Comment on enhancement of forbidden nuclear beta decay by high-intensity radio-frequency fields

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A recent claim that forbidden nuclear beta decay can, by the application of a high-intensity radio-frequency field, be enhanced by many orders of magnitude is contested. The effect is shown to be nonexistent, at least within the theoretical model which has been adopted thus far.

I. INTRODUCTION

In two interesting recent publications[1,2] Reiss claims that forbidden nuclear beta decay can, in the presence of an intense, but readily achievable radio frequency field, be enhanced by many orders of magnitude, the more so the higher forbidden the decay is. Of course, this would be a fascinating effect, both from a theoretical and a practical point of view. Unfortunately, we find, at least in the framework of the approximations and idealizations underlying this work, his conclusions to be incorrect and the effect nonexistent.

The approach of Ref. 1 to nuclear beta decay in the presence of a strong external electromagnetic field treats the weak coupling to first order of perturbation theory, but is supposed to take into account the coupling with the external field to all orders, at least approximately. Hence in Reiss's model the state of the emitted electron is described by the Volkov wave function,[3] whereas the nuclear states are to be well approximated by the so-called momentum translation approximation (MTA) wave function.[4] We shall show in Sec. II by explicit calculation that within this particular model the total beta-decay rate is essentially independent of the external electromagnetic field. Our argument turns out to be independent of whether or not we employ a relativistic description of the decay. Hence, for the sake of simplicity, we shall first turn to a completely nonrelativistic description within a long wavelength approximation for the applied field. Once we have made our point, the generalization to the fully relativistic problem is conceptually straightforward and will be presented in Sec. III. Our conclusion that the lifetime of a nucleus cannot be influenced by an external field in the framework of the model adopted by Reiss leads us to suspect that Ref. 1 includes a calculational error. In Sec. IV we point out such an error. In order to understand why Reiss's model cannot yield an enhancement of the nuclear decay rate we critically review the derivation and justification of the MTA wave function in Sec. V. By comparing the "$\vec{r}\cdot\vec{E}$" vs "$\vec{p}\cdot\vec{A}$" form of the interaction Hamiltonian we obtain an indication of why the MTA method is an unjustified approximation. We then point out with the help of gauge arguments[5] that the MTA wave function is nothing but the correct unperturbed state in the Coulomb gauge.[6] This statement can be corroborated by writing the S matrix for the beta decay in different gauges. In this way we see that in Reiss's theory only the interaction of the field with the electron is incorporated exactly, whereas the interaction with the nucleus is completely neglected. This fact is in strict contrast to the statements and intentions of Ref. 1 and leads to a completely different interpretation of the model. If the interaction of the nucleus with the external field is not incorporated into the model, a modification of the total decay rate can only originate from the coupling of the electron to the field. It was recently shown[7] that decays of neutral particles are unaffected by the application of optical and, even more so, radio frequency fields, to an excellent approximation. Consequently, since Reiss's approach to the problem does not contain any genuine interaction of the nuclei with the field, it cannot yield any impact on the nuclear lifetime. In Sec. VI we summarize our various criticisms of Ref. 1.

II. FIELD INDEPENDENCE OF THE TOTAL TRANSITION PROBABILITY

The starting point of the formalism of Refs. 1 and 2 is the S matrix for nuclear beta decay. If we denote the weak interaction, which causes the transition, by (gV), the nonrelativistic limit to the S-matrix element to first order in the weak coupling reads

$$S_{fi}=-i\int d^3r\int dt\,\Psi^{(f)*}_{\rm MTA}(\vec{r},t)\Psi^{*}_{(e)}(\vec{r},t;\vec{p})\Psi^{*}_{(\nu)}(\vec{r},t;\vec{q})(gV)\Psi^{(i)}_{\rm MTA}(\vec{r},t)\ . \tag{2.1}$$

The various terms in Eq. (2.1) are discussed below. $\Psi_{(\nu)}$ denotes the neutrino wave function, which is a plane wave with momentum $\vec{q}$. Following the procedure outlined in Ref. 1, we take the wave functions of the charged particles, i.e., the electron and the nucleus in the initial and final state, to be in the Coulomb gauge. The electron wave function is then a solution of the Schrödinger equation

$$i\frac{\partial}{\partial t}\Psi_{(e)}(\vec{r},t)=\frac{1}{2m}[\vec{p}-e\vec{A}(t)]^2\Psi_{(e)}(\vec{r},t)\ . \tag{2.2}$$

We use here natural units $\hbar=c=1$. Unlike the convention in Ref. 1 where $e=|e|$, we denote the electric charge of a particle by e, so that, for instance, the electron charge is $e = -|e|$. The exact solution of Eq. (2.2) is the nonrelativistic Volkov wave function with momentum $\vec{p}$,

$$\Psi_{(e)}(\vec{r},t;\vec{p}) = \exp\left[-i\left(Et - \vec{p}\,\vec{r} - \frac{1}{2m}\int^{t}[2e\vec{A}(\tau)\vec{p} - e^2\vec{A}^2(\tau)]d\tau\right)\right] , \tag{2.3}$$

where $E = p^2/2m$, so that our nonrelativistic theory is only consistent for low energy emitted electrons.

