Quasi-shell closure for even $Z=58$ to 70, $N=82$ and the similarity of these nuclei

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Diverse experimental information is analyzed to show that all the $N=82$ isotones with $Z=58$, 60, 62, 64, 66, 68, and 70 are doubly "magic" nuclei. Also, nuclei in this region are shown to exhibit identical behavior, i.e., they are similar in many ways. The implications of this discovery for the interacting boson model and other areas of physics are discussed.

[NUCLEAR STRUCTURE $Z=58$ to 70, $N=82$ isotones, stability and magicity, seniority mixing.]

The shell closure at nucleon numbers 2, 8, 20, 50, 82, and 126 has long been a basis for a better understanding of nuclear physics. Nature is, however, prepared to exhibit much more order than this. One unique manifestation of such an order is presented in this paper.

Data from diverse experiments are collected and analyzed in this paper. It is shown that all the isotones with $N=82$ and $Z=58$, 60, 62, 64, 66, 68, and 70 are very stable in neutron and proton structure, and as such, are doubly "magic." Not only that, in many ways they look alike, too. The persistence of magicity and similarity, over such a large region as this, is an amazing phenomenon. Both these properties are in fact interlinked. At the end of this paper some physical implications of this inertia are pointed out.

Recently the $Z=64$, $N=82$ nucleus ^{146}Gd has attracted a lot of attention.[1-8] An analysis of high spin p-h excitations in this nucleus indicates that there is a large gap in the single particle spectrum and ^{146}Gd is now very commonly used as a doubly closed core to specify multiparticle high spin states in neighboring nuclei.[1] Lawson,[6] using this fact, made predictions for the low lying spectra associated with the $\pi h_{11/2}{}^n$ configuration when $n=4$, 5, and 6, i.e., for the nuclei ^{150}Er, ^{151}Tm, and ^{152}Yb. He assumed that ^{148}Dy is described by a $\pi h_{11/2}{}^2$ configuration outside an inert ^{146}Gd core. If one is calculating excitation energies, the residual interaction

$$E_0=\langle h_{11/2}{}^2 \,|\, v \,|\, h_{11/2}{}^2\rangle_{J=0}$$

is unimportant. The experimentally observed energies provided him with the complete set of $(\pi h_{11/2})^2$ matrix elements (see Fig. 1). These are (in MeV): $E_0=0$, $E_2=1.678$, $E_4=2.427$, $E_6=2.731$, $E_8=2.832$, and $E_{10}=2.918$. Subsequently, good agreement was obtained with the experimental observation of levels in ^{150}Er.[8] Good agreement was also obtained for yrast $\pi h_{11/2}{}^n$ excitations in other proton-rich nuclei with $N=82$.[7]

This success should not make us complacent. There is a basic problem with the $(\pi h_{11/2})^2$ matrix elements taken from the positive parity yrast levels of ^{150}Er. Let us look at the identical particle $0d_{5/2}{}^2$, $0f_{7/2}{}^2$, and $0g_{9/2}{}^2$ spectrum plotted in Fig. 2. The decrease of level spacing as a function of mass number had been noted by Talmi.[9] He pointed out that this is due to a decrease in the interaction energy in the $J=0$ state. Physically it can be understood as due to the outer nucleons being farther apart in larger nuclei. As one goes from orbit d to f to g, the 2^+ state falls from 1.982 to 1.461 to 1.058 MeV. As such, one would expect the 2^+ state arising from $h_{11/2}{}^2$ to be considerably lower than 1 MeV. On the other hand, the corresponding value taken in the above-mentioned calculations is 1.6777 MeV; a much higher value. This prompts a further investigation into the problem.

