[1-(c+b)^2]^{1/2}}\right](-i)^lJ_l(e\vec{a}\cdot\vec{r}-e\vec{a}\cdot\vec{r}\,')\ . \tag{4}$$

Implicit in Eq. (94) of I and in Eq. (4) above are the constraints that the saddle points in the steepest-descent procedure lie on the real axis. This implies $|b\pm c|\leq 1$. This constraint is not necessary, but it is analytically convenient to adopt it here, and it is consistent with a broad range of physically practical parameters. The last line in Eq. (4) follows from the previous line by a phase-averaging procedure identical to that described in connection with Eq. (108) of I. It should be noted that the first and last of the three inequalities in Eq. (92) are not at all necessary, and they are not enforced here. The sum over l in the last line of Eq. (4) can be accomplished in closed form, since

$$\sum_l(-i)^l\cos l\theta_1J_l(e\vec{a}\cdot\vec{r}-e\vec{a}\cdot\vec{r}\,')=\exp[-i\cos\theta_1(e\vec{a}\cdot\vec{r}-e\vec{a}\cdot\vec{r}\,')]\ , \tag{5}$$

with a similar expression for the term involving θ_2. Equations (4) and (5) above, when employed in Eq. (64) of I, give the induced beta decay transition probability

$$W_1=\frac{G^2}{(2\pi)^5}\int dE_e E_e (E_e^2-m^2)^{1/2}\int d\Omega_e \sum_n (E_0-E_e-\eta\omega+n\omega)^2\frac{1}{\eta c}$$
$$\times\left[\frac{1}{[1-(c-b)^2]^{1/2}}\left(\left|\exp\left[-i(c-b)z^{1/2}\frac{r}{R_0}\cos\theta\right]\right|_{fi}^2+\kappa^2\left|\exp\left[-i(c-b)z^{1/2}\frac{r}{R_0}\cos\theta\right]\vec{\sigma}\right|_{fi}^2\right)\right.$$
$$\left.+\frac{1}{[1-(c+b)^2]^{1/2}}\left(\left|\exp\left[i(c+b)z^{1/2}\frac{r}{R_0}\cos\theta\right]\right|_{fi}^2+\kappa^2\left|\exp\left[i(c+b)z^{1/2}\frac{r}{R_0}\cos\theta\right]\vec{\sigma}\right|_{fi}^2\right)\right] . \tag{6}$$

The result in Eq. (6) bears a strong resemblance to Eq. (114) in I. In particular, the $\cos\theta$ factors in the exponentials in the squared nuclear matrix elements provide the angular momentum and/or parity which overcomes forbiddenness in a forbidden beta decay. The most essential difference between Eq. (6) above and Eq. (114) of I is in the extra $(c\pm b)$ factors which occur in the exponential here. Since $c\pm b$ can be shown [from the defining Eqs. (89) and (91) in I] to be of order $z_f^{-1/2}$, then the magnitude of the argument of the exponential is

$$(z/z_f)^{1/2}=2^{1/2}mR_0 ,$$

where m is the electron mass and R_0 is the nuclear radius. The contribution of a squared matrix element in a beta decay transition of forbiddenness of order L is therefore of magnitude $(z/z_f)^L$, or $[2(mR_0)^2]^L$. In the conventional theory of forbidden beta decay, the factor by which forbidden decays are inhibited with respect to allowed decays is precisely $(mR_0)^{2L}$ for Lth order forbiddenness. Therefore, plane-wave electromagnetic field interactions can make major changes in forbidden beta decays.

III. REVIEW OF THE MTA

BSS have misconstrued the MTA so fundamentally that a brief review of the essentials is indicated.

The MTA is an approximation method for plane-wave electromagnetic field-induced transitions. It is carried out entirely in Coulomb gauge. There are no gauge transformations. The field is, at all times, described by the vector potential $\vec{A}$. The interaction Hamiltonian is always

$$H'=-e\vec{A}\cdot\vec{p}/m+e^2\vec{A}^2/2m . \tag{7}$$

Within this context, one seeks an approximate solution of the equation of motion. The device employed is to use the approximate integrating factor $\exp(ie\vec{A}\cdot\vec{r})$. That is, the substitution is made that

$$\psi=e^{ie\vec{A}\cdot\vec{r}}\phi , \tag{8}$$

where ϕ is the solution of the equation of motion without the field. This is purely a mathematical procedure carried out within Coulomb gauge. No gauge transformation is implied by Eq. (8). The field is described by $\vec{A}$, and the interaction Hamiltonian is given by Eq. (7). Substitution of Eq. (8) in the equation of motion does not satisfy it exactly, but leaves a residual term $-e\vec{E}\cdot\vec{r}$. This term is *not* a potential. The potential describing the field is $\vec{A}$. The MTA will give useful results when the magnitude of the residual term (which has units of energy) is small as compared with a characteristic energy of the problem. A meaningful comparison is with all or the first term of the transition matrix element involving the interaction Hamiltonian, Eq. (7). That is, if the ratio