The underlying nuclear model of Ref. 1 is the shell model with an inert 0^+ core, which is affected neither by the beta decay nor by the electromagnetic field, and one or more valence nucleons in an angular momentum coupled state. The nuclear wave functions $\Psi^{(i)}$ and $\Psi^{(f)}$ are then derived from a one-particle Hamiltonian. The initial and final nuclear states are approximated in Ref. 1 by the so-called momentum translation approximation (MTA) wave function. This wave function is given by

$$\Psi_{\rm MTA}(\vec{r},t) = \exp[i\tilde{e}_{\rm N}\vec{A}(t)\,\vec{r}\,]\Phi_0(\vec{r},t) , \tag{2.4}$$

where $\tilde{e}_{\rm N}$ is the reduced nuclear charge [Eq. (6) of Ref. 1] and Φ_0 denotes the nuclear wave function in the absence of the external electromagnetic field, i.e.,

$$i\frac{\partial}{\partial t}\Phi_0(\vec{r},t) = H_0\Phi_0(\vec{r},t) , \tag{2.5}$$

with

$$H_0 = \frac{1}{2m_r}\vec{p}^2 + V(\vec{r}) . \tag{2.6}$$

$V(\vec{r})$ denotes the nuclear binding potential, and m_r is the reduced nuclear mass [Eq. (5) of Ref. 1]. Since we consider the nucleus initially and finally to be in an eigenstate $\phi_n(\vec{r})$ $(n = i,f)$ of H_0 with energy E_n, we shall later use Φ_0 in the form

$$\Phi_0^{(n)}(\vec{r},t) = e^{-iE_nt}\phi_n(\vec{r}) . \tag{2.7}$$

For the time being we shall adopt the MTA wave function in the same way, as it is used in Ref. 1, and postpone a detailed discussion on this subject to Sec. V.

The total transition probability per unit time is calculated from the S-matrix element (2.1) by

$$\Gamma = \int\frac{d^3q}{(2\pi)^3}\int\frac{d^3p}{(2\pi)^3}\lim_{T\to\infty}\frac{1}{T}|S_{fi}|^2 . \tag{2.8}$$

If we insert the S-matrix element (2.1) into Eq. (2.8) using the wave functions (2.3), (2.4), (2.7), and plane waves for the neutrino, the total rate Γ takes the form

$$\begin{aligned}\Gamma = \frac{1}{(2\pi)^6}\lim_{T\to\infty}\frac{1}{T}\int d^3r\,d^3r'\int_{-T/2}^{T/2}dt\,dt'\exp[iE_0(t'-t)]\exp\{-ie[\vec{A}(t')\,\vec{r}\,'-\vec{A}(t)\,\vec{r}\,]\}\\ \times G_{(\nu)}(\vec{r}\,',t';\vec{r},t)G_{(e)}(\vec{r}\,',t';\vec{r},t)\\ \times[\phi_f^*(\vec{r})(gV)\phi_i(\vec{r})][\phi_i^*(\vec{r}\,')(gV)^*\phi_f(\vec{r}\,')] .\end{aligned} \tag{2.9}$$

Here $E_0 = E_i - E_f$ denotes the nuclear energy released in the beta decay. We furthermore used the relation $\tilde{e}_i - \tilde{e}_f = e$ [see Ref. 1, Eq. (49), and our convention for the sign of e]. In fact, this relation is only approximate, since the minor impact of the external field on the nuclear core is neglected. The sum over the neutrino states is expressed in terms of a plane wave Green's function with a dispersion $E_\nu = |\vec{q}|$,

$$G_{(\nu)}(\vec{r}\,',t';\vec{r},t) = \int d^3q\,\Psi^*_{(\nu)}(\vec{r},t;\vec{q})\Psi_{(\nu)}(\vec{r}\,',t';\vec{q}) = \int d^3q\,\exp[-i|\vec{q}|(t'-t)]\exp[i\vec{q}(\vec{r}\,'-\vec{r})] . \tag{2.10}$$

All the electronic contributions to the total rate Γ are contained in the nonrelativistic Volkov Green's function

$$\begin{aligned}G_{(e)}(\vec{r}\,',t';\vec{r},t) &= \int d^3p\,\Psi^*_{(e)}(\vec{r},t;\vec{p})\Psi_{(e)}(\vec{r}\,',t';\vec{p})\\ &= \exp\left[-i\frac{e^2}{2m}\int_t^{t'}\vec{A}^2(\tau)d\tau\right]\int d^3p\exp\left\{-i\left[\frac{t'-t}{2m}\vec{p}^2 - \left(\vec{r}\,'-\vec{r}+\frac{e}{m}\int_t^{t'}\vec{A}(\tau)d\tau\right)\vec{p}\right]\right\} .\end{aligned} \tag{2.11}$$

The integral (2.11) can be reduced to a Gaussian integral and we find

$$\begin{aligned}G_{(e)}(\vec{r}\,',t';\vec{r},t) = \left(\frac{2m\pi}{i(t'-t)}\right)^{3/2}\exp\left(i\frac{m}{2}\frac{(\vec{r}\,'-\vec{r})^2}{t'-t}\right)\exp\left(ie\frac{\vec{r}\,'-\vec{r}}{t'-t}\int_t^{t'}\vec{A}(\tau)d\tau\right)\\ \times\exp\left\{-i\frac{e^2}{2m}\left[\int_t^{t'}\vec{A}^2(\tau)d\tau - \frac{1}{t'-t}\left(\int_t^{t'}\vec{A}(\tau)d\tau\right)^2\right]\right\} .\end{aligned} \tag{2.12}$$

The $\vec{A}^2$ term in Eq. (2.11) contributes to an effective mass (see Sec. III).

