Comments

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Comment on a discrepancy between proton- and alpha-induced cluster knockout reactions on ^{16}O

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The possibility is discussed that the enhanced clustering in the nuclear surface of some nuclei observed by Samanta *et al.* [Phys. Rev. C 26, 1379 (1982)] is due to the modification of the wave function of a discrete nuclear state at the nuclear surface by unbound states. The modification is formulated in a model with unified description of bound and unbound states. Experimental data from knockout reactions on odd nuclei are necessary in order to test the theoretical statements.

[NUCLEAR REACTIONS Cluster knockout, light nuclei, spectroscopic factors, continuum shell model, nuclear surface.]

Recently, absolute values for spectroscopic factors of clusters in light and medium mass nuclei have been extracted from (p, px) and $(\alpha, \alpha x)$ reactions.[1-3] The results obtained are somewhat surprising. The spectroscopic factors extracted from the alpha-induced knockout reactions are one to two orders of magnitude larger than those from the proton-induced reactions. The last ones agree more or less with the theoretical values. The conclusion drawn from these data is the notion of greatly enhanced clustering in the extreme nuclear surface.[1] But the relative spectroscopic factors for ground-state transitions in nuclei with $16 \leq A \leq 64$ from both (p, pα) and $(\alpha, 2\alpha)$ reactions are fairly consistent with each other and with (^{6}Li, d) data.[1,2]

The experimental spectroscopic factors are obtained in conventional distorted wave impulse approximation using cluster-core wave functions with rms radii close to the target radii deduced in electron scattering. Smaller cluster spectroscopic factors from $(\alpha, \alpha x)$ reactions can be obtained by introducing apparently excessive cluster-core rms radii.[3] This fact suggests consideration of the problem of enhanced clustering in the nuclear surface together with the question of which radii are really extracted from the experimental data by use of the traditional methods of analysis. A similar question is raised in the interpretation of the pion scattering data on ^{40}Ca, ^{48}Ca, and ^{51}V (Refs. 4 and 5) in order to obtain the proton and neutron distribution in these nuclei.

The wave function of an isolated nuclear state consists, actually, of the wave function of the discrete state and of an additional term which takes into account the modification of the state by the continuum, i.e., by the finite lifetime of most of the excited states. It is[6]

$$\Omega_R = \phi_R + \omega_R \quad . \tag{1}$$

Here, ϕ_R is the wave function of the discrete state calculated in a traditional nuclear structure model. In the shell model, it is the eigenfunction of the shell model Hamiltonian H_{QQ},

$$(E_R - H_{QQ})\phi_R = 0 \quad . \tag{2}$$

In the continuum shell model[6] in which the coupling of the discrete shell model states to the continuum is taken into account in a straightforward manner, H_{QQ} is described by a finite radial potential, e.g., a Woods-Saxon potential. The additional term

$$\omega_R = \sum_c \int_{\epsilon_c}^{\infty} dE' \xi_{E'}^c \frac{1}{E^+ - E'} \langle \xi_{E'}^c | H | \phi_R \rangle \tag{3}$$

takes into account that the discrete states are coupled to the continuum via the matrix elements $\langle \xi_E^c | H | \phi_R \rangle$ irrespective of whether they are decaying states or not (ξ_E^c and ϕ_R are orthogonal). In the continuum shell model (CSM),[6] the wave functions ξ_E^c are solutions of the coupled-channels equations

$$(E^+ - H_{PP})\xi_E^c = 0 \tag{4}$$

calculated by the finite radial potential used in Eq. (2), where $H_{PP} \equiv PHP$ is the projection of the Hamiltonian operator H onto the subspace of unbound (scattering) states (in analogy to $H_{QQ} \equiv QHQ$ where Q projects onto the subspace of discrete states). The sum in Eq. (3) runs over the channels c irrespective of whether they are open ($E > \epsilon_c$) or closed ($E < \epsilon_c$). The functions ω_R are solutions of coupled-channels equations with source term

$$(E^+ - H_{PP})\omega_R = H_{PQ}\phi_R \tag{5}$$

according to Eq. (3). All three functions ϕ_R, ξ_E^c, and ω_R can be calculated[6] by traditional methods with the unique Hamiltonian operator $H = H_0 + V$.

