$(\alpha, 2\alpha)$ reaction at intermediate energies

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The distortion effects are investigated for the $(\alpha, 2\alpha)$ reaction in the framework of the distorted-wave impulse approximation at intermediate energies (>100 MeV) for various nuclei. It has been found that the main effect of the distortion is to reduce the cross section. This reduction factor decreases with the energy and, at a particular energy, increases with the mass number. The distortion is also found to change the shape of the "distorted" momentum distribution around the secondary maxima. Comparison of the calculated results with the available data at 850 MeV on ^{16}O gives the spectroscopic factor in excellent agreement with the theoretically estimated value.

[NUCLEAR REACTIONS Intermediate energies, $(\alpha, 2\alpha)$, DWIA.]

In the context that the bulk of the properties of the atomic nucleus can be described in the general framework of the shell and collective models, the phenomenon of clustering in nuclei becomes a side issue. It is nevertheless important and interesting, because this aspect of the nucleus is distinctly different and complementary to the other models. These clusters, most stable of them being α particles, due to weakening of the cluster disrupting forces, and less inhibition due to the Pauli blocking, are thought to exist, in general, on the surface of the nuclei. Theoretically, apart from those in light nuclei ($A \lesssim 24$), there does not seem to exist any method, at present, to determine the probability of these clusters in medium and heavy nuclei in a consistent and systematic manner.[1] It therefore appears necessary that this probability should be extracted from the experiments. Due to surface localization of this quantity, the most suitable reaction for this purpose for α clusters seems to be the quasifree $(\alpha, 2\alpha)$ reaction. Recognizing this potential, experiments have been done for this reaction at 90 MeV and below on various targets and at 140 MeV on ^{16}O (Ref. 2). Meager $(\alpha, 2\alpha)$ data, which as a result of experimental difficulties are limited in scope, also exist at 650 and 850 MeV on ^{16}O and ^{28}Si (Ref. 3). These experimental data at and below 140 MeV have been analyzed in terms of the distorted wave impulse approximation (DWIA).[2] Unfortunately, these efforts did not achieve much success. Apart from not reproducing the shape, the calculated cross sections underestimate the measured values by about two orders of magnitude. However, recently, identifying the shortcoming of these analyses,[4] the present authors have proposed an alternative approach.[5] The predicted results of this approach agree very well, both in magnitude and shape, with the measured data at 90 and 140 MeV. Encouraged by this and since the basic assumption of the eikonal approximation for the α distorted waves in this approach is expected to be even more reliable at higher energies, in the present paper we present results for the intermediate energy α particles (i.e., >200 MeV).

As shown earlier,[4] beyond about 100 MeV incident energy the differential cross section for the $(\alpha, 2\alpha)$ reaction can be written in the factorized on-shell version of the distorted wave impulse approximation, i.e.,

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} = F_K \sigma_{\alpha\alpha}(\bar{E}, \bar{\theta}) \sum_M S_\alpha(L)(2L+1)^{-1} |P_{LM}(\vec{Q})|^2 , \quad (1)$$

where S_α is the spectroscopic factor, F_K is the kinematic factor, and $\sigma_{\alpha\alpha}$ is the free α-α cross section in its center of mass at the appropriate laboratory energy $\bar{E}$ and scattering angle $\bar{\theta}$. In the eikonal approximation and by use of the surface localization of the $(\alpha, 2\alpha)$ reaction, the "distorted" momentum distribution $P(Q)$ of the initially bound α particle in the nucleus can be written as (for details see Ref. 5)

$$P_{LM}(\vec{Q}) = (2\pi)^{-1} \int e^{i\lambda z} J_M(K^{\perp} b) f_{NL}(r) \Theta_{LM}(\theta) D_{\vec{k}_0}(yb, yz) D_{\vec{k}_1}(b,z) D_{\vec{k}_2}(b,z)\, b\, db\, dz , \quad (2)$$

with the nuclear part of the distortion factor D_k given by

$$D_{\vec{k}}(b,z) = \exp\{\tfrac{1}{2} i\chi(b)[1+\mathrm{erf}(z/\sqrt{2ab})]\} , \quad (3)$$

where $\chi(b)$ $[=2\delta(b)]$ are the α-nucleus phase shifts for the angular momentum l ($\simeq kb - \frac{1}{2}$) and a is the measure of the diffuseness of the nuclear surface. λ and $K^{\perp}$ are the longitudinal and transverse components of the recoil momentum $\vec{Q}$, respectively. f_{NL} is the radial part of the α-wave function in the bound state and is normalized to unity. The Coulomb part of the distortion is evaluated using the appropriate Coulomb potential.