The essential field dependence of the total rate Γ is concentrated in the factors

$$\exp\{ie[\vec{A}(t')\,\vec{r}\,'-\vec{A}(t)\,\vec{r}\,]\}$$

and the Volkov Green's function $G_{(e)}$. In Ref. 1 the entire effect of changing the degree of forbiddenness of the nuclear decay is derived from the factors $\exp(ie\vec{A}\,\vec{r})$ in Eq. (2.4). We shall now show explicitly that these factors actually cancel out of the total decay rate against corresponding terms in $G_{(e)}$ and hence cannot give rise to the effects calculated in Ref. 1.

By inserting Eqs. (2.12) and (2.10) into the total decay rate (2.9), we see that in the field-free limit the major contribution to the time integral in Eq. (2.9) comes from time differences $t'-t$, for which the phase of the integrand is stationary:

$$|t'-t| = \left[\frac{m(\vec{r}\,'-\vec{r})^2}{2(E_0-E_\nu)} \right]^{1/2} . \tag{2.13}$$

The estimate (2.13) is not significantly affected by the external field, in particular, when there is no energy transfer from the field to the electron, as Reiss assumes.

It was shown in Ref. 7 that in the classical limit $\hbar \to 0$ only times t' infinitesimally close to t contribute to the total transition rate Γ. In this limit the field-dependent exponentials in the integrand of Eq. (2.9) are unity and only the field-independent part survives. Thus in the classical limit the total decay rate would be unaffected by the external field. This is in agreement with the results of Ref. 7.

The remaining field dependence in Γ is then only due to quantum effects. It will be found to be very small for the parameters of Ref. 1, as we shall now demonstrate. We can rewrite the space- and field-dependent exponential in the electron Green's function (2.12) with the help of integration by parts and find

$$\exp\left[ie\frac{\vec{r}\,'-\vec{r}}{t'-t}\int_t^{t'} d\tau\,\vec{A}(\tau)\right] = \exp\{ie[\vec{A}(t')\,\vec{r}\,'-\vec{A}(t)\,\vec{r}\,]\}\exp\left[ie\int_t^{t'} d\tau\,\vec{R}(\tau)\vec{E}(\tau)\right] , \tag{2.14}$$

with

$$\vec{R}(\tau) = \vec{r}\,' - \frac{\vec{r}\,'-\vec{r}}{t'-t}(t'-\tau) . \tag{2.15}$$

By inserting Eq. (2.12) with Eq. (2.14) into Eq. (2.9), we realize that the phase factors $\exp(ie\vec{A}\,\vec{r})$ cancel in the total decay rate and we obtain

$$\begin{aligned}\Gamma = \frac{1}{(2\pi)^6}\lim_{T\to\infty}\frac{1}{T}\int d^3r\,d^3r'\int dt\,dt'\exp[iE_0(t'-t)]G_{(\nu)}(\vec{r}\,',t';\vec{r},t)\\ \times\left[\frac{2m\pi}{i(t'-t)}\right]^{3/2}\exp\left[i\frac{m}{2}\frac{(\vec{r}\,'-\vec{r})^2}{t'-t}\right]\exp\left[ie\int_t^{t'} d\tau\,\vec{R}(\tau)\vec{E}(\tau)\right]\\ \times\exp\left\{-i\frac{e^2}{2m}\left[\int_t^{t'} d\tau\,\vec{A}^2(\tau)-\frac{1}{t'-t}\left(\int_t^{t'} d\tau\,\vec{A}(\tau)\right)^2\right]\right\}\\ \times[\phi_f^*(\vec{r})(gV)\phi_i(\vec{r})][\phi_i^*(\vec{r}\,')(gV)^*\phi_f(\vec{r}\,')] .\end{aligned} \tag{2.16}$$

In Ref. 1, Eq. (74), the nuclear intensity parameter z,

$$z = (e|\vec{A}|R_0)^2 , \tag{2.17}$$

which specifies the magnitude of the phase $\exp(ie\vec{A}\,\vec{r})$, was assumed to be of order of unity. The actual remaining exponential with a field and space dependence in Eq. (2.16) contains the integral

$$\begin{aligned}\left|e\int_t^{t'}\vec{R}(\tau)\vec{E}(\tau)d\tau\right| \sim eR_0|\vec{E}|\,|t'-t|\\ \sim eR_0\omega|\vec{A}|\left[\frac{mR_0{}^2}{E_0}\right]^{1/2} \sim \frac{1}{80}\frac{\omega}{E_0} \ll 1 .\end{aligned} \tag{2.18}$$

Here R_0 denotes the nuclear radius, and the length of the time interval was estimated by Eq. (2.13). We furthermore determined the amplitude of the electric field by $|\vec{E}| = \omega|\vec{A}|$, corresponding to a monochromatic plane wave with frequency ω, and applied Eq. (82) of Ref. 1. Equation (2.18) shows that the field- and space-dependent exponential in Eq. (2.16) is unity to an excellent approximation, i.e., the factors $\exp(ie\vec{A}\,\vec{r})$, which are the origin of the large enhancement obtained in Refs. 1 and 2, cancel. The $\vec{A}^2$ terms in Eq. (2.16) do not depend on $\vec{r}$ and can therefore not change the degree of forbiddenness of the decay.