$$\frac{|e\vec{E}\cdot\vec{r}|}{|\langle f|e\vec{A}\cdot\vec{p}/m|i\rangle|}=\frac{|e\omega\vec{A}\cdot\vec{r}|}{|\langle f|e\vec{A}\cdot\vec{r}\Delta E|i\rangle|}\approx\frac{\omega}{\Delta E} \tag{9}$$

is small, then the residual term is of little importance, and the ansatz (8) gives a good approximation to the exact solution. [There are other provisos for validity of the MTA (see Ref. 3), but none of them enters into the beta decay problem.] The numerator on the right-hand side of Eq. (9) is the energy of a single photon of the electromagnetic field, and the denominator is the total transition energy of the beta decay. By hypothesis, the ratio expressed in Eq. (9) is extremely small in the present problem. The MTA is ideally suited to the treatment of low-frequency electromagnetic interaction with the beta decay process.

BSS lose sight of the Coulomb gauge nature of the MTA, and they become confused with gauge transformations and inverses thereof. As final evidence of this confusion, they conclude that ψ as given in Eq. (8) is a noninteracting solution in Coulomb gauge. However, by construction, ϕ is the noninteracting solution in Coulomb gauge, and so ψ obviously contains interaction with the field. The BSS conclusion is self-contradictory.

IV. THE BSS CALCULATION

BSS have attempted to repeat the calculation of I using a different procedure. They are unable to complete the process, however, and they are left with two extremely difficult integrations which they cannot carry out. As a result, they are forced to attempt order-of-magnitude estimates without having an analytical framework which is sufficiently explicit for the task. In particular, the integrations they cannot perform are over time parameters, and so they are unable to extract the energy delta functions that reside within their integrals. Order-of-magnitude estimates with expressions that contain singular functions hidden somewhere within them are risky indeed.

As an example of the last point mentioned above, one of the authors of BSS made objection[7] that the factor $\exp(ie\vec{p}_e\cdot\vec{r})$ was set aside (i.e., it was replaced by unity) in I on the grounds that this factor contributed to the usual forbidden beta decay, but the investigation of I was concerned with other channels for the decay opened up by the applied field. It was remarked[7] that the $\exp(ie\vec{p}_e\cdot\vec{r})$ factor is absolutely essential in the BSS calculation. That is certainly true for BSS. In I, the $\exp(ie\vec{p}_e\cdot\vec{r})$ factor is neglected only after extraction of the delta function, when the magnitude of $|\vec{p}_e\cdot\vec{r}|$ can be assessed and it can be ascertained that its effect is small. In BSS, that delta function is never extracted, and so it is essential to retain all terms. Totally fallacious results would certainly obtain in BSS if $\exp(ie\vec{p}_e\cdot\vec{r})$ were replaced by unity. Other approximations in BSS must also be approached with caution.

One fallacy which follows from the inability of BSS to accomplish the t and t' integrations is their resort in their Eq. (2.13) to a result of a stationary phase calculation done with *no* field present. Such a result is not relevant to the field-dependent case. First of all, the result they quote is for $|t'-t|$, since the field-free Green's function has time-translation invariance. The presence of the field destroys this property. Furthermore, the location of stationary phase points relating to field effects in the actual field-dependent problem will depend upon the field itself, and such dependence is crucial. It can be seen from Eq. (6) above that field quantities and dynamical quantities for the nucleus and beta particle become mixed very intimately.

It must be remembered that field-induced beta decay is a small effect as compared with a true allowed beta decay. The formalism in I and in BSS share the property that allowed beta decay is included. In I, this part of the amplitude can be set aside unambiguously for those decays where the nuclear matrix element will vanish because angular momentum and/or parity selection rules are not met. Such is not the case in BSS. Phase factors relating to energy contributions, to momentum contributions, and to field contributions are intimately mixed in BSS. Possibly as an outcome of this masking effect, or possibly as the outcome of an error, BSS have lost entirely the conventional electron retardation term that occurs in the usual theory of forbidden beta decay. It exists in the initial statement of the formalism, but it is gone by the time BSS seek $\vec{r}$ dependence from electromagnetic contributions. The $\vec{r}$ dependence contributed by the electron retardation term is missing completely at this point. Only allowed beta decays can be identified. If one applied the arguments used by BSS to an analysis of conventional forbidden beta decay, one would (falsely) conclude it could not occur. Yet the necessary electron retardation term exists in the initial statement of the formalism.