Apart from $P(Q)$, the calculation of the differential cross section in Eq. (1) requires the knowledge of the free α-α cross section ($\sigma_{\alpha\alpha}$). This cross section, apart from that in limited angular range at 650 and 850 MeV, in general, does not exist at intermediate energies. Therefore, for the study of the systematics, our discussion and results would be restricted to the "distorted" momentum factor $P(Q)$ only. This would provide us with the knowledge of the presently unknown and important distortion effects in the $(\alpha, 2\alpha)$ reaction. At 850 MeV, where some limited experimental information on the $(\alpha, 2\alpha)$ reaction on ^{16}O exists, we of course compute the differential cross section at the quasielastic point and compare it with the available number.

Results are presented from 250 MeV to 1 GeV for ^{16}O. In order to have a relative measure of the distortion with the mass number A, calculations are also done at 850 MeV for ^{20}Ne, ^{24}Mg, ^{28}Si, ^{40}Ca, and ^{66}Zn. The input quantities required for these calculations, as we see from Eqs. (2) and (3) are (i) the radial bound state wave function $f_{NL}(r)$, and (ii) the α-nucleus phase shifts $\chi(b)$. They are generated in the following manner:

(i) $f_{NL}(r)$: As in our earlier work[5] f_{NL} are generated in the Woods-Saxon potential whose radius parameters are fixed by making reference to the folding model potential.[6] In order to take account of the antisymmetrization, the folding model half value radius is arbitrarily increased by 1.0 fm for all nuclei. The potential parameters are listed in Ref. 5 except those for ^{20}Ne and ^{28}Si. For these nuclei $r_0(=R_{1/2}/A_c^{1/3})$ is taken equal to 1.76 and 1.67 fm, respectively.

(ii) $\chi(b)$: For generating $\chi(b)$ we have taken guidance from the work of Kirzon and Dar[7] on the elastic scattering of α and heavier ions on nuclei. In this work it has been demonstrated that right from 65 MeV α energy onward the elastic scattering cross section can be well described by the Glauber model formalism. For our purpose we assume that this prescription is applicable for α energy between 250 MeV and 1 GeV. $\chi(b)$ is thus given by[7,8]

$$\chi(b)=\frac{1}{2\pi k}\int d^2q\, e^{-i\vec{q}\cdot\vec{b}} f_{\rm NN}(q)F_\alpha(q)F_A(q) \ , \tag{4}$$

where $F_\alpha(q)$ and $F_A(q)$ are the one-body form factors for α and the nucleus. They are defined as

$$F(q)=A\int e^{i\vec{q}\cdot\vec{r}}\rho(\vec{r})d\vec{r} \ , \tag{5}$$

with the density $\rho(\vec{r})$ normalized to unity. Using the optical theorem the NN scattering amplitude $f_{\rm NN}(q)$ can be written as

$$f_{\rm NN}(q)=(k/4\pi)\sigma_{\rm NN}^T(i+\beta)g(q) \ , \tag{6}$$

TABLE I. Nucleon-nucleon parameters.

Laboratory energy (MeV)	$\sigma_{\rm NN}^T$ (mb)	β	ν^2 (fm^2)
14.1	461	0.85	0
19.8	325	0.95	0
27.0	200	1.15	0.2
41.0	125	1.30	0.25
52.0	100	1.30	0.30
60.0	83	1.50	0.40
70.0	68	1.55	0.55
78.5	60	1.62	0.70
92.0	54	1.60	0.80
100.0	50	1.40	1.00
125.0	40	1.20	1.00
150.0	37	1.00	1.00
175.0	34	0.86	0.92
200.0	32	0.75	0.86
225.0	31	0.62	0.80
250.0	30	0.50	0.75

TABLE II. Single particle density parameters for various core nuclei.