We also see from the estimate (2.18) that the actual parameter that governs the field impact on the nuclear lifetime is

$$z' = (e\omega|\vec{A}|R_0^2)^2 = z(\omega R_0)^2 , \tag{2.19}$$

which is much smaller than z. This means that one needs a much higher field intensity, or fields with a much shorter wavelength than the one applied in Refs. 1 and 2, to produce a noticeable effect of the external field on the nuclear lifetime.

III. RELATIVISTIC THEORY

We shall now address ourselves to a relativistic treatment of the electron in analogy with Ref. 1. For a quantitative analysis this is indispensable because the electron energy will, in general, be relativistic, and because the entire decay process is intrinsically relativistic. However, the crucial point of the preceding analysis, the actual replacement of the superficial appearing gauge factors $\exp(ie\vec{A}\,\vec{r})$ by $\exp[ie\int d\tau\,\vec{R}(\tau)\vec{E}(\tau)]$ in the total decay rate, will proceed completely analogously in the relativistic treatment. We then shall have to examine the remaining expressions for possible additional terms incorporating $\vec{A}\vec{r}$ which might yield the large enhancements claimed by Reiss. These will have to be relativistic quantum terms induced by a radio frequency field. Hence it is not likely that they will play any significant role, and, in fact, they will not turn out to do so.

The starting point will now be the relativistic S-matrix element S_{fi} as given in Eq. (7) of Ref. 1. As we did in Eq. (2.8), we shall again sum over the electron's final momenta p and spins s, obtaining

$$\begin{aligned} R &= \int \frac{d^3\vec{p}}{2p_0} \sum_{\text{spins}} |S_{fi}|^2 \\ &= \int d^4p\, \theta(p_0)\delta(p^2-m^2) \sum |S_{fi}|^2 \\ &= \frac{G^2}{4m} \int d^4x\, d^4x' [\bar{\Psi}_f(x)\gamma_\mu(1-\kappa\gamma_5)\Psi_i(x)] \\ &\quad \times [\bar{\Psi}_i(x')(1+\kappa\gamma_5)\gamma_\rho\Psi_f(x')] \\ &\quad \times [\bar{\Psi}_{(\nu)}(x')(1+\gamma_5)\gamma^0 G^{(+)}(x',x) \\ &\quad \times \gamma^\mu(1-\gamma_5)\Psi_{(\nu)}(x)] \ . \end{aligned} \tag{3.1}$$

Here

$$G^{(+)}(x',x) = \int d^4p\, \theta(p_0)\delta(p^2-m^2) E(x',p)(\not{p}+m)\bar{E}(x,p) \tag{3.2}$$

is the homogeneous positive frequency Volkov Green's function replacing Eq. (2.11), and we have written the Volkov solution in the form

$$\Psi_{(e)}(x,p) = E(x,p)u(p,s) \ ,$$

where $u(p,s)$ is the free Dirac spinor so that

$$(\not{p}-m)u(p,s)=0 \ .$$

We are again using the Coulomb gauge so that the nuclear wave functions are given by Eq. (2.4).

The quantity R, when summed over the final states of the nucleus and the neutrino, yields the total decay rate of Eq. (2.8). The decisive point is that any Green's function in the presence of an external field $A^\mu(x)$ can be split as[8]

$$G^{(\,)}(x',x) = \phi(x',x)\tilde{G}^{(\,)}(x',x) \ , \tag{3.3}$$

where

$$\phi(x',x) = \exp\left[-ie\int_x^{x'} d\bar{x}_\mu A^\mu(\bar{x})\right] \tag{3.4}$$

is Schwinger's line integral which is to be integrated over a straight line connecting x with x' [the definition can be given in a path-independent way (see Ref. 8)], and $\tilde{G}^{(\,)}(x',x)$ is gauge invariant, i.e., depends only on $\vec{E}$ and $\vec{B}$. Hence the entire gauge dependence is isolated in the line integral (3.4). Collecting now all the $\vec{r}\vec{A}$-dependent exponentials in Eq. (3.1) we obtain

$$\exp\left[ie[\vec{r}\vec{A}(t) - \vec{r}'\vec{A}(t')] - ie\int_x^{x'} d\bar{x}_\mu A^\mu(\bar{x})\right] \ . \tag{3.5}$$

Evaluating the line integral yields

$$\int_x^{x'} d\bar{x}_\mu A^\mu(\bar{x}) = -\frac{\vec{x}'-\vec{x}}{k(x'-x)} \int_{kx}^{kx'} d(k\bar{x})\vec{A}(k\bar{x}) \ , \tag{3.6}$$

where k_μ denotes the wave vector of the field $A_\mu(kx)$. In the long wavelength approximation, which is finally also adopted in the relativistic treatment of Ref. 1, Eq. (3.6) reproduces the expression which we already encountered in the nonrelativistic treatment. The exponential (3.5) is then identical to the corresponding exponential in Eq. (2.9) with the nonrelativistic Green's function (2.12), and the arguments pointed out, following Eq. (2.12), apply to the relativistic case as well.