For the closed shell nucleus ^{56}Ni, the lowest excited state is 2^+ at 2.702 MeV. ^{54}Fe can be regarded as having a $f_{7/2}{}^{-2}$ configuration with an inert ^{56}Ni. The 2^+ state at

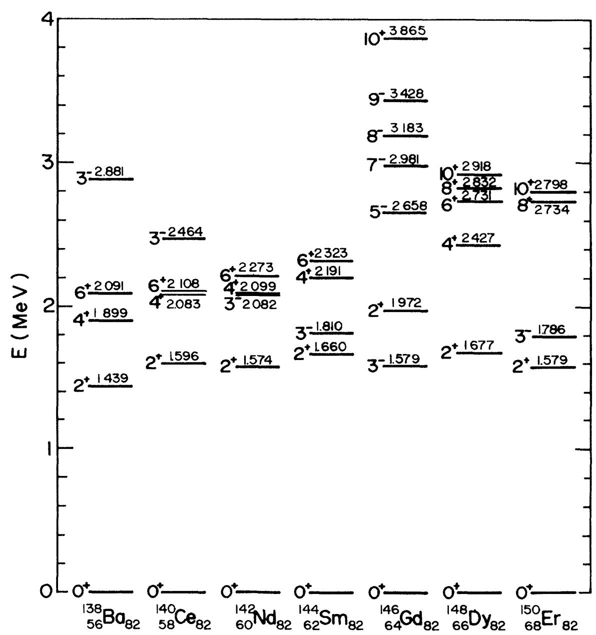

FIG. 1. Positive parity yrast states for some Z even, $N=82$ isotones, including the lowest 3^- state and the negative parity yrast states of ^{146}Gd. Most of the levels are from Refs. 35 and 36. Many high lying states from Ref. 37. ^{150}Er levels from Ref. 8.

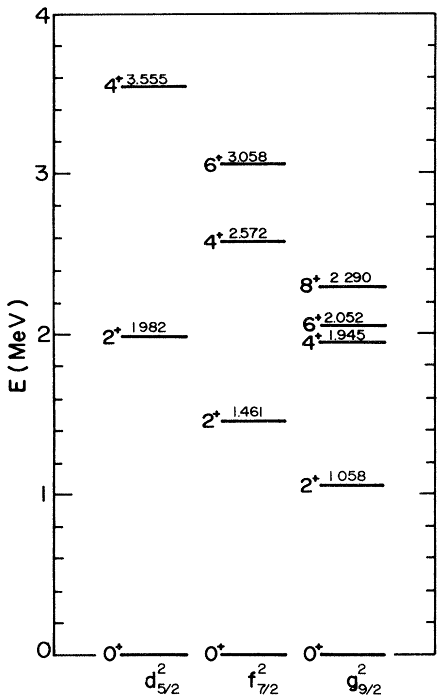

FIG. 2. $d^2{}_{5/2}$, $f^2{}_{7/2}$, and $g^2{}_{9/2}$ matrix elements from Ref. 16. $d^2{}_{5/2}$ is from the ^{18}O spectrum; $f^2{}_{7/2}$ and $g^2{}_{9/2}$ are best fit values.

1.4077 therein arises from this configuration. Note that this is close to the corresponding empirically determined value of 1.461 MeV (see Fig. 2). As far as excitation energies over the ground state are concerned, the $\pi f_{7/2}{}^{-2}$ spectrum is the same as the $\pi f_{7/2}{}^{2}$ spectrum. We therefore note that the 2^+ in a closed shell is about twice as high as the 2^+ arising from the $f_{7/2}{}^2$ configuration. To give another example, the 2^+ state at 1.0569 MeV due to the $g_{9/2}{}^{-2}$ configuration in ^{88}Zr (or the best 2^+ fit due to the $g_{9/2}{}^2$ configuration—see Fig. 2) is twice as low as the lowest excited states, the 0^+ (1.7607 MeV) and 2^+ (2.1865 MeV) in the doubly closed shell nucleus ^{90} Zr. Earlier it had been indicated that the 2^+ state due to the $\pi h_{11/2}{}^2$ configuration should come much lower than 1 MeV. For purposes of discussion, let us take it to be $\sim$0.8 MeV. (Later more will be said about the location of the 2^+ state.) Then in ^{150}Er the lowest 2^+ (1.5789 MeV) state is about twice as high as this number. Does this mean that ^{150}Er is some sort of a doubly closed shell nucleus? Note that in this region ^{146}Gd has already been elevated to the level of a doubly closed shell nucleus.[1,6–8]