The partial width $\Gamma_{R,c}$ of a decaying state is defined by

$$\Gamma_{R,c}^{1/2} = (2\pi)^{1/2} \langle \Omega_R | H | \chi_E^c \rangle |_{E = E_R} \quad , \tag{6}$$

where the function χ_E^c describes the observed final state formed by the particle and the residual nucleus. It differs from the ξ_E^c by channel coupling which is taken into account in the ξ_E^c but not in the χ_E^c. It should be underlined

$$H = H_{QQ} + H_{PP} + H_{PQ} + H_{QP} \quad . \tag{7}$$

In the expression (6), the term

$$\gamma_2 = \langle \omega_R | H | \chi_E^c \rangle |_{E=E_R} \tag{8}$$

appears in addition to the expression

$$\gamma_1 = \langle \phi_R | H | \chi_E^c \rangle |_{E=E_R} \tag{9}$$

that is believed, generally, to be the exact expression of the amplitude of a partial width. The term (8) has an averaging influence due to the integral expression (3) of ω_R. It plays a role for partial widths being small in absolute value in that it may enhance these values by one or more orders of magnitude.[7] For large partial widths (corresponding to spectroscopic factors near the Wigner limit), it need not be considered in agreement with the general statements.

Because of[8]

$$\langle \Omega_R | H | \chi_E^c \rangle = \langle \phi_R | H | \xi_E^c \rangle \quad , \tag{10}$$

the matrix elements appearing in Eq. (3) are, at $E = E_R$, amplitudes of the partial widths. However, the matrix elements $\langle \phi_R | H | \xi_E^c \rangle$ and $\langle \Omega_R | H | \chi_E^c \rangle$ are defined not only at $E = E_R$ for decaying states but for all energies and also for the nondecaying ground states.

The numerical solution of the Eqs. (2), (4), and (5) of the CSM is described in Ref. 6. It is performed by rewriting the many-particle wave functions in products of single-particle wave functions and using the method of second quantization.

As an example, the results obtained for the excitation of the isospin forbidden analog resonance $J^\pi = \frac{3}{2}^-$, $T = \frac{3}{2}$ at 15.1 MeV in the mirror nuclei ^{13}N and ^{13}C will be given here. In the numerical calculations, the state is considered as isolated. The calculations are performed with the nucleon-nucleon interaction

$$V(r_1 - r_2) = -V_0(a + bP_{12}^\sigma)\delta(r_1 - r_2) \quad ,$$

where $V_0 = 650$ MeV fm^3, $a = 0.8$, $b = 0.2$, and parameters of the Woods-Saxon potential similar to those used usually for the $A = 16$ system: $V(l=0) = 56.36$ MeV, $V(l=1) = 57.67$ MeV, $V(l=2) = 54.65$ MeV and $V_{ls}(l=1) = 9.76$ MeV, $V_{ls}(l=2) = 5.27$ MeV for both neutrons and protons. The configuration space of the ^{12}C, ^{13}C, and ^{13}N nuclei is the whole $1p$ shell.