Finally, in order to make sure that the gauge-invariant remainder $\tilde{G}^{(+)}$ in Eq. (3.3) does not, so to speak by the back door, reintroduce corresponding exponentials, we shall now write down the complete Green's function $G^{(+)}(x',x)$. The easiest way to obtain it is by straightforward evaluation of Eq. (3.2), given the Volkov wave functions $E(x,p)$. The analogous approach has been followed, e.g., in Ref. 9, in the case of the Feynman propagator. The result (for arbitrary polarization, i.e., $A^\mu = \sum_{i=1}^{2} e_i^\mu A_i(\xi)$, $\xi = kx$, $ke_i=0$, $e_ie_j = -\delta_{ij}$) is very closely related to the former and reads

$$\begin{aligned} \tilde{G}^{(+)}(x',x) = \int_{-\infty}^{\infty} \frac{ds}{s^2} \exp\left[-is(m^2+T) - i\frac{(x-x')^2}{4s}\right] \\ \times \Bigg\{ \theta[s(\xi'-\xi)] \Bigg[\frac{1}{2s}(\not{x}'-\not{x}) + m - 2ms\not{k}\not{e}_iM_i \\ - \not{e}_i(\xi'-\xi)L_i + i(\xi'-\xi)\gamma_5\not{e}_i\epsilon_{ij}M_j + \not{k}\left[-(x'^i-x^i)L_i + s(\xi'-\xi)(L_i^2-M_i^2) - \frac{sT}{\xi'-\xi}\right] \\ + i\gamma_5\not{k}[-\epsilon_{ij}(x'^i-x^i)M_j + 2s(\xi'-\xi)\epsilon_{ij}L_iM_j] \Bigg] + i\,\text{sgn}(s)\not{k}\delta(\xi'-\xi) \Bigg\} \ . \end{aligned} \tag{3.7}$$

Here x^i denotes the two vector components of x^μ transverse to k^μ. We adopt a summation convention for the indices i and j extending from 1 to 2, and $\epsilon_{ij} = -\epsilon_{ji}$, $\epsilon_{12}=1$. The functions L, M, and T are given by

$$T(\xi',\xi)=\frac{e^2}{\xi'-\xi}\int_{\xi}^{\xi'} d\eta\, A_i(\eta)\left[A_i(\eta)-\frac{1}{\xi'-\xi}\int_{\xi}^{\xi'} d\eta' A_i(\eta')\right] , \tag{3.8}$$

$$M_i(\xi',\xi)=-\frac{e}{2(\xi'-\xi)}[A_i(\xi')-A_i(\xi)] , \tag{3.9}$$

$$L_i(\xi',\xi)=\frac{e}{2(\xi'-\xi)}\left[A_i(\xi')A_i(\xi)-\frac{2}{\xi'-\xi}\int_{\xi}^{\xi'} d\eta\, A_i(\eta)\right] . \tag{3.10}$$

We incidentally note that the quantity $T(\xi',\xi)$ in view of Eq. (3.7) specifies a space-time-dependent mass correction. It already showed up in the nonrelativistic treatment [cf. the last exponential in Eq. (2.12)].

We now have to check whether Eq. (3.7) contains any terms similar to $\cos(e\vec{a}\,\vec{r}-e\vec{a}\,\vec{r}')$, which is the sole cause of the enhancement in Ref. 1 [cf. Eq. (110)]. There are two candidates in Eq. (3.7):

$$-\not{k}(x'^i-x^i)L_i, \quad -i\gamma_5\not{k}\epsilon_{ij}(x'^i-x^i)M_j .$$

We notice that they are both only linear in (x'^i-x^i). Hence they can at best reduce the order of forbiddenness by one. Since they are proportional to $\not{k}$, their order of magnitude is $\omega e|\vec{A}|R_0=\omega\sqrt{z}$, which has to be compared with the electron mass m in the square bracket in Eq. (3.7). Since $\omega/m \ll 1$, in particular, for a radio frequency field, whatever effect is caused by these terms will be very small, as was to be expected.

