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Relation between pairing correlations and two-particle space correlations

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We study the spatial correlations between two particles (or two holes) around a closed shell core in terms of the probability distribution expressed as a function of the c.m. coordinate R of the two particles and of their relative coordinate r. We find that the mixing of configurations induced by the pairing force leads to a probability distribution centered in regions corresponding to larger values of R and smaller values of r, as compared to the case of a pure $(j)_0^2$ configuration. This tendency to a "surface clustering" is mainly due to the interference of the contributions coming from levels with different parity. However, even with the inclusion of a large number of configurations, the size of the localized "cluster" is much larger than that of a free dinucleon system.

[NUCLEAR STRUCTURE Pairing correlations, correlation in space, two-particle transfer reactions.]

Two-particle transfer reactions are usually considered a typical tool for the study of particle-particle correlations in nuclei.[1] In such reactions [we are thinking, in particular, of reactions induced by light ions, such as (p,t), (t,p), or (^{3}He, n)] a dinucleon system with $I=0$, very confined in space, is transferred on to (or from) the nuclear surface. It is, therefore, of interest to study to what extent the pairs of particles in the nucleus move closely in space toward each other, in particular in the surface region, where the Pauli principle is less effective and the probability of formation of few-nucleon correlated substructures should increase. We want to clarify the effect[2-4] on this correlation in space of the particle-particle residual interaction, which is known to strongly enhance the two-particle transfer cross section. This should shed some light on the more general problem of the relation between correlations in spin, isospin, and angular momentum and clusterization in space.

We consider the case of two identical particles (or two holes), coupled to angular momentum $I=0$ and moving in single particle orbitals around a closed-shell core. The problem has been approached in Refs. 2 and 4 by selecting the $S=0$ part of the two-particle wave function and assuming the two particles at equal distance from the center of the nucleus, and then studying the behavior of the two-particle wave function as a function of the relative angle between the particles. We remove here these restrictions, both taking the full two-particle wave function and letting the two particles move in the full coordinate space. Let

$$\Psi(\vec{r}_1,\chi_1;\vec{r}_2,\chi_2)=\sum_\alpha B_\alpha[\Psi_\alpha(\vec{r}_1,\chi_1)\otimes\Psi_\alpha(\vec{r}_2,\chi_2)]_0 \tag{1}$$

be the general antisymmetrized wave function describing the two-particle system. The index α stands for the set $\{n_\alpha l_\alpha j_\alpha\}$ of quantum numbers characterizing the single particle wave function

$$\Psi_{nljm}(\vec{r},\chi)=\phi_{nlj}(r)[Y_l(\hat{r})\chi_{1/2}(\chi)]_{jm} \; . \tag{2}$$

In order to study the spatial correlations between the two particles we introduce the coordinates $R=|\vec{r}_1+\vec{r}_2|/\sqrt{2}$ and $r=|\vec{r}_1-\vec{r}_2|/\sqrt{2}$ associated with the center of mass and relative motion, respectively, and consider the probability distribution

$$P(r,R)=\int|\Psi(\vec{r}_1,\chi_1;\vec{r}_2,\chi_2)|^2 r^2R^2 d\hat{r}\, d\hat{R}\, d\chi_1 d\chi_2 \; . \tag{3}$$

In the case of harmonic oscillator (HO) wave functions the coordinate transformation and the integration over the angular variables can be performed analytically and the probability distribution (3) assumes the explicit expression

$$P(r,R)=r^2R^2\sum_{\alpha_1\alpha_2}B_{\alpha_1}B_{\alpha_2}\sum_L \hat{L}^4\hat{j}_{\alpha_1}^2\hat{j}_{\alpha_2}^2\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & L\\ l_{\alpha_1} & l_{\alpha_1} & L\\ j_{\alpha_1} & j_{\alpha_1} & 0\end{Bmatrix}\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & L\\ l_{\alpha_2} & l_{\alpha_2} & L\\ j_{\alpha_2} & j_{\alpha_2} & 0\end{Bmatrix}\sum_{\substack{nn'\lambda\\ NN'\Lambda}}\langle n_{\alpha_1}l_{\alpha_1}n_{\alpha_1}l_{\alpha_1}L|n\lambda N\Lambda L\rangle\langle n_{\alpha_2}l_{\alpha_2}n_{\alpha_2}l_{\alpha_2}L|n'\lambda N'\Lambda L\rangle$$
$$\times\phi_{n\lambda}(r)\phi_{n'\lambda}(r)\phi_{N\Lambda}(R)\phi_{N'\Lambda}(R) \; ,$$

where $\langle|\rangle$ are the Talmi-Moshinsky transformation brackets and all the HO wave functions are characterized by the same frequency as the initial single particle wave functions.