The calculations [solutions of Eq. (2)] give a small isospin mixing (about 10^{-3}) of the wave functions ϕ_R. According to this high isospin purity of both the states of the $A = 13$ system and of the residual nucleus ^{12}C, γ_1 is small. But the coupling of the open isospin forbidden channel to closed isospin allowed channels enlarges the width of the $\frac{3}{2}^-$, $\frac{3}{2}$ level by at least one order of magnitude (Table I). The considered channels correspond to two low-lying $T \approx 0$ states and three $T \approx 1$ states of ^{12}C in addition to the channel corresponding to the ground state of ^{12}C. The reason for the large influence of channel coupling on the partial width in the case considered is, of course, the small value of γ_1, Eq. (9), due to the isospin selection rule. Therefore, γ_2 is important. It is worth mentioning that the comparable large experimental value of the widths of the $\frac{3}{2}^-$, $\frac{3}{2}$ levels could not be explained theoretically without taking into account the continuum in spite of much effort (see Ref. 7). Furthermore, experimental investigations with high accuracy[16] showed that the wave functions of the lowest $T = \frac{3}{2}$ states in the mirror nuclei ^{13}N and ^{13}C are no longer related by the isospin raising or lowering operator. This agrees with the interpretation of Ω_R instead of ϕ_R as the resonance state wave function.

The wave function of the final state of $A \rightarrow B + b$ is[6]

$$\psi_E^c = \xi_E^c + \sum_R \Omega_R \frac{1}{E - \tilde{E}_R + (i/2)\tilde{\Gamma}_R} \langle \phi_R | H | \xi_E^c \rangle \quad . \tag{11}$$

It is the wave function of the continuous unbound state $B + b$ modified by the discrete bound states R of A [compare Eqs. (1) and (3) for bound states modified by the continuum]. It is the solution of the Schrödinger equation $H\psi_E^c = E\psi_E^c$. The functions $\tilde{E}_R - (i/2)\tilde{\Gamma}_R$ are eigenvalues of the operator

$$H_{QQ}^{\text{eff}} = H_{QQ} + H_{QP}(E - H_{PP})^{-1}H_{PQ} \quad , \tag{12}$$

and $E_R = \tilde{E}_R(E = E_R)$, $\Gamma_R = \tilde{\Gamma}_R(E = E_R)$ are the energy and width of the resonance state R.[6]

The transition of the ground state of A to the unbound state $B + b$ which is induced by the interaction operator W can be described[11] in first-order Born approximation by the

TABLE I. Width Γ_R of the (isolated) $\frac{3}{2}^-$, $\frac{3}{2}$ resonance state at 15.1 MeV in ^{13}N.

Number of channels	Corresponding states of ^{12}C: $(0^+,0)_1$	$(0^+,0)_2$	$(0^+,1)_1$	$(2^+,0)_1$	$(2^+,1)_1$	$(2^+,1)_2$	Γ_R/keV
1	x						0.19
2	x	x					0.37
3	x	x	x				0.38
3	x	x		x			0.42
4	x	x	x	x			1.01
5	x	x		x	x	x	4.69
6	x	x	x	x	x	x	6.44

transition matrix elements $\langle \psi_E^\xi | W | \phi_T \rangle$. These matrix elements consist of three parts according to the three functions ξ_E^ξ (direct part), ϕ_R (resonance part), and ω_R (channel-resonance scattering part) on the right-hand side of Eq. (11). The spectral function

$$S_R = |(A_R^{(\mathrm{res})} + A_R^{(\mathrm{chr})}) \frac{1}{E - \tilde{E}_R + (i/2)\tilde{\Gamma}_R} \langle \phi_R | H | \xi_E^c \rangle|^2 \quad (13)$$

for the (isolated) state R contains two terms: (i) the resonance term $S_R^{(\mathrm{res})}$ proportional to

$$A_R^{(\mathrm{res})} = \langle \phi_R | W | \phi_T \rangle \ , \quad (14)$$

and (ii) the channel-resonance scattering term $S_R^{(\mathrm{chr})}$ proportional to

$$A_R^{(\mathrm{chr})} = \langle \omega_R | W | \phi_T \rangle \ , \quad (15)$$

where

$$\langle \omega_R | W | \phi_T \rangle = \sum_{c'} \int_{\epsilon_{c'}}^{\infty} dE' A_{E',c'}^{(\mathrm{dir})} (E^+ - E')^{-1} \langle \xi_{E'}^{c'} | H | \phi_R \rangle \quad (16)$$

and

$$A_{E',c'}^{(\mathrm{dir})} = \langle \xi_{E'}^{c'} | W | \phi_T \rangle \ . \quad (17)$$

The interaction operator W describes the interaction of the nucleus A with an external field, e.g., electron scattering, proton or alpha scattering at intermediate energy, or quasifree scattering.