IV. AN ALGEBRAICAL ERROR IN REF. 1

The above considerations strongly suggest that there is a computational error in Ref. 1. Hence we have cursorily checked some of the calculations. In fact, it appears that factors $(-i)^j$ and i^m are missing on the right-hand side of Eqs. (56) and (57) of Ref. 1, respectively, which should read

$$\exp(-ie\vec{A}\,\vec{r})=\exp[-ie\vec{a}\,\vec{r}\cos(kx+\rho)]$$
$$=\sum_{j=-\infty}^{\infty} J_j(e\vec{a}\,\vec{r})\exp[-ij(kx+\rho)](-i)^j \tag{4.1a}$$

and

$$\exp(ie\vec{A}'\,\vec{r}')=\exp[ie\vec{a}\,\vec{r}'\cos(kx'+\rho)]$$
$$=\sum_{m=-\infty}^{\infty} J_m(e\vec{a}\,\vec{r}')\exp[im(kx'+\rho)]i^m . \tag{4.1b}$$

These factors seem to be consistently missing. If they are included, the crucial equation (110) of Ref. 1,

$$\sum_q(-)^q J_{2q}(e\vec{a}\,\vec{r}-e\vec{a}\,\vec{r}')=\cos(e\vec{a}\,\vec{r}-e\vec{a}\,\vec{r}') , \tag{4.2}$$

loses the factor $(-)^q$ on the left-hand side and is changed into

$$\sum_q J_{2q}(e\vec{a}\,\vec{r}-e\vec{a}\,\vec{r}')=1 , \tag{4.3}$$

and there is no field induced enhancement of forbidden beta decay, in agreement with the arguments previously put forward in this paper.

V. REVIEW AND CRITICISM OF THE MTA WAVE FUNCTION

For a review of the MTA wave function from Reiss's point of view we refer to Ref. 4. The MTA method claims to be "gauge specific" for the Coulomb gauge (C gauge; vector potential $\vec{A}$ and vanishing scalar potential $A_0=0$). The Schrödinger equation in the long-wavelength approximation for the vector potential $\vec{A}$ then has the form

$$i\frac{\partial}{\partial t}\Psi(\vec{r},t)=[H_0+H_1(t)]\Psi(\vec{r},t) , \tag{5.1}$$

with

$$H_0=\frac{1}{2m}\vec{p}^2+V(\vec{r}),\quad H_1(t)=-\frac{e}{m}\vec{A}(t)\vec{p}+\frac{e^2}{2m}\vec{A}^2(t) . \tag{5.2}$$

If one writes an ansatz for $\Psi(\vec{r},t)$ in the form

$$\Psi(\vec{r},t)=\exp[ie\vec{A}(t)\,\vec{r}]\Phi(\vec{r},t) , \tag{5.3}$$

then the new wave function $\Phi(\vec{r},t)$ obeys the equation of motion

$$i\frac{\partial}{\partial t}\Phi(\vec{r},t)=[H_0+H_2(t)]\Phi(\vec{r},t) , \tag{5.4}$$

with

$$H_2(t)=-e\vec{E}(t)\,\vec{r} . \tag{5.5}$$

The so-called "momentum translation approximation" consists of neglecting the perturbation H_2 in Eq. (5.4) and replacing Φ in Eq. (5.3) by the unperturbed wave function Φ_0, given by

$$i\frac{\partial}{\partial t}\Phi_0(\vec{r},t)=H_0\Phi_0(\vec{r},t) . \tag{5.6}$$

We then obtain the MTA wave function as an approximation of the exact solution Ψ:

$$\Psi_0(\vec{r},t)=\exp[ie\vec{A}(t)\,\vec{r}]\Phi_0(\vec{r},t) . \tag{5.7}$$

This procedure is considered to be justified if H_2 is much smaller than H_1, in particular, when H_1 is too large to be treated as a perturbation with respect to H_0, whereas H_2 still is a small perturbation compared with H_0 (i.e., the magnitude of H_2 is small as compared with a characteristic energy of the field-free problem).[10] The condition "H_2 small compared with H_1" means that the transition matrix elements from an initial unperturbed state $|i\rangle$ to a final unperturbed state $|f\rangle$ are much smaller when the transition is induced by the residual interaction H_2 instead of H_1. The states $|i\rangle$

and $|f\rangle$ are eigenstates of H_0:

$$H_0|n\rangle = E_n|n\rangle \quad . \tag{5.8}$$

By using the commutator relation

$$[\vec{r}, H_0] = i\frac{\vec{p}}{m} \quad , \tag{5.9}$$

as well as

$$|\vec{E}| = \omega|\vec{A}| \quad , \tag{5.10}$$

we find

$$\frac{|\langle f|H_2(t)|i\rangle|}{|\langle f|H_1(t)|i\rangle|} \sim \frac{\omega}{E_i - E_f} \quad . \tag{5.11}$$

Hence in situations where the field frequency is much smaller than the considered transition energy, the MTA procedure might seem to be well justified.

As we have noted, central to the MTA is the condition that the transition matrix elements for the interaction H_2 are much smaller than for its counterpart H_1. This applies, in particular, to nuclear transitions in the presence of an optical or even radio frequency field. But one must be careful in drawing conclusions from or making approximations based upon the estimate (5.11). Instead of transition matrix elements, one actually has to compare transition probabilities, which are the directly measurable quantities. As we shall see below, the factor $\omega/(E_i - E_f)$ does not occur after integration over time and the two interactions H_1 and H_2 give the same transition probabilities for any ratio of ω over $(E_i - E_f)$.