With the identification of the lowest excited state in ^{146}Gd as being 3^- rather than 2^+, as was believed earlier,[1] similarities with ^{208}Pd, which is a doubly closed shell nucleus and, where too, the lowest excited state is 3^-, were pointed out. The first excited state in even-even nuclei is generally 2^+. Doubly closed shell nuclei like ^{16}O, ^{40}Ca, and ^{208}Pd are the exceptions. It was argued that since the 3^- was the lowest state and that the yrast 2^+ state was pushed up a little higher (see Fig. 1), this was spectroscopic evidence of shell closure[14,19] at $Z=64$. The 3^- states in ^{208}Pd and ^{146}Gd are, however, very different in their basic structures. Whereas the 3^- state in ^{208}Pd is composed of collective mixtures of several particle-hole states, the 3^- in ^{146}Gd has a simple $\pi d_{5/2}{}^{-1}\ \pi h_{11/2}$ structure.[10] This indicates that in spite of subshell closure, ^{146}Gd exhibits simple configurations of a spherical shell model. ^{146}Gd can be considered to exhibit a subshell closure not because the lowest state is 3^-, but because the lowest 3^- state, as well as the 2^+ state, is almost twice as high as the corresponding 2^+ ($\sim$0.8 MeV) state arising from the $h_{11/2}{}^2$ configuration.

Let us look at the low yrast positive and negative parity states for some Z even, $N=82$ isotones shown in Fig. 1. One is struck by the fact that the lowest state 2^+ (3^- for ^{146}Gd) remains steady around 1.6 MeV as one goes from ^{140}Ce to ^{150}Er (^{152}Yb data are not available). This value is rather high from a spectroscopic point of view,

$$\langle h_{11/2}{}^2 | v | h_{11/2}{}^2 \rangle_{J=2} \approx 0.8 \text{ MeV} ,$$

indicating some sort of shell closure for protons. The fact that in ^{146}Gd the 3^- state is lower than the 2^+ state, while in other neighboring nuclei, say ^{144}Sm, 3^- is higher than the 2^+ state, is not significant and can be explained by looking at the corresponding proton single particle structure. In Fig. 3, low lying states in odd Z, $N=82$ nuclei in this neighborhood are plotted. The fact that $\frac{11}{2}^-$ in ^{143}Pm lies higher than the corresponding state in ^{145}Eu will push the 3^- in ^{144}Sm higher than in ^{146}Gd (note that this 3^- state arises from the configuration $\pi d_{5/2}{}^{-1}$ $\pi h_{11/2}$).[10] Similarly, the relative location of the 3^- and 2^+ states can be explained in other nuclei too.

On the basis of a simple shell model, the low lying states $\frac{7}{2}^+$, $\frac{5}{2}^+$, $\frac{11}{2}^-$, $\frac{3}{2}^+$, and $\frac{1}{2}^+$ could be identified with $0g_{7/2}$, $1d_{5/2}$, $0h_{11/2}$, $1d_{3/2}$, and $2s_{1/2}$ proton states, respectively. In single proton transfer studies on the $N=82$ isotones[11] the spectroscopic factors suggest strong single particle components for $\frac{7}{2}^+$, $\frac{5}{2}^+$, and $\frac{11}{2}^-$ orbitals. However, for nuclei $N=82$ to have proton shell closure, large gaps should exist between the unoccupied and the occupied single particle states. A large gap of 2.792 MeV between $h_{11/2}$ and $g_{7/2}$, and of 1.829 MeV between $h_{11/2}$ and $d_{5/2}$ orbitals is found in ^{133}Sb.[12] This large gap persists as one increases the number of protons.[13] The shell closure