The contribution of the channel-resonance scattering term to cross section and spectral function has been numerically investigated in two cases.

(i) The ^{16}O (γ, n_0) photoabsorption cross section with excitation of the giant dipole resonance:[12] The calculation shows that the sum of direct and channel resonance scattering parts is small in comparison with the resonance reaction part. Consequently, the approximation of the traditional shell model $\sigma_{\mathrm{tot}} \approx \sigma_{\mathrm{res}}$ is justified in accordance with all the results obtained for many years.

(ii) The spectral function for the inelastic scattering of electrons on ^{12}C:[13] In these calculations, the relative contribution of the channel resonance scattering to the spectral function rises with the impulse q transferred in the reaction. For $q \gtrsim 1$ fm^{-1} its contribution cannot be neglected.

The analyses of proton- and alpha-induced cluster knockout reactions have been performed by taking into account only the resonance term according to the conventional assumption that ϕ_R is the wave function of a discrete bound state. It is, at present, difficult to calculate the channel-resonance scattering term in order to show numerically the differences between proton- and alpha-induced reactions at about 100 MeV. They arise, mainly, from the matrix elements (16), i.e., from all the energy-dependent matrix elements $A_{E',c'}^{(\mathrm{dir})}$, Eq. (17), for open as well as closed channels. Numerical calculations can be performed only by restricting to the most important channels. The selection of important channels should be based[9] not only on energy considerations (position of the threshold ϵ_c of the channel c) but also on structure considerations (spectroscopic factor of ϕ_T in relation to ξ_E^ξ). Since very little is known about the properties of closed channels, experimental hints are necessary in order to select the channels for the calculations on concrete nuclei.

In cluster knockout reactions the spectral function of the ground state T of the nucleus A is investigated in order to draw conclusions on the amplitudes of the partial widths $\langle \phi_T | H | \xi_E^\xi \rangle$, Eqs. (10) and (13). The matrix elements $A_T^{(\mathrm{res})}$, Eq. (14), are assumed to be proportional to those of the corresponding two-body scattering cross section in both alpha- and proton-induced reactions. The remaining nucleons in the nucleus are considered as spectators. But the matrix elements $A_T^{(\mathrm{chr})}$, Eq. (15), usually neglected, contain all the matrix elements $A_{E',c'}^{(\mathrm{dir})}$, Eq. (17), and $\langle \phi_T | H | \xi_{E'}^{c'} \rangle$. The latter contain the parentage connection of ϕ_T to all the channels c'.

Generally, the channel-resonance scattering term in the spectral function has the following properties:

(i) It takes into account channel coupling not only to open but also to closed channels. This type of channel coupling differs from both the inclusion of higher shell model states, which are in reality unbound, in the traditional shell model approximation and the interpretation as projectile induced by inelastic coupling to excited states. The channel coupling considered here determines the properties of the very state investigated since it is contained in the wave function Ω_R. Therefore it is characteristic of the state.

(ii) It has averaging properties since it contains a sum over all channels and an integral over energy, Eq. (16). Therefore, the sum $A_R^{(\mathrm{res})} + A_R^{(\mathrm{chr})}$ in Eq. (13) is more or less constant for nuclei with similar cluster structure and similar energy ϵ_c of cluster thresholds. Neglecting the channel resonance term $A_R^{(\mathrm{chr})}$ in the analysis of the data will falsify, consequently, the absolute values of the extracted spectroscopic factors of these nuclei but not so much the relative ones.

