Four-particle–four-hole states in ^{4}He

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Deformed states in ^{4}He, in which four nucleons are placed in the $1p$ shell, are considered. An interaction consisting of a Gaussian attraction and zero range density dependent repulsion is employed. The parameters are chosen so that the ground state energy of ^{4}He is -28 MeV, and the oscillator length parameter b is 1.4 fm. The method of variation after projection is employed, using the techniques of Evans and Elliot, to obtain the energies of $J=0^+$, 2^+, and 4^+ states. The results are studied as a function of the range of the Gaussian interaction. Connection is made with the more extensively studied problem of four-particle–four-hole states in ^{16}O.

INTRODUCTION

In this work we wish to consider the possibility of identifying four-particle–four-hole states in ^{4}He using methods which are analogous to those used in ^{16}O. In the latter nucleus, Brown and Green[1] described the $J=0^+$ first excited state at 6.03 MeV as mainly a four-particle–four-hole highly deformed state. The assumption of large deformation was essential to get the state to come at a sufficiently low energy, and to explain the large intraband electric quadrupole transition rates. We shall see that deformation is also extremely important for ^{4}He.

In medium-heavy mass nuclei density dependent interactions have often been used to perform Hartree-Fock calculations. We shall here use them for the very light nucleus ^{4}He. Following the philosophy of Brink and Boeker,[2] we choose the parameters so that the ground state properties come out reasonably well. Then the interaction is used to explore excited states.

Because there are uncertainties in the precise form of the interaction, we study the problem of four-particle–four-hole states with a variable parameter—the range of an attractive Gaussian force. We shall see that the results are extremely sensitive to this range.

One reason for studying ^{4}He in this way is that this nucleus is amenable to exact or nearly exact calculations, e.g., in the recent work of Ballot,[3] and of Ayoub.[4] Furthermore, ^{4}He is the lightest nucleus which in some sense behaves like a usual closed shell nucleus, e.g., it has a binding energy per particle of about 8 MeV.

This nucleus may then provide a good testing ground for the phenomenological approaches which are used in heavier nuclei, e.g., the use of density dependent interactions to simulate the importance of the tensor force in second order.

Because of the uncertainties in the interactions, we try to correlate to some extent the results of ^{4}He with those of ^{16}O. We shall see that a form of interaction which brings the $J=0^+$ 4p-4h state sufficiently low in ^{16}O will also bring the state in ^{4}He sufficiently low so as to associate it either with the first excited 0^+ state at 20.1 MeV, or the state at 25.5 MeV (it is not certain if this is a 0^+ state).

In the course of the work, we make several approximations. Deformed oscillator wave functions are used as are approximate projection procedures owing to Elliot and Evans.[5] Experience suggests that the exact calculations would lower the excitation energy relative to the results we obtain here. It is also hoped that these calculations will stimulate the search for rotational bands in ^{4}He.

THE INTERACTION AND GROUND STATE PROPERTIES—^{4}He

The interaction used is

$$-V_0[(1+P^M)/2+\xi(1-P^M)/2]e^{-r^2/a^2}+t_3/6\rho(R)\delta(\vec{r})\ ,$$

where

$$\vec{r}=\vec{r}_1-\vec{r}_2\ ,\quad \vec{R}=(\vec{r}_1+\vec{r}_2)/2\ ,$$

and P^M is the Marjorana exchange operator.

One should say at the outset, that for the particular problems considered here, four particles in the $1s$ shell (ground state) and four particles in the $1p$ shell, there can be no odd state interactions. Hence the Gaussian term can be simplified to

$$-V_0e^{-r^2/a^2}\ .$$

We use harmonic oscillator wave functions. We impose the condition that the ground state energy is -28 MeV and that the oscillator length parameter has a value $b=1.4$ fm. It is convenient to introduce T_3 such that

$$T_3=3t_3/(144\sqrt{3}\pi^3)\ .$$

A Hamiltonian is used in which the center of mass energy is removed

$$\bar{H}=H-\frac{1}{2Am}\left[\sum_i\vec{P}_i\right]^2\ .$$

This has the effect of changing the kinetic energy of four $1s$ nucleons from

$3\hbar^2/mb^2$ to $2.25\hbar^2/mb^2$.

The expression for the energy of four 1s nucleons is then

$$E(b)=2.25\hbar^2/mb^2-6V_0\alpha^3+64T_3/b^6 ,$$

where

$$\alpha^2=1/(1+2b^2/a^2) .$$

We also demand that

$$\frac{dE(b)}{db}=0 \text{ at } b=1.4 \text{ fm} .$$

Solving these equations, we find

$$V_0=9.8963/[\alpha^3(1-\alpha^2b^2/a^2)] ,$$

$$T_3/b^6=-1.1729+0.9278/(1-\alpha^2b^2/a^2) .$$

It is of interest to consider the zero range limit of this interaction. In that case, the interaction becomes identical to a (velocity independent) Skyrme interaction[6]

$$-t_0\delta(\vec{r})+t_3/6\rho(R)\delta(\vec{r}) ,$$

with

$$\lim_{\substack{a\to o\\ V_0\to\infty}} V_0a^3=t_0/\pi^{3/2} .$$

In this limit,

$$\alpha\to\frac{a}{\sqrt{2}b} ,$$

$$V_0a^3\to 153.614 \text{ MeV fm}^3 ,$$

and

$$T_3/b^6\to 0.6826 \text{ MeV} .$$

As the range a is increased, there is a point where T_3 vanishes. This occurs at $a=2.336$ fm.

THE FOUR-PARTICLE—FOUR-HOLE STATE

Four nucleons are now put in the deformed $1p$ shell. It is convenient to use Cartesian coordinates x, y, and z and frequencies ω_x, ω_y, and ω_z. We classify the states by (N_x, N_y, N_z), where N_x is the number of nodes in the x direction, etc. It is also convenient to introduce Σ_x, Σ_y, Σ_z, where Σ_x is sum $(N_x+\frac{1}{2})$ over the occupied state. The kinetic energy of the deformed intrinsic state is

$$\frac{3}{4}\left[\frac{\hbar\omega_x}{2}\Sigma_x+\frac{\hbar\omega_y}{2}\Sigma_y+\frac{\hbar\omega_z}{2}\Sigma_z\right] .$$

(The factor of $\frac{3}{4}$ is particular to ^{4}He and results from taking out the center of mass energy.) The four nucleons are put in the state $|0,0,1\rangle$. Thus $\Sigma_x=2$, $\Sigma_y=2$, and $\Sigma_z=6$. The energy of the intrinsic state is

$$\left\langle H-\frac{1}{2Am}\left[\sum_i\vec{P}_i\right]^2\right\rangle=\tfrac{3}{4}(\hbar\omega_x+\hbar\omega_y+3\hbar\omega_z)$$
$$-\tfrac{1}{2}V_0\alpha_x\alpha_y\alpha_z(9-6\alpha_z^2+9\alpha_z^4)$$
$$+T_3/b^6 35.5555 .$$

We assume axial symmetry and define t via

$$\omega_x=t\omega_0 ,$$

$$\omega_y=t\omega_0 ,$$

$$\omega_z=t^{-2}\omega_0 .$$

We treat t as a variational parameter. In the zero range limit, the potential energy of the intrinsic state becomes

$$-12T_0/b^3+35.5555T_3/b^6$$

with

$$T_0=\frac{3}{16\sqrt{2}}(V_0a^3)_{a\to 0} ,$$

where

$$b_0^3=b_xb_yb_z .$$

Hence the potential energy is independent of deformation. In that case the deformation is obtained by minimizing the kinetic energy alone. As noted by Ripka,[7] this leads to the Mottelson conditions[8]

$$\hbar\omega_x=\hbar\omega_0(\Sigma_x\Sigma_y\Sigma_z)^{1/3}/\Sigma_x ,$$

$$\hbar\omega_y=\hbar\omega_0(\Sigma_x\Sigma_y\Sigma_z)^{1/3}/\Sigma_y ,$$

$$\hbar\omega_z=\hbar\omega_0(\Sigma_x\Sigma_y\Sigma_z)^{1/3}/\Sigma_z .$$

In this limit the parameter t becomes equal to 1.4422.

PROJECTION OF ANGULAR MOMENTUM USING THE ELLIOT-EVANS METHOD—^{4}He

Elliot and Evans[5] have devised an approximate method of angular momentum projection which is much easier to use than the exact method. Instead of projecting out a state of definite angular momentum J, one is content to construct a state for which the average value of J^2 is $J(J+1)$.

Let us consider the nucleus ^{4}He for which the 4p-4h state is axially symmetric. Let H be the Hamiltonian. One constructs $H'=H-\lambda J^2$. The variational ground state solution of H' is designated by Φ. One eliminates the Lagrange multiplier by imposing the constraint

$$\langle\Phi J^2\Phi\rangle=J(J+1) .$$

The variational solution for the *intrinsic* state was of the form $(|001\rangle)^4$. In the variational solution of H', we replace $|001\rangle$ by

$$\psi_\gamma=\cos\gamma\,|001\rangle+i\,\sin\gamma\,|010\rangle .$$

The expectation value of the Hamiltonian becomes

$$\langle H\rangle=\tfrac{3}{4}[\hbar\omega_x+\hbar\omega_y+3\hbar\omega_z+2(\hbar\omega_y-\hbar\omega_z)\sin^2\gamma]-\tfrac{1}{2}V_0\alpha_x\alpha_y\alpha_z(9-6\alpha_z^2+9\alpha_z^4)+T_3/b^6 35.5555$$
$$+\sin^2 2\gamma[\tfrac{3}{4}V_0\alpha_x\alpha_y\alpha_z(2-\alpha_y^2-3\alpha_z^2+3\alpha_z^4-\alpha_y^2\alpha_z^2)-21.333\,33T_3/b^6] .$$

The expectation value of $\langle J^2 \rangle$ is

$$\langle J^2 \rangle = J(J+1) = (3\sin^2 2\gamma + 2)(\omega_y/\omega_z + \omega_z/\omega_y + 2) .$$

We introduce

$$p = \sin^2 2\gamma .$$

There are two possible solutions for $\sin^2\gamma$,

$$\sin^2\gamma = \tfrac{1}{2}(1 \pm \sqrt{1-p}) .$$

We choose the solution which gives the lowest energy, namely

$$\sin^2\gamma = \tfrac{1}{2}(1 - \sqrt{1-p}) .$$

One can express p in terms of J and thus the expectation value of H will depend explicitly on J. For each J we can obtain a different deformation parameter t_J which minimizes the expectation value. The results are presented in Table I. The first column contains different values of the range parameter a. The second column gives the energy of the intrinsic state in parentheses, the deformation parameter $t = \omega_x/\omega_0$. The next three columns give the energies of the $J = 0$, 2, and 4^+ states, respectively, which have been obtained by *variation after projection* using the Elliot-Evans approximation.[5]

A striking feature of the results is that the energies increase with the range a. For example, the energy of the $J = 0^+$ state changes from 16.27 MeV to 41.43 MeV as the range is varied from $a = 0$ to $a = 2.2$. This is undoubtedly an effective mass effect. For zero range, the effective mass is 1. As the range increases, m'/m becomes less than 1.

Thus we see that for small a, the energies are such that we could associate the 4p-4h state with the 20.1 MeV state in ^{4}He or possibly the 25.5 MeV state. But for large ranges, the state comes out too high. Also, the deformation decreases as a increases. We note that there is a sharp rise of E with J especially with small values of a.

The magnitude of the energy difference between the intrinsic state and the projected $J = 0$ state is largest when a is smallest. For example when $a = 0$ this difference is 14.93 MeV. For $a = 1.8$ fm the difference is only 6.99 MeV.

We also note that the deformation parameter $t = \omega_x/\omega_0$ is larger for the case of variation after projection than it is for the intrinsic state. This is true for each value of J. Indeed, the deformation *increases* with J.

The values of $\sin^2 2\gamma$ and $\sin^2\gamma$ are listed in Table II for $J = 0$, 2, and 4 and for various values of the range a. Note that for $J = 0$, the value of $\sin^2 2\gamma$ is $-\frac{2}{3}$ independent of a. This is evident from the equation relating the angular momentum to γ. The fact that $\sin^2 2\gamma$ can be negative has been discussed by Elliot and Evans.[5]

EFFECTIVE MASS

The contribution from the Gaussian interaction to the ground state energy of ^{4}He is $-6V_0\alpha^3$. We define the finite range energy $E_{\rm FR}$ as the difference of this and the energy obtained in the zero range limit, in which case α approaches $a/(\sqrt{2}b)$. Thus we obtain

$$E_{\rm FR} = -6V_0\alpha^3 + \frac{6V_0a^3}{2\sqrt{2}b^3} .$$

The effective mass is then given by the expression (E_K is the kinetic energy)

$$\frac{m}{m^*} = (1 + E_{\rm FR}/E_K) .$$

Thus

$$\frac{m}{m^*} = 1 + [6V_0a^3/(2\sqrt{2}b^3) - V_0\alpha^3]/2.25\hbar\omega .$$

Clearly for $a = 0$ $m^*/m = 1$. We find that for $a = 0.5$ fm $m^*/m = 0.81$ while for $a = 1$ fm $m^*/m = 0.54$. Currently, values of m^*/m of about 0.7 to 0.75 are favored. With $a = 0.5$ fm, the energy of the 4p-4h 0^+ state comes at 21.55 MeV, while with $a = 1$ fm it comes at 31.07 MeV.

CONNECTIONS WITH ^{16}O

The four-particle—four-hole states in ^{16}O have been extensively studied by many authors, including Engeland,[9] Brown and Green,[1] Bassichis and Ripka,[10] Kelson,[11] Talmi and Unna,[12] Brink and Boeker,[2] Boeker,[13] Stephenson and Banerjee,[14] Volkoff,[15] Zamick,[16] Arima,[17] Bertch,[18] Krieger,[19] Irvine *et al.*,[20] Halika *et al.*,[21] Harvey and Khanna,[22] and undoubtedly many others.

Brink and Boeker[2,13] were the first to constrain the interaction used in the four-particle—four-hole calculation to fit the binding energy and radius of the ground state. We have adopted this philosophy here. Boeker[13] and Stephenson and Banerjee[14] noted that the lowest Hartree-

TABLE I. The calculated energies and deformed parameters of the four-particle—four-hole states in ^{4}He.

Range a (fm)	Intrinsic state energy ($t = \omega_x/\omega_0$) (MeV)	Variation after projection $J = 0$	$J = 2$	$J = 4$
0	31.12(1.45)	16.27(1.55)	23.30(1.70)	37.40(2.10)
0.1	31.29(1.45)	16.46(1.55)	23.51(1.70)	37.60(2.05)
0.5	34.87(1.45)	21.55(1.55)	28.19(1.60)	42.49(1.95)
1.0	41.53(1.40)	31.07(1.50)	36.59(1.55)	49.91(1.75)
1.5	45.99(1.35)	37.90(1.50)	42.38(1.50)	54.28(1.65)
1.8	47.14(1.35)	40.15(1.45)	44.17(1.45)	55.35(1.55)
2.2	47.29(1.30)	41.43(1.40)	44.98(1.40)	55.34(1.40)

TABLE II. The values of $\sin^2 2\gamma$ and $\sin^2\gamma$.

Range	$J=0$		$J=2$		$J=4$	
a (fm)	$\sin^2 2\gamma$	$\sin^2\gamma$	$\sin^2 2\gamma$	$\sin^2\gamma$	$\sin^2 2\gamma$	$\sin^2\gamma$
0	$-\frac{2}{3}$	-0.1455	-0.3856	-0.0886	-0.0803	-0.0197
0.1	$-\frac{2}{3}$	-0.1455	-0.3856	-0.0886	-0.0454	-0.0112
0.5	$-\frac{2}{3}$	-0.1455	-0.3512	-0.0812	0.0314	0.0079
1.0	$-\frac{2}{3}$	-0.1455	-0.3329	-0.0773	0.2168	0.0575
1.5	$-\frac{2}{3}$	-0.1455	-0.3140	-0.0732	0.3262	0.0896
1.8	$-\frac{2}{3}$	-0.1455	-0.2947	-0.0689	0.4459	0.1278
2.2	$-\frac{2}{3}$	-0.1455	-0.2752	-0.0646	0.6384	0.1993

Fock state was triaxial.

This can be most easily seen in the limit of a zero range interaction. To construct the four-particle—four-hole state one destroys four $1p$ shell quanta in the y direction and one creates $2s$-$1d$ shell quanta in the z direction. In that case

$$\Sigma_x=12,\ \ \Sigma_y=8,\ \ \Sigma_z=20\ .$$

(The removal of the center of mass energy can be achieved by changing these numbers to $\Sigma_x=11.5$, $\Sigma_y=7.5$, and $\Sigma_z=19.5$.) Then, since the deformation of the intrinsic state is independent of the potential energy, the frequencies are given, as previously mentioned, by the Mottelson conditions. Hence, $\omega_x=1.034\omega_0$, $\omega_y=1.586\omega_0$ and $\omega_z=0.610\omega_0$.

With the zero range interaction $-t_0\delta(\vec{r})+t_3/6\rho(R)\delta(\vec{r})$ we find that the energies of the ground state and 4p-4h intrinsic states are

$$\text{(ground)}\ \ E=\hbar^2/mb^2(17.25)-124T_0/b^3 +824.8889T_3/b^6\ ,$$

$$\text{(4p-4h)}\ \ E=\hbar^2/mb_0^2 17.8384-122.25T_0/b^3 +834.37037T_3/b^6$$

$$[T_0=3t_0/(8(2\pi)^{3/2})]\ .$$

Setting the oscillator length parameter to a value $b=1.76$ fm and the ground state energy to $E=-128$ MeV, one finds

$$T_0/b^3=4.51959\text{ MeV}\ ,$$

$$T_3/b^6=0.247436\text{ MeV}\ .$$

The intrinsic energy of the 4p-4h state comes out at -109.957 MeV. That is to say the excitation energy of the intrinsic state is 18.043 MeV (if we allow b_0 to vary, we find $b_{\text{4p-4h}}/b_{\text{ground}}=1.017$ and the excitation energy now becomes 17.60 MeV).

Lamme and Boeker[23] found for the $B1$ interaction that the $J=0^+$ state came down 8.8 MeV below the intrinsic state. However, as noted for ^{4}He, one gets a much larger difference when the range of the Gaussian a is smaller.

COMMENTS ON THE KINETIC ENERGY

Bertsch[18] has pointed out that the kinetic energy of the 4p-4h deformed state should be nearly the same as that of the ground state. If we treated the 4p-4h state as spherical, then the kinetic energy would be $\frac{3}{4}(5\hbar\omega)$ as compared with $\frac{3}{4}(3\hbar\omega)$ for the ground state (remember that the factor of $\frac{3}{4}$ comes from removing the center of mass energy).

Taking into account deformation and projection, the kinetic energy of the 4p-4h state is

$$\frac{3}{4}\hbar\omega\left[2t+3/t^2+2\left[t-\frac{1}{t^2}\right]\sin^2\gamma\right]\ .$$

For $a\to 0$ we have $t=1.45$ and for $J=0$ $\sin^2\gamma=0.1455$. The kinetic energy is then $\frac{3}{4}(3.092)\hbar\omega$, which is close to the ground state value, thus confirming Bertsch's conjecture.

Actually, if we allow b_0 to vary in the 4p-4h intrinsic state, we find $b_{\text{4p-4h}}/b_0=1.055$. This decreases the kinetic energy of the 4p-4h state by a factor $(1.055)^2$. Thus, we

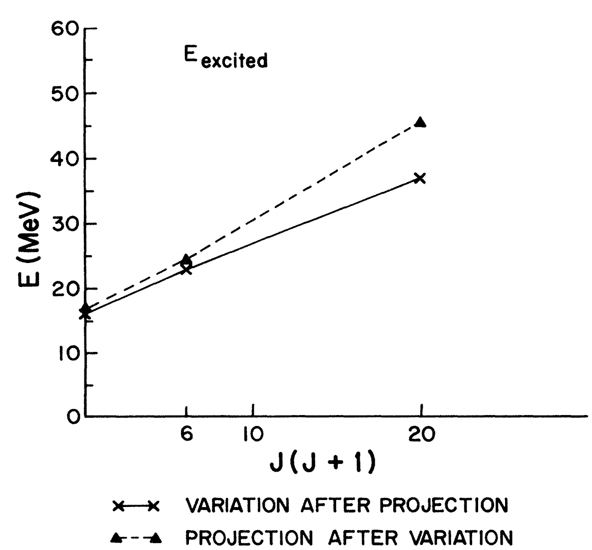

FIG. 1. The energies of the $J=0^+$, 2^+, and 4^+ 4p-4h states, variation after projection versus projection from a fixed intrinsic state. The results are for a zero range interaction.

obtain a value of $\frac{3}{4}(2.78\hbar\omega_0)$ which is even less than that for the ground state.

VARIATION AFTER PROJECTION COMPARED WITH PROJECTION AFTER VARIATION

In Fig. 1, we show the energies plotted against $J(J+1)$ for two methods. First, the presumably superior method of variation after projection (VAP), and second, the easier and hence more often employed method of projection from an intrinsic state, i.e., projection *after* variation (PAV).

We note that the energies of the $J=0^+$, 2^+, and 4^+ states are lower with the VAP method than with the PAV method. This is particularly true for the 4^+ state, which is 7.5 MeV lower in the VAP method. The energy levels in the VAP method follow the $J(J+1)$ law more closely than in the PAV approach. It would appear then that variation after projection is important especially for the high spin states in the band.

ADDITIONAL REMARKS

We may consider other forms of the interaction. For example, the density dependent term can be generalized to $-t_3/6\rho^{\sigma}(R)\delta(\vec{r})$ where σ is the power of the density. Indeed, it was shown by one of us[24,25] that the nuclear compressibility is very sensitive to σ. For example, with a zero range attraction ($a=0$) the formula is

$$K=\frac{1}{A}[\langle T\rangle+9E_B+\sigma(3\langle T\rangle+9E_B)]\ ,$$

where $E_{B/A}$ is the binding energy per particle and $\langle T\rangle/A$ the kinetic energy per particle. Thus if we take 8 MeV and 20 MeV, respectively, for these quantities, we obtain $K=(92+132\sigma)$ MeV.

Up to now the calculations have been performed for $\sigma=1$. We have obtained the energies of the intrinsic states of ^{4}He and ^{16}O for $\sigma=2$. We find that the energies in both cases are about 1.5 MeV higher than for $\sigma=1$. The accepted nuclear compressibilities are consistent with small values of σ, i.e., $\sigma=\frac{1}{6}$. By extrapolation we expect the results to be about 1 or 2 MeV lower than for $\sigma=1$, although the explicit calculations have not been done.

We should mention that the removal of center of mass energy is important for ^{4}He. The calculated 4p-4h intrinsic state energy is 6.6 MeV lower than it would have been had we ignored the center of mass energy.

Note that for the state considered here, four particles in the $|0,0,1\rangle$ states, the density vanishes at the center. Of course admixtures of other configurations, e.g., 2p-2h, will modify this. Still, it will be of interest to explore the implications of a vanishing central density.

The experimental situation concerning a deformed rotational band is at present not clear. The most comprehensive survey on ^{4}He was done in 1973 by Fiarman and Meyerhof.[26] We show in Fig. 2 the energy levels from the compilation of Lederer and Shirley.[27] Recently, Gruebler *et al.* reported overwhelming evidence for a 1^- level at 24.1 MeV and strong indication of a 4^+ level at 24.6 MeV.[28] It is difficult to understand theoretically how a 4^+ level could occur below the first 2^+ level. Note that neither of these levels are in the above compilation.[26]

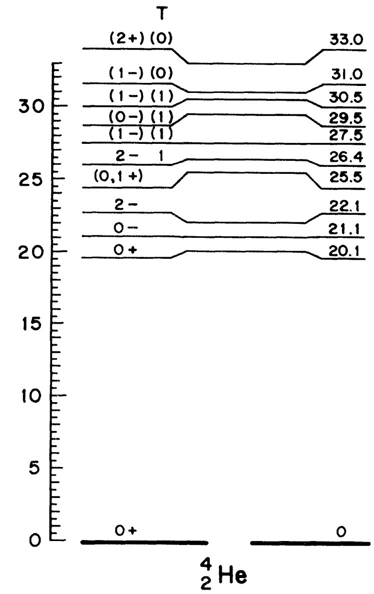

FIG. 2. The energy levels of ^{4}He (experiment).

The positive parity states which are of concern here are (0^+) at 20.1 MeV, (4^+) at 24.6 MeV, $(0,1^+)$ at 25.5 MeV, and (2^+) at 33.0 MeV. It would clearly be of help to have more definite spin assignments and to make sure that all the levels have been found. Also in our calculations the energy of the 4^+ state ranges from 37.4 MeV for $a=0$ to 55.34 MeV for $a=2.2$. Therefore a search for 4^+ resonances in this energy range would be in order.

To summarize then, we have made not an ironclad but a plausible case that four-particle–four-hole deformed states could exist in ^{4}He at a sufficiently low energy so as to be detectable by experiment. The uncertainty in the calculated energy is here parametrized by the range of the Gaussian attractive interaction. It is noted that the same interaction which brings the $J=0^+$ state in ^{4}He to a sufficiently low energy (20–25 MeV), also brings the 4p-4h state in ^{16}O to a reasonable energy so that it could be identified with the 6.05 MeV state. On the other hand in ^{16}O one has one additional parameter, the strength of the p state repulsion, which does not enter for ^{4}He.

Clearly, further work has to be done both in theory and experiment. The fact that ^{4}He is amenable to nearly exact calculations, suggests that this nucleus is a good testing ground for the approximate many body techniques that are employed in heavier nuclei.

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