Let us repeat a supposedly well known argument. In the interaction picture the transition amplitude to first order of perturbation theory reads

$$F(T) = -i\int_0^T dt\, \exp[-i(E_i - E_f)t]\langle f|H_{\rm int}(t)|i\rangle \quad . \tag{5.12}$$

If we use the interaction $H_{\rm int} = H_2$, we find

$$F(T) = ie\langle f|\vec{r}|i\rangle \int_0^T dt\, \exp[-i(E_i - E_f)t]\vec{E}(t) \quad . \tag{5.13}$$

For the case $H_{\rm int} = H_1$, let us consider the situation where the field is switched on at time $t = 0$ and that we are only interested in the transition probability at time T, when the field is already switched off, so that

$$\vec{A}(0) = 0, \quad \vec{A}(T) = 0 \quad . \tag{5.14}$$

This is the usual experimental situation. We can then rewrite the integral in Eq. (5.12) by partial integration

$$\int_0^T \exp[-i(E_i - E_f)t]\vec{A}(t)\,dt = \frac{i}{E_i - E_f}\int_0^T \exp[-i(E_i - E_f)t]\vec{E}(t)\,dt \quad . \tag{5.15}$$

We have now obtained a factor $(E_i - E_f)$ by replacing $\vec{A}$ by $\vec{E}$ in the integrand, instead of the factor ω, which entered the relation (5.11) via Eq. (5.10). By using Eqs. (5.9) and (5.15) we find for $H_{\rm int} = H_1$ again the transition amplitude (5.13).

This simple calculation shows that the transition probability is the same whether one uses the interaction H_1 or H_2. This holds true to any order in the perturbation series.[11] We can furthermore drop the restriction (5.14) and obtain equal transition amplitudes $F(T)$ at any time T, if we take into account that the same physical state is represented by different state vectors in different gauges if $\vec{A}(0) \neq 0$ or $\vec{A}(T) \neq 0$ (see next paragraphs). The error will therefore be of the same order of magnitude whether one neglects the interaction H_2 in Eq. (5.4) or H_1 in Eq. (5.1) for any ratio of the field frequency ω over the typical transition energy $E_i - E_f$. Since the approximate solution Φ_0 of Eq. (5.4) is an unperturbed wave function, these arguments indicate that also the MTA wave function represents a noninteracting state.

In order to see that the MTA wave function represents nothing but a noninteracting state, we must consider the different gauges involved in the problem. The wave functions of all charged particles participating in any reaction or decay process must be taken consistently in the same gauge. The Coulomb gauge is convenient for the calculation of the Volkov wave function of a free particle in the presence of an external electromagnetic field. To derive the properties of the nonrelativistic MTA wave function it is more instructive to begin with the electric field gauge [E gauge; vanishing vector potential $\vec{A} = 0$ and scalar potential $-e\vec{E}(t)\vec{r}$]. Wave functions are transformed from the C gauge (notation Ψ) to the E gauge (notation Φ) by the unitary transformation

$$\Phi(\vec{r}, t) = \exp[-ie\vec{A}(t)\vec{r}]\Psi(\vec{r}, t) \quad , \tag{5.16}$$

where $\vec{A}$ is the vector potential in long wavelength approximation for the C gauge.

The Schrödinger equation in the E gauge has the form (5.4), which was derived at the beginning of this chapter in a different context. If we entirely neglect the interaction of the particle with the field to a zeroth approximation, we obtain from Eq. (5.6) as an approximate solution the wave function Φ_0 in the absence of the external field.

In the present example of beta decay, the nuclear state has to be determined from the Schrödinger equation in the C gauge, i.e., from Eq. (5.1), since the electron wave function is given in the C gauge. If we approximate the solution of Eq. (5.1) by the noninteracting state Φ_0 in the E gauge and use the gauge transformation (5.16), we obtain the wave function Ψ_0 of Eq. (5.7), the MTA wave function. Although Ψ_0 depends on the vector potential $\vec{A}$, it still represents a noninteracting state since a state vector which does not include any interaction with the external field in a particular gauge (here the E gauge), neither does so when transformed to any other gauge (here the C gauge).

It should be mentioned that the wave function Φ_0 is not the correct noninteracting solution in the C gauge. In the C gauge the operator of the canonical momentum $\vec{p} = -i\vec{\nabla}$ differs from the operator of the kinetical momentum. Hence in the C gauge the Hamiltonian H_0 in Eq. (5.2) does not describe the situation, where the interaction of the particle with the field is entirely neglected. The correct procedure is first to transform from the C gauge via (5.16) to the E gauge, where the vector potential vanishes and the operators of the kinetical and canonical momentum are identical, so that $p^2/2m$ is the operator of the kinetical energy. Hence in this gauge the Hamiltonian in Eq. (5.6) really specifies the field-free motion, and Φ_0 represents the state in the absence of the field. The wave function in the C

gauge, which completely neglects the interaction with the field, is then given by Eq. (5.7), i.e., by the MTA wave function. For a lucid and thorough discussion of gauge invariance in quantum mechanics see Ref. 5.