Nuclei	R (fm)	a (fm)	w
^{16}O	2.608	0.513	−0.051
^{20}Ne	2.805	0.571	0.0
^{24}Mg	3.192	0.604	−0.249
^{36}Ar	3.73	0.62	−0.19
^{62}Ni	4.4425	0.5386	−0.209

where $\sigma_{\rm NN}^T$ is the average nucleon-nucleon total cross section, β is the ratio of the real to imaginary part of the scattering amplitude, and $g(q)$ describes the q dependence of the $f_{\rm NN}(q)$ around the forward direction. For a given α-particle energy these quantities, of course, need to be taken at the relevant nucleon-nucleon energy. This energy in the laboratory can be approximated by

$$E_{\rm N}=[E_0-(A+4)/AU_{\rm Coul}(R_c)]/4 \ , \tag{7}$$

where $U_{\rm Coul}(R_c)$ is the representative α-nucleus Coulomb potential at the surface of the nucleus. For example, R_c can be taken equal to the sum of rms radii for α and the nucleus. E_0 is the incident laboratory energy of the α particle.

For the incident energy range of our interest (250–1000 MeV), the minimum kinetic energy of the slower α particle

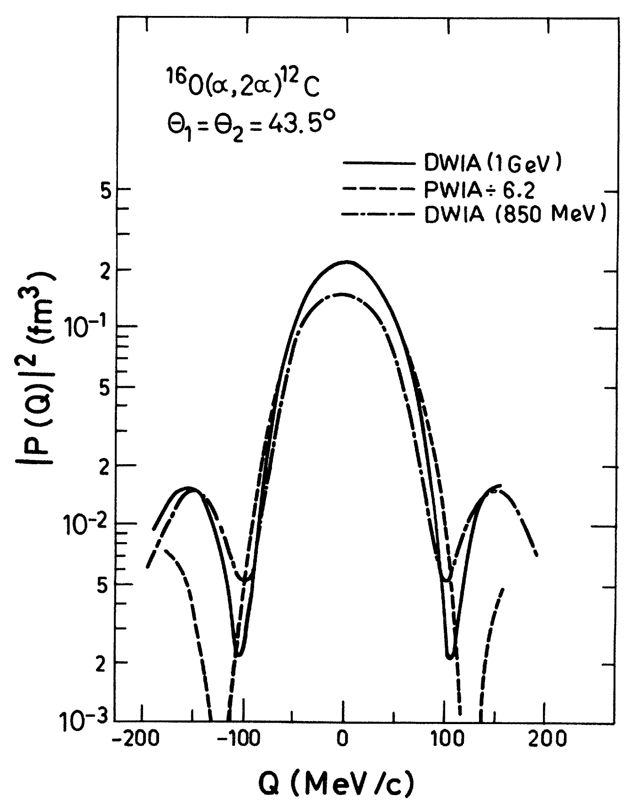

FIG. 1. "Distorted" momentum distribution vs recoil momentum Q for ^{16}O at 850 MeV and 1 GeV along with the plane wave results.

TABLE III. The ratio $|P(Q)|^2_{\mathrm{PWIA}}/|P(Q)|^2_{\mathrm{DWIA}}$ for various nuclei at different values of incident energy.

Energy (MeV) / Target	^{16}O	^{20}Ne	^{24}Mg	^{28}Si	^{40}Ca	^{66}Zn
90	300		240		720	1300
140	57.0					
250	34.0					
450	18.4					
650	12.1					
850	8.3	4.13	6.5	7.5	16.3	12.84
1000	6.2					

in the energy sharing kinematics is limited to around 80 MeV. This covers the recoil moment Q at quasielastic angles from −200 to +200 MeV/c. The required nucleon-nucleon scattering parameters [Eq. (6)] beyond 100 MeV are taken from the CERN report by Schwaller *et al.*[9] Below this energy they are extracted from the available information on the pp and pn scattering cross section.[10] All these parameters are listed in Table I. For the scattering amplitude f_{NN}, instead of distinguishing between pp and pn parameters, we have taken the average values. The slope parameter for $g(q)$ around $q=0$ is taken by assuming the $\exp(-\nu^2 q^2/2)$ form.