(iii) It is important in the nuclear periphery since it contains the wave functions of the unbound states. An analysis of the data without the channel-resonance scattering term should be done, therefore, with a radius larger than the rms radius. The actual value of the radius depends on the ratio of $A_R^{(\mathrm{res})}$ to $A_R^{(\mathrm{chr})}$.

(iv) Its contribution to the spectral function depends on the energy ϵ_c of the thresholds and parentage connection for decay into those channels c' for which the matrix elements $A_{E',c'}^{(\mathrm{dir})}$, Eq. (17), are large. By this, differences in the absolute values of spectroscopic factors obtained by analyzing different reactions with neglect of channel-resonance scattering may appear.

It follows from these properties that the virtual enhanced clustering in the nuclear periphery observed in the alpha-induced reaction data may be connected with the term ω_R in the wave function of the nuclear states which describes virtual particles coupled to closed channels. Such an interpretation is supported by the fact that an analysis with radii larger than the rms radius gives no enhancement and that the relative spectroscopic alpha factors for ground-state transitions of nuclei with low-lying thresholds for alpha decay are fairly consistent with each other.

Also the experimental result that the clustering is observed in alpha-induced reactions but not in proton-induced ones supports the interpretation given here, although very little is known on the matrix elements $A_{E',c'}^{(\mathrm{dir})}$. It can be assumed, however, that they are altogether larger at the energies considered for alpha-induced knockout reactions, due to their stronger surface character, than for proton-induced reactions. The virtual alpha particles in the nuclear surface described by the term ω_R in the wave function Ω_R of the nuclear states are expected, therefore, to be visible in alpha-induced reactions but not so much in nucleon-induced reactions.

The difference in the absolute values of the spectroscopic factors from proton- and alpha-induced knockout reactions is observed for nuclei with relative low-lying alpha decay channels but high lying nucleon decay channels. In ^{16}O, e.g., the threshold $^{12}C_{g.s.}+\alpha$ is at 7.2 MeV and for $^{12}C_{4.4\ MeV}+\alpha$ at 11.6 MeV while the nucleon channels open only at higher energy. Furthermore, the corresponding alpha spectroscopic factors of $^{16}O_{g.s.}$ are large in comparison with the proton and neutron spectroscopic factors.[14] Consequently, alpha channels are important in ω_R which means that the surface of the nucleus ^{16}O contains alpha clusters.

In order to prove the statements given here it would be very interesting to have experimental data not only for nuclei with low-lying cluster thresholds but also for other ones. An example is ^{13}N with the $^{12}C + p$ threshold at 1.9 MeV and the $^{9}B + \alpha$ threshold at 9.5 MeV. In this case, alpha channels in ω_R should be less important than nucleon channels. Consequently, the nuclear surface is expected to contain single nonclustered nucleons. The ratio between alpha spectroscopic factors extracted from alpha- and proton-induced reactions by using rms radii is expected, therefore, to be smaller for this nucleus than for ^{16}O.

Summarizing, it can be stated that the enhanced clustering in the nuclear surface observed in alpha-induced cluster knockout reactions may be caused by the term ω_R in the wave function of the nuclear state, Eq. (1). It describes virtual particles in the nuclear surface and is characteristic of the properties of the very state. A measure of the enhanced clustering is the ratio between the cluster spectroscopic factors extracted by using rms radii from knockout reactions with different projectiles. More systematic data on different nuclei are necessary before numerical calculations can be performed and conclusions on clustering effects in the nuclear surface, the term ω_R of the wave function, and the radius extracted really from the data can be drawn. A study of the term ω_R by means of knockout reactions allows one to draw conclusions not only on cluster enhancement effects in the nuclear surface but also on other unsolved problems such as, e.g., the problem of absolute values of alpha widths in heavy nuclei according to Eqs. (6), (8), (9),[15] the problem of extracting the neutron and proton distribution radii in nuclei such as ^{40}Ca, ^{48}Ca, ^{51}V from pion scattering data, and some problems in heavy ion induced reaction processes.

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