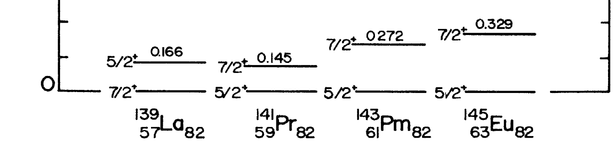

FIG. 3. Low lying single particle states of some $N=82$ isotones (Ref. 35).

at $Z=64$ has been studied in greater depth.[14] A gap of ~ 2 MeV is expected around $Z=64$ nuclei,[14] but is found in other nuclei in this region too. The lowest 8^- state in ^{146}Gd is at 3.18 MeV. This could be identified with $d_{5/2}$-$h_{11/2}$ spacing if there were no residual interaction. Of course the residual interaction brings it down to ~ 2 MeV. The point to emphasize here is that an 8^- state similarly exists at 3.37 MeV in ^{144}Sm and possibly at 3.456 MeV in ^{142}Nd. One should also bear in mind the fact that most of the low lying yrast states in these even Z, $N=82$ nuclei seem to have almost pure proton configurations[14] with practically no neutron admixtures.

What does one mean by shell closure here? The relevant proton orbitals are $0g_{7/2}$, $1d_{5/2}$, $2s_{1/2}$, $0h_{11/2}$, and $1d_{3/2}$. So possible subshell closures could arise at $Z=58$ ($0g_{7/2}$ full), $Z=64$ ($1d_{5/2}$ full), $Z=66$ ($2s_{1/2}$ full), and $Z=70$ ($1d_{3/2}$ full). But the effect of subshell closure is not evidenced by the experimental data.[15] However, all even Z nuclei have a high lying lowest excited state, indicating some sort of quasi-shell closure for all the even-even nuclei ^{40}Ce to ^{150}Er, and possibly ^{152}Yb and ^{138}Ba. ^{138}Ba seems to behave like these other nuclei. A definite word on this would, however, have to wait until further investigation. It seems as if the neutron closure at $N=82$ is the dominating factor which forces all sets of two protons to pair off strongly, leading to an effective quasi-closed shell behavior, so much so that even numbers which are not standard subshell closures, like $Z=60$, 62, behave in this manner. It should be noted that a precursor of this amazing phenomenon is observed in another region too. Nucleon numbers 38 and 40, in general, do not lead to shell closures. However, $Z=38$, 40 in the presence of neutron number $N=50$ behaves quite differently. ^{90}Zr has been accepted as a genuine doubly closed shell nucleus. In ^{88}Sr the $Z=38$ acts as a stable structure, enabling a good shell model description of ^{90}Zr excited states.[16] So in ^{90}Zr and ^{88}Sr, as in $N=82$ and $Z=58$ to 70, there are indications of the overwhelming influence that the neutron shell closure at $N=50$ and 82 exerts on nuclei.

So far the possibility of subshell closure for even Z around 58 to 70 in terms of proton structure only has been discussed. The unique inertia to addition of proton pairs was pointed out. This large pairing correlation between protons may be the reason why even isotones in this region behave as quasi-magic nuclei. The role of the pairing force for nuclei in this region was studied in Refs. 2 and 13. Now, if there were no proton quasi-shell closure, removal of one or two neutrons from the $N=82$ closed shell would lead to drastic changes in the energy levels of the residual nucleus. However, if all the even nuclei under consideration did form a stable structure for protons, removal of one or two neutrons would not change the level scheme of these nuclei. A look at Figs. 4–7 for the low lying states in $N=81$, 80, 79, and 78 isotones would convince us of the indicated stability. As one, two, and possibly three and four neutrons are picked up, the protons maintain an inert core for $Z=58$, 60, 62, 64, 66, 68, and possibly 70. This is an amazing phenomenon. Let us look at it in greater depth.