Applying this discussion to the nuclear beta decay, we see that the S matrix (2.1) includes the exact electron state and noninteracting nuclear states in the C gauge. In the nonrelativistic approach the S matrix could also be written in the E gauge. The unperturbed nuclear state is then given by the field-free wave function Φ_0, and the gauge phase $\exp(ie\vec{A}\,\vec{r})$ is incorporated in the corresponding Volkov wave function, which we obtain from Eq. (2.3) and the transformation (5.16). The entire field dependence is then concentrated in the electronic wave function. This procedure yields the same analytical form of the S-matrix element as Eq. (2.1).

The factor $\exp(ie\vec{A}\,\vec{r})$ always appears when one consistently combines the nuclear state in the absence of the field, which is related to the E gauge, with a Volkov state, which can be calculated in a simple way for the C gauge. This factor can be shifted from the nuclear to the electronic wave function in different gauges and has nothing to do with the interaction of the nucleus with the field. The identification of the factor $\exp(ie\vec{A}\,\vec{r})$ in Eq. (5.3) as a gauge transformation was in the present context first pointed out in Ref. 6. In Ref. 4, Reiss does not consider this factor as a gauge transformation but rather calls it a "unitary transformation within the Coulomb gauge." He then claims that due to this phase factor the MTA wave function fairly represents the effects of the applied field on the particle to any order of interaction. Relying on this interpretation, Reiss attempts in Ref. 1 to derive the entire effect of changing the degree of forbiddenness of a nuclear decay and of enhancing nuclear decay rates from this factor $\exp(ie\vec{A}\,\vec{r})$.

If the interaction of the nucleus with the field is neglected, only the coupling of the electron with the field can still modify the nuclear lifetime. But as it was shown in Ref. 7, this can only happen for a field which is much stronger than the one considered in Ref. 1, as long as the field frequency is very small as compared with the nuclear decay energy. Therefore the model used in Ref. 1 cannot result in an appreciable change of the nuclear lifetime.

V. SUMMARY

The basic formalism developed at length in Ref. 1 is essentially the same as in Refs. 12 and 13 (cf. Ref. 14). The theory of Ref. 1 only differs from this previous work by using the correct noninteracting wave function Ψ_0 (2.4) instead of Φ_0 (2.5) (but with the misguided intention of incorporating the interaction with the field) and by considering linear polarization of the field instead of circular polarization. Whether the field is linearly or circularly polarized is rather immaterial for the effects in question. Considering linear in place of circular polarization mainly increases the calculational labor without adding additional insight.

We would like to concentrate on two central objections to Refs. 1 and 2: (i) Contrary to its intentions, Reiss's model does not include any interaction between the nucleus and the field. Notably, the so-called momentum-translation (MTA) wave function is nothing but the correct free wave function in the Coulomb gauge, as was shown in Sec. V in three different ways. (ii) The total decay rate cannot be influenced by an external field as chosen in Ref. 1, if one describes the electron by the relativistic Volkov wave function and the nucleus by the nonrelativistic MTA wave function. This can be derived by general arguments from point (i) and Ref. 7. We also prove this explicitly in Secs. II and III. There we show that in a correct treatment the gauge factors $\exp(ie\vec{A}\,\vec{r})$ (from which Ref. 1 derives its entire effect) mutually cancel in the total decay rate. Our argument shows that this cancellation is independent of the polarization and pulse shape of the external field. The dramatic enhancements of Refs. 1 and 2 seem to be due to an algebraic error. When this is corrected no enhancement remains, in agreement with the results of Secs. II and III.

The physical concept underlying Refs. 1 and 2 differs from the previous work (Refs. 12 and 13) by intending to concentrate on forbidden beta decay in the presence of an intense radio frequency field rather than an optical field. Some final remarks on these two new aspects: (a) Changing the degree of forbiddenness of a nuclear decay by modifying the nuclear states under the impact of an external field is an exciting idea, but cannot be achieved by Reiss's model, which only uses noninteracting nuclear states. In particular, it should be emphasized that Reiss tries to treat *forbidden* beta decay along the same lines as *allowed* beta decay, i.e., a multipole expansion of the lepton wave functions and relativistic corrections to the nuclear wave functions, both of which normally enable forbidden decays to take place, are not considered, since the dominant mechanism of the enhancement is supposed to originate from the gauge factors $\exp(ie\vec{A}\,\vec{r})$. (b) The advantage of radio frequencies as compared with optical frequencies seems to lie in the fact that larger values of the parameter $(ea/m)^2$ can be achieved at much lower field strengths. But if very high intensity radio frequency fields with a wavelength of $\lambda \sim 100$ m (Ref. 2) are applied, the applicability of the Volkov solution, which assumes a plane wave field of *infinite extent* in space and time, seems very doubtful and certainly requires some justification. A relativistic electron which moves freely in such a field performs an oscillatory motion over a distance of λ. Such electromagnetic fields also raise experimental problems, since the atomic electrons tend to shield an external low frequency field, reducing its field strength at the site of the nucleus by orders of magnitude.[15]

We would finally like to emphasize that it remains an open question as to whether properly including the interaction between the nucleus and the field might yet lead to some enhancement of forbidden beta decay, although we believe that the orders of magnitude stated in Refs. 1 and 2 are very unlikely to be achieved by the latter mechanism.

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