The remainder of the analysis in Sec. II of BSS depends upon the field-free estimate in Eq. (2.13) plus recourse to the arguments of another paper[6] by the same authors, quoted as Ref. 7 in BSS. However, that paper refers only to intense-field processes involving free charged particles. That is, the charged particles must have plane-wave character. Reference 6, however, has no relevance at all to I. The work in I involves bound-state nuclear wave functions which are most certainly not plane wave in nature. Furthermore, since the beta decay formalism is that of a four-fermion interaction, the spatial coordinates associated with the decay electron become the same as the coordinates of the beta-decay nucleon. Therefore, the Volkov solution used for the decay electron is also deprived of its plane-wave character, and so it is not subject to the conditions of the theorem given in Ref. 6. The BSS argument is irrelevant.

Section III of BSS contains nothing not already discussed in the context of their Sec. II.

Section IV of BSS points out the omission of the i factor in the generating function for the Bessel function. They draw from this an unwarranted conclusion. As discussed in Sec. I above, after the i factors are inserted properly, a completely straightforward application of the procedures and results presented in I leads to consistent and nontrivial results.

The context of Sec. V of BSS has already been commented upon in Sec. III above. BSS have simply misunderstood the MTA, and nothing they say on the subject has any meaning. Furthermore, it is possible to carry out the analysis of I without the use of the MTA, and still achieve the same results. One way to do this is to employ a first-order perturbation theory for the electromagnetic field, so that one actually must deal with a second-order perturbation calculation in which one order is the weak interaction and the other is electromagnetic. Although this approach confines one to analysis only of first-forbidden decays, and the results are thereby less general and also less conveniently achieved than with the MTA, they are nonetheless equivalent.

Another way to avoid the use of the MTA and still achieve the same results as given in I involves transformation to another gauge. However, in view of the confusion engendered by the introduction of gauge transformations in BSS, it is best to avoid any discussion of gauge transformation here. Not only is it customary to employ Coulomb gauge in nuclear physics in connection with electromagnetic transitions, Coulomb gauge is also the gauge employed in the standard derivations of the Volkov solution used in I. It is best, therefore, to retain the Coulomb gauge for this entire discussion. Coulomb gauge is the only gauge used in I.

The Summary section in BSS is defective for all the reasons detailed above. In particular, the introductory paragraph in their Sec. V is completely false. The theories developed in Ref. 4 by Baranov and in Ref. 5 by Becker, Louisell, McCullen, and Scully (Refs. 12 and 13 in BSS) contain effects of the field only by considering the decay electron to be represented by a free-particle Volkov solution. As such, these theories are, therefore, entirely negated by the no-go theorem of Ref. 6. The theory presented in I, by contrast, considers electromagnetic field interaction with all charged particles in the problem. As pointed out above, this means that bound states enter in an essential way, and the theorem of Ref. 6 is inapplicable to I.

V. SUMMARY

BSS present a calculation which is suspended when critically incomplete. Their results are extremely complicated, implicit, and contain a hidden singular function. No reliable conclusions about a small, sensitive effect such as the influence of electromagnetic fields on forbidden beta decay can be extracted from such a partial calculation as presented by BSS. Either through the incompleteness of their calculation or through error, BSS have lost not only electromagnetic contributions to beta decay, they have lost as well the electron retardation term of conventional forbidden beta decay theory. Nevertheless, they draw conclusions, and attempt to explain them by entering into misinterpretations of one of the techniques (the MTA) employed in I.

The results presented in I are complete, explicit, and demonstrate effects of important magnitude. The derivation of the techniques and procedures employed are exhibited in full detail. The examination of this paper by BSS has uncovered one algebraic oversight which is immediately correctible within the context of the procedures and results given in I. The final results remain consistent and important after the algebraic change in incorporated. BSS have found no other errors.

[1]W. Becker, R. R. Schlicher, and M. O. Scully, preceding Comment, Phys. Rev. C 29, 1150 (1984).
[2]H. R. Reiss, Phys. Rev. C 27, 1199 (1983).
[3]H. R. Reiss, Phys. Rev. A 23, 3019 (1981).
[4]I. G. Baranov, Izv. Vyssh. Uchebn Zared. Fiz. 4, 115 (1974) [Sov. Phys. J. 533 (1974)].
[5]W. Becker, W. H. Louisell, J. D. McCullen, and M. O. Scully, Phys. Rev. Lett. 47, 1262 (1981).
[6]W. Becker, G. T. Moore, R. R. Schlicher, and M. O. Scully, Phys. Lett. 94A, 131 (1983).
[7]W. Becker (private communication).