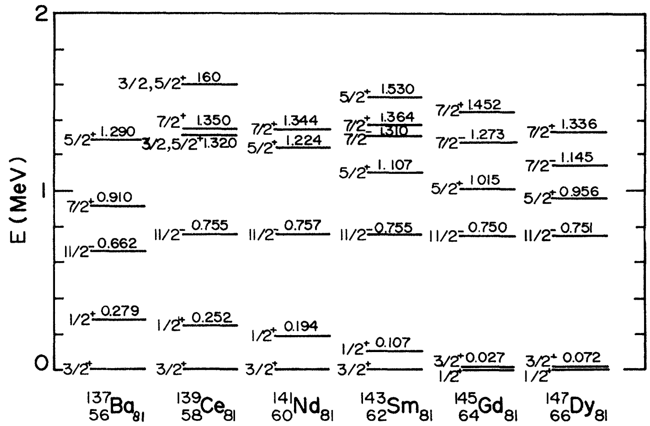

FIG. 4. Low lying single particle states of some even Z, $N=81$ isotones (Ref. 35).

The important neutron orbitals can be assessed from the study of one neutron hole state, i.e., $N=81$ isotones (Fig. 4). The low lying states in this region are associated with $1d_{3/2}$, $2s_{1/2}$, $0h_{11/2}$, $1d_{5/2}$, and $0g_{7/2}$ orbitals.[17] Jolly and Kashy[18] studied the one neutron hole structure in ^{143}Sm and ^{141}Nd through the (p,d) reaction on ^{144}Sm and ^{142}Nd. If there were no proton quasi-shell closure, one would have expected some low lying collective core excitations. They[18] did not observe any such excitation below 1.5 MeV. All the low lying states were associated with the pure neutron orbitals indicated above. It should be pointed out that these two nuclei have $Z=60$ and 62. Inertia of the proton structure can only be understood in terms of some quasi-shell closure. Of course one may be more willing to accept subshell closure at $Z=58$ and 64. As such, all these low lying levels for $N=81$ isotones should be

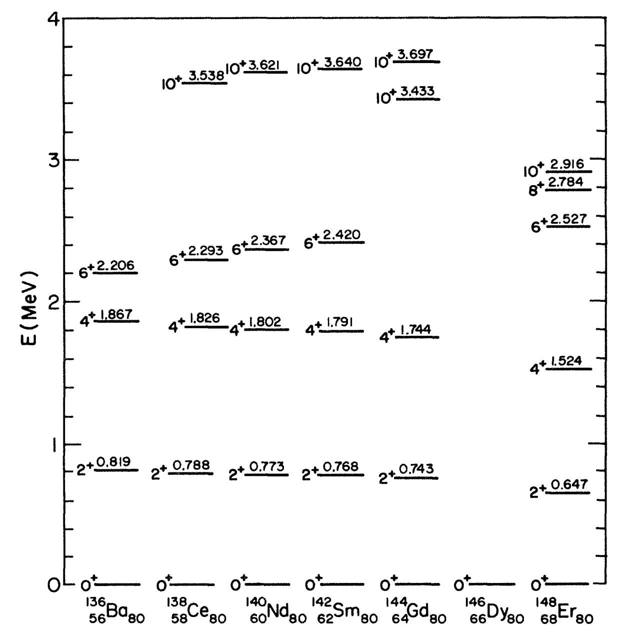

FIG. 5. Positive parity yrast states of some even Z, $N=80$ isotones, mostly from Refs. 35 and 36. Levels in ^{148}Er from Ref. 8. Some high lying states are from Refs. 25, 27, and 34. ^{146}Dy data were not available.