We have performed the calculation in the case of the ground state and excited 0^+ states of ^{206}Pb, i.e., two neutron holes in the $N=126$ closed shell ($\nu=0.166$ fm^{-1}). The wave functions describing the two-hole states, i.e., the amplitudes B_α in Eq. (1), have been obtained by diagonaliz-

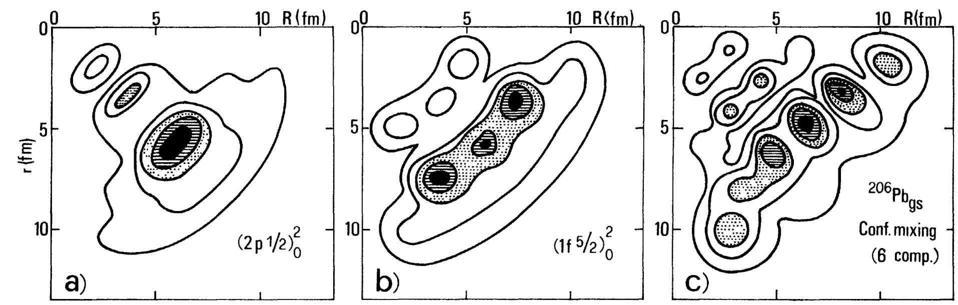

FIG. 1. Contour plots of the probability distribution $P(r,R)$ as a function of the coordinates $R=|\vec{r}_1+\vec{r}_2|\sqrt{2}$ and $r=|\vec{r}_1-\vec{r}_2|/\sqrt{2}$. The contours are drawn in steps of 0.2 times the maximum value, starting from 0.1. (a) and (b) refer to the case of the pure configurations $(2p\frac{1}{2})^2_0$ and $(1f\frac{5}{2})^2_0$, respectively. (c) gives the probability distribution for $^{206}Pb_{g.s.}$ in the case of configuration mixing. The wave function has been obtained by diagonalizing a schematic pairing force in the first six neutron levels below the Fermi surface. The energies of the single particle levels were obtained in the Hartree-Fock approximation using the Skyrme III interaction, while the pairing coupling constant was fixed to reproduce the correct binding energy of $^{206}Pb_{g.s.}$.

ing a residual schematic pairing force, whose coupling constant G has been fixed to reproduce the binding energy of $^{206}Pb_{g.s.}$. Equivalent results have been obtained in the case of ^{210}Pb.

In Figs. 1(a) and 1(b) the probability distribution $P(r,R)$ is given for the cases of the pure configurations $(2p\frac{1}{2})^{-2}_0$ and $(1f\frac{5}{2})^{-2}_0$, which are the main components of the $^{206}Pb_{g.s.}$ wave function (65% and 11%, respectively). Because of the symmetry properties of the Talmi-Moshinsky transformation, for a pure configuration $(j)^2_0$ the probability distribution is symmetric with respect to the two coordinates r and R. It also shows a systematic pattern, which can be easily correlated with the number n of nodes of the radial wave function and with the orbital angular momentum l [see also Figs. 2(a) and 2(b)]. The probability turns out to be mainly concentrated in $(n+1)$ circular shells, with a dominance of the outer shell. In this region it has a number of maxima equal to the orbital angular momentum quantum number l, in the case of antiparallel spin $(j=l-\frac{1}{2})$, and equal to $l+1$, in the case of parallel spin $(j=l+\frac{1}{2})$.

The situation changes when we introduce the mixing of configurations [Fig. 1(c)]. The probability distribution is now clearly asymmetric, being shifted toward larger values of R and, correspondingly, smaller values of r. This effect is due to the interference of contributions coming from orbitals with different parity. In fact, with the phase coming from the pairing interaction, for the ground state wave function the different parity orbitals contribute constructively at large R (and small r) and destructively in the opposite region (small R and large r). In the case considered, the whole effect is produced by the "intruder" state $(0i\frac{13}{2})$, coming from the higher major shell, even though it has the small amplitude $(B_{i13/2}=0.31)$. It is not, therefore, the use of a larger configuration space that leads *per se* to a surface "clustering," but the fact that this larger configuration space introduces components with different parity. In fact, if one has a larger mixing of opposite parity states (as it occurs in very heavy nuclei) in the wave function, even with few components one will obtain a more pronounced effect. As an example, in Fig. 2(c) is displayed the probability distribution associated with the fictitious wave function

$$1/\sqrt{2}(0h\tfrac{11}{2})^2_0+1/\sqrt{2}(0i\tfrac{13}{2})^2_0\ .$$

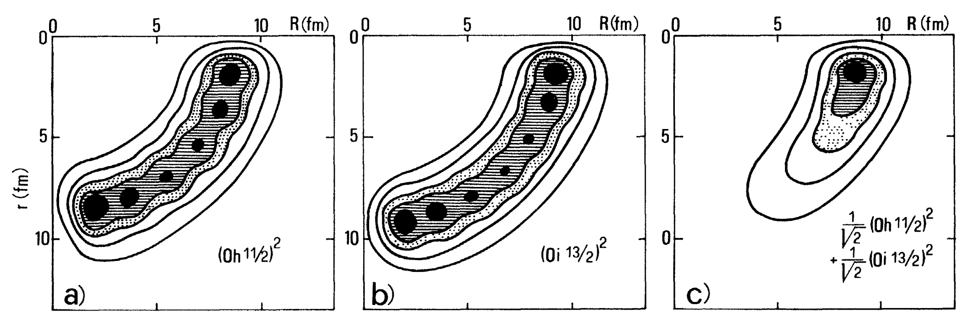

FIG. 2. Contour plots of the probability distribution $P(r,R)$ for the pure configurations (a) $(0h\frac{11}{2})^2_0$ and (b) $(0i\frac{13}{2})^2_0$, respectively. (c) refers to the case of the mixed wave function

$$\frac{1}{\sqrt{2}}(0i\tfrac{13}{2})^2_0+\frac{1}{\sqrt{2}}(0h\tfrac{11}{2})^2_0\ .$$

For more details see caption to Fig. (1).

The interference is in this case dramatic and leads to a really localized dineutron system on the nuclear surface [compare with the two pure configuration cases shown in Figs. 2(a) and 2(b)].

A better understanding of the problem may be obtained by selecting the contributions of different parts of the wave function, characterized by different intermediate quantum numbers. This is done in Fig. 3. Figure 3(b) shows the probability $P(r,R)$ associated with the $S=0$ part of the wave function (1). The bulk of the probability is further shifted to larger R (and smaller r), showing that the above described interference between orbitals with different parity is more effective in the $S=0$ channel. This part of the wave function is, however, the relevant one in the case of two-particle reactions such as a (t,p) or (p,t) reaction, since an $S=0$ state is assumed for the transferred dineutron. Furthermore, in the description of these reactions a 0*s* state is usually assumed for the relative motion of the two neutrons in the triton. Under this assumption, only the component of the two-particle wave function (1) corresponding to an *s* state for the relative motion is excited in the reaction. This corresponds to picking up only the terms with $\lambda=\Lambda=0$ in (4), and this leads to the results displayed in Fig. 3(c), characterized by a further enhancement of the surface clusterization. We remark for comparison that for a pure $(j)^2_0$ configuration also the probability distributions associated with both the $S=0$ part and the $\lambda=\Lambda=0$ part remain symmetric with respect to r and R. The mixing of configurations has instead enhanced the $n=n'=0$ component in (4), corresponding to a more localized wave function in the r variable. In fact it has practically shifted most of the probability similarly to the case in which we force the relative motion to be described by a 0*s* state, by picking up only the terms $L=\lambda=\Lambda=0$ and $n=n'=0$ in the sum (4), as shown in Fig. 3(d).

We can examine now whether the pairing interaction has noticeable effects on the excited 0^+ states. As seen from Fig. 4, the probability distributions $P(r,R)$ associated with these states do not show any surface enhancement, but rather closely resemble the ones associated with pure configurations. This is a direct consequence of the small collectivity of these states and of the relative phases of the amplitudes which are not all coherent, as in the case of the ground state. This does not rule out the possibility that, owing to the shell structure, definite excited states may be characterized by a mixture of states with different parity, leading to situations such as the one displayed in Fig. 2, but such as "accident" cannot, of course, be traced back to the pairing interaction.

We can conclude that the pairing interaction leads, in the case of the ground state, to pairs of valence particles which are localized more closely on the nuclear surface than un-

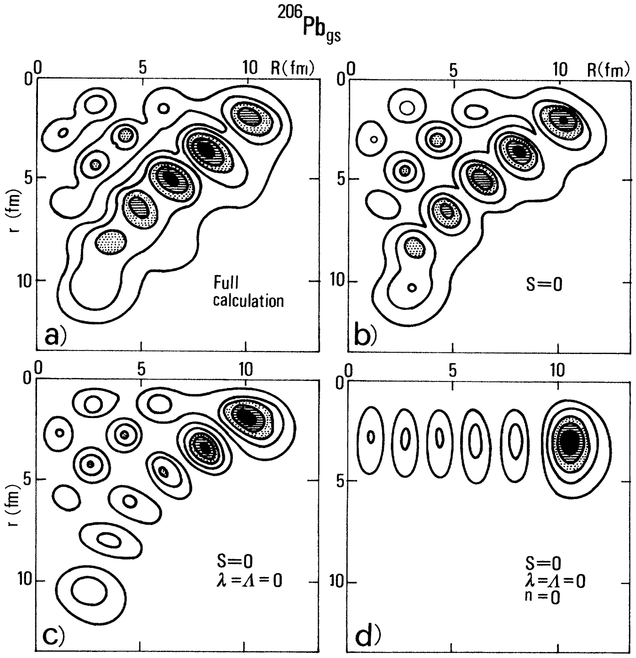

FIG. 3. Contour plots of the probability distribution $P(r,R)$ for $^{206}Pb_{g.s.}$, associated with different parts of the two-hole wave function. (a) gives the results associated with the full wave function (2). In this case the two holes are correlated in the whole set of 22 filled neutron hole states and the use of a larger configuration space has further shifted the probability distribution towards larger values of r [compare Fig. 1(c), where only 6 levels were used]. (b) shows the probability distribution associated to the $S=L=0$ part of the wave function (2). The integrated probability amounts to about 72% of the one obtained in the full case. In (c) we introduce the further restriction $\lambda=\Lambda=0$ in the expansion (4). The total integrated probability has now decreased to be 32% of the one shown in (a). Finally (d) corresponds to the terms with $n'=n=\lambda=\Lambda=S=L=0$ in the expansion (4) (with a total integrated probability which now amounts to 16% of the full one).

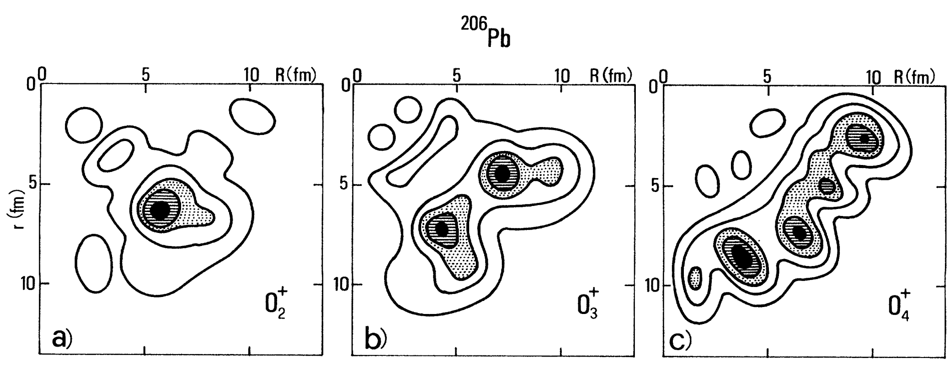

FIG. 4. Contour plots of the probability distribution $P(r,R)$ associated with the excited 0^+ states of ^{206}Pb. For more details see caption to Fig. (1).

correlated pairs. Although this effect is noticeable and will reflect itself in larger transfer cross sections, the size of the cluster, even with the inclusion of a large configuration space, is much larger than that of a free dinucleon system. This is an obvious consequence of the well known fact that the pairing matrix elements (~ 1 MeV) are too small to overcome the dominant shell model structure. The same reason is responsible for the fact that the excited states are only very weakly affected by the pairing correlation. We therefore do not agree with the statement of Ref. 4 that the inclusion of a very large configuration space and a subsequent proper treatment of the continuum would give rise to a real δ-like cluster. This would mean, in fact, that the pairing interaction has been able to completely overcome the shell structure.

The problem studied in this paper also has been studied with different techniques by Broglia, Dasso, Ferreira, Liotta, and Winther.[5] We are indebted to all of them for several discussions which stimulated the present work. We acknowledge financial support from INFN, Italy.

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