Clustering of nucleons in the nuclear surface

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The numerical results of a recent paper on nn and pp clustering are explained in terms of a simple schematic model which emphasizes the role of positive and negative parity single particles.

In a recent article[1] by Janouch and Liotta on alpha cluster formation and alpha decay, numerical results were presented illustrating how configuration mixing produces pair correlations. Shell model calculations were made for the ground states of ^{210}Pb and ^{210}Po. For fixed values, $r_1=r_2=7.5$ fm, the wave functions of the valence nucleons were plotted against Θ, the angle, between the radius vectors r_1 and r_2. As the number of configurations was increased these "angular distribution functions" became sharply peaked about $\Theta=0°$ and small at back angles, $\Theta>90°$. The purpose of this Comment is to show that these results are easily understood in terms of a simple, schematic model published earlier.[2]

As in Ref. 1, only the singlet ($S=0$) component of the wave function is considered. For a *jj*-coupling configuration $(n_1l_j,n_2l_j,J=0^+)$, the singlet part of the wave function is given by

$$\psi_0(n_1l_j,n_2l_j)=(-)^l(j+\tfrac{1}{2})^{1/2}\frac{1}{4\pi}R_{n_1l}(r_1)R_{n_2l}(r_2)P_l[\cos\Theta] \ . \quad (1)$$

This is easily shown by transforming from *jj* to *LS* coupling and using the spherical harmonic addition theorem. The schematic model consists of a single zero-order configuration plus higher-order configurations included in perturbation theory,

$$\Psi=\psi((nlj)^2)+\sum_{n_1n_2l'j'}\psi(n_1l'j',n_2l'j')\frac{\langle n_1l'j',n_2l'j'|V|(nlj)^2\rangle}{2\epsilon(nlj)-\epsilon(n_1l'j')-\epsilon(n_2l'j')} \ . \quad (2)$$

For the ground state of the system, the energy denominators in Eq. (2) are all negative. A surface-delta interaction is chosen for V,

$$\langle n_1l'j',n_2l'j0|V|(nlj)^20\rangle=-G\sigma_{12}[(j+\tfrac{1}{2})(j'+\tfrac{1}{2})]^{1/2}(-)^{l+l'}R_{nl}{}^2R_{n_1l'}R_{n_2l'} \ , \quad (3)$$

in which $\sigma_{12}=1$, for $n_1=n_2$ and $\sigma_{12}=\sqrt{2}$ for $n_1\neq n_2$. The quantity G is positive, and the radial functions are evaluated at the nuclear surface. Let the distribution in angle Θ of the function Ψ, Eq. (2), be evaluated at the same r_1, r_2 as in the surface-delta interaction. Substituting Eqs. (1) and (3) in Eq. (2) results in a sum of Legendre polynomials P_l; *the coefficients of all these P_l have the same sign.* Since $P_l(1)=1$, all configurations tend to produce constructive interference for $\Theta\approx0°$. Similarly P_l of even and odd values of l interfere destructively for $\Theta>90°$. This model not only gives a simple interpretation of the numerical results plotted in Fig. 1 of Ref. 1, it shows the importance of configurations built from single particle states of parity opposite to those of the low-lying valence orbitals. A superposition of states of the form, Eq. (1), all with even l (odd l), will be an even (odd) function of $\cos\Theta$. The probability for $\Theta=180°$—maximum distance between the nucleons—would be the same as for $\Theta=0°$. One cannot achieve a high degree of two-particle clusterization without single-particle excitations of $1\hbar\omega, 3\hbar\omega\ldots,$.

This constructive interference of $P_l(\cos\Theta)$ values from all configurations is exactly the same as that which results in large enhancements of two-nucleon transfer form factors, as discussed in Ref. 2.

[1]F. A. Janouch and R. J. Liotta, Phys. Rev. C 27, 896 (1983).

[2]W. T. Pinkston, Nucl. Phys. A295, 345 (1978).