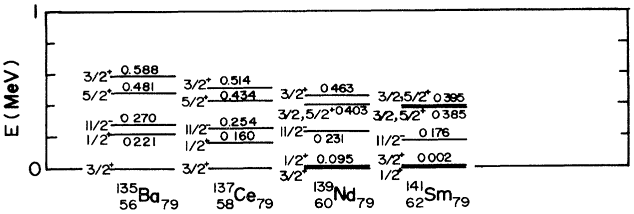

FIG. 6. Low lying states of some even Z, $N=79$ isotones (Ref. 35).

pure one neutron hole states in the above-mentioned orbitals. It is interesting to note that the $h_{11/2}$ orbital does not budge from the value of 0.75 MeV even though the proton number changed drastically from $Z=58$ to $Z=66$. It was as if all these nuclei were identical. As a warning, one does not find such a phenomenon for the $N=49$ isotones.

Further evidence for proton quasi-shell closure is provided by two neutron hole states in $N=82$ nuclei, i.e., by the $N=80$ isotones. Oelert *et al.*[20] investigated the (p,t) reaction on even samarium isotopes. Inter alia, they analyzed the $^{144}Sm(p,t)^{152}Sm$ data in terms of the quadrupole pairing vibrational model.[21] The pairing vibrational states are expected to figure prominently near closed shells. To their[20] surprise, they found that the strongly excited 2^+ state (0.768 MeV) in ^{142}Sm in their (p,t) experiment was explained very well by the quadrupole pairing vibrational model. This is not at all surprising in light of what is being stated here. ^{144}Sm is indeed a quasi-doubly-closed-shell nucleus with $Z=62$, $N=82$, and the 2^+ state (0.768 MeV) in ^{142}Sm has a pure $\nu h^{-2}{}_{11/2}$ configuration. This also agrees with the assertion made earlier that the 2^+ state due to the $h^2{}_{11/2}$ configuration should come considerably below 1 MeV. A similar analysis done earlier[22,23] for ^{138}Ce and ^{140}Nd confirms the view on shell closure expressed here. The positive parity yrast states in all the even Z, $N=80$ isotones are very strongly excited in the (p,t) and other two neutron pickup reactions.

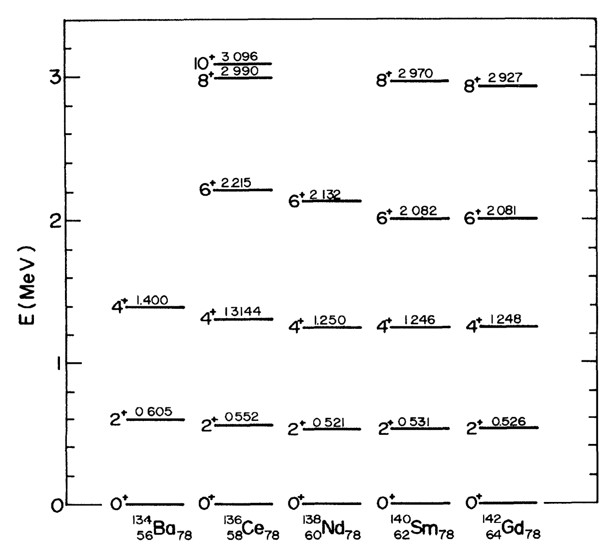

FIG. 7. Positive parity yrast states of some even Z, $N=78$ isotones (Refs. 35 and 36).

What is being stated here is that all the even Z (58–70), $N=80$ isotones are basically two neutron hole nuclei. The low lying positive parity yrast states could arise from two neutron holes in the orbitals $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$. The $0h_{11/2}$ orbital, however, has a larger diagonal pairing energy term[23] since

$$\langle j^2 | v | j^2 \rangle_J \sim 2j+1 \ .$$

This will lower the energy of the $0h^{-2}{}_{11/2}$ configuration relative to the other two neutron hole states in ^{138}Ce, ^{140}Nd, ^{142}Sm, ^{144}Gd, ^{146}Dy, and ^{148}Er. As such, the positive parity yrast states in all these even-even nuclei are expected to have a simple $\nu h^{-2}{}_{11/2}$ configuration. This point will be discussed in detail later.