The calculation of $\chi(b)$ also requires the α and nuclear form factors, $F(q)$ [Eq. (4)]. After correcting for the finite proton size they are taken from the electron-scattering analyses. The proton form factor $f_{\mathrm{p}}(q)$ is described by[8,11]

$$f_{\mathrm{p}}(q)=\exp(-0.13q^2) \quad . \tag{8}$$

The α and ^{12}C form factors are taken from the analysis of Frosch *et al.*[12] and are given by

$$F(q)=[1-(cq^2)^{n}]\exp(-dq^2) \quad , \tag{9}$$

with $c=0.0999$ fm^2, $d=0.464$ fm^2, and $n=6$ for α, and $c=0.296$ fm^2, $d=0.681$ fm^2, and $n=1$ for ^{12}C. For other nuclei they are obtained from the following density form:[13]

$$\rho(r)=\rho_0(1+wr^2/R^2)/\{1+\exp[(r-R)/a]\} \quad . \tag{10}$$

The parameters for various nuclei are listed in Table II.

For ^{16}O the distorted momentum distribution $|P(Q)|^2$ is calculated at 250, 450, 650, 850, and 1000 MeV. Comparison of these results with the plane wave results shows that the main effect of distortion is to reduce the cross section. The reduction factors $|P(Q)|^2_{\mathrm{PW}}/|P(Q)|^2_{\mathrm{DW}}$ for various energies are listed in Table II. This table also contains the reduction factor at 90 and 140 MeV from our earlier work.[5] As we see, the reduction factor is very large (~300 at 90 MeV) at lower energies and decreases with the energy. By 1 GeV it comes down to around 6. This decrease in the reduction factor is understandable as the σ^T_{NN}, in terms of which the $\chi(b)$ is described, comes down with E_{N} (see Table I) in the range of α energy up to 1 GeV.

The distortion also changes the shape of the distribution $|P(Q)|^2$ with respect to Q. However, this change is not as large as that in the magnitude. The distorted momentum distribution at 1 GeV and 850 MeV along with the plane wave results are displayed in Fig. 1. The plane wave results are normalized to the distorted wave $|P(Q=0)|^2$ at 1 GeV. As these curves show, the main effect with respect to plane wave results is to increase the ratio of the cross section maximum at $Q=0$ to its secondary maxima.

In order to show that the distortion effects calculated in the present work are realistic we compared the available experimental data[3] at 850 MeV on ^{16}O with the corresponding calculated results. The experimental number pertains to $(d\sigma/dE,d\Omega_1,d\Omega_2)$ at the quasielastic point ($\sigma_\alpha = \pm 43.5°$). Comparison of this number with the calculated $|P(Q=0)|^2$ multiplied by the available $\sigma_{\alpha\alpha}$ and the kinematic factor F_K [see Eq. (1)] gives the spectroscopic factor S_α equal to 0.22 for the ^{16}O ground state. This is in excellent agreement with the theoretical estimate of 0.23 (Ref. 14).

Calculations are also done for ^{20}Ne, ^{24}Mg, ^{28}Si, ^{40}Ca, and ^{66}Zn at 850 MeV. Qualitatively, the results are similar to those on ^{16}O. Quantitatively, the reduction factors are listed in Table III. As for ^{16}O, these factors are also given at 90 MeV from our earlier work. As we see, in general, the reduction factor increases with the mass number. This trend is also understandable because the $(\alpha,2\alpha)$ reaction is surface localized; and with the increase of A surface to volume ratio of the nucleus comes down.

In conclusion, the results of the present paper on the $(\alpha,2\alpha)$ reaction from 90 to 1000 MeV for various nuclei from ^{16}O to ^{66}Zn show that the main effect of the distortion is to reduce the cross section. It also changes shape of $|P(Q)|^2$ around the secondary maxima. The reduction factor goes down with the energy and, at a particular energy, increases with the mass number.

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