The similarity in the 2^+ and 4^+ and possibly the 6^+ levels (Fig. 5) should be seen in light of the steadiness of the $0h_{11/2}$ orbital in the $N=81$ isotones (Fig. 4). All these are two neutron hole nuclei, indicating also that the $N=82$ isotones with $Z=56$, 58, 60, 62, and possibly beyond these too, exhibit a doubly quasi-shell closure. It also indicates that in many ways these nuclei are alike. The inertia persists even for even Z, $N=79$ isotones (Fig. 6) and even Z, $N=78$ isotones (Fig. 7) which can be regarded as three neutron hole and four neutron hole nuclei, respectively. Such a property is not exhibited by $N=49$, 48 isotones.

It is known that seniority is conserved for identical nucleon configurations in semimagic nuclei.[16,24] As Talmi[24] has emphasized, this is manifested as constant separation between, say 2^+, 4^+ states and the ground state over a range of even-even nuclei for a semiclosed shell nucleus. This is beautifully illustrated by the isotopes of tin ($Z=50$).[24] For $N=80$ isotones (Fig. 5), we too observe this phenomena. The yrast 2^+ and 4^+ states (and possibly 6^+ and 10^+ states, 8^+ not having been established yet) remain practically stationary as one goes from ^{138}Ce to ^{148}Er (and perhaps ^{136}Ba, too). Obviously seniority is being conserved here. It is known[24] that neutron-proton interaction breaks seniority. There are no free protons here; all of them having paired off to produce quasi-closed shells. The points that are being emphasized here are that all these are seniority two states arising from a practically pure $\nu h^{-2}{}_{11/2}$ configuration and that in the manner that they manifest their low energy shell properties, all these even Z nuclei are amazingly identical. So, to say they exhibit both the properties of isotones and isotopes at the same time, is as if to say they are "isotopic isotones."

A plot of two nucleon separation energy also provides some evidence for a shell closure.[24] A sudden drop in the smooth decrease of this quantity as a particular shell is crossed towards increasing mass number, is taken to be an indication of magicity. Spanier *et al.*[4] plotted the two proton separation energy for even Z, $N=82$ isotones to study the shell closure at $Z=64$. They obtained only a slight kink in the plot. Schmidt-Ott *et al.*[5] made the same plot with improved empirically determined masses in the vicinity of $Z=64$. The drop at $Z=64$ is more apparent, but due to the absence of an experimentally determined mass of ^{147}Tb, they could not quote any specific number. At best, however, it appears to be $\sim$0.4 MeV. This is to be contrasted with a drop of ~ 5 MeV at $Z=82$ for $N=126$ isotones (^{208}Pb).[4] It should, however, be pointed out that the presence or absence of a gap in the nucleon separation energy is not a sufficient criterion for shell closure. This point has been discussed in Ref. 2. One can plot the two proton separation energies a little differently, as has been done in Fig. 8. This was done to emphasize the discontinuities at $Z=56$, 58, 60, 62, and 64. The discontinuities are perhaps there, though not well pronounced.

Because of the quasi-shell closure and the close similarity with each other, ^{138}Ce, ^{140}Nd, ^{142}Sm, ^{144}Gd, and ^{148}Er (for which data are available) should each provide the matrix elements for the $\nu h^{-2}{}_{11/2}$ configuration, these being the positive parity yrast states in these nuclei. If one is only interested in the study of the excited states, the $\nu h^{2}{}_{11/2}$ spectrum is the same as that of $\nu h^{-2}{}_{11/2}$.

Nolte *et al.*[8] have recently identified many excited states in ^{150}Er and ^{148}Er. They claimed to have found a complete $h^{2}{}_{11/2}$ spectrum with seniority two in ^{148}Er. In light of the discussion in this paper, these can be treated as a complete set of states for the $\nu h^{-2}{}_{11/2}$ configuration. Here these will be taken as such to do a simple shell model calculation.

The isomeric 10^+ states in ^{138}Ce, ^{140}Nd, and ^{142}Sm, which occur at 3.538, 3.621, and 3.640 MeV, respectively (Fig. 5), have been shown to have a predominant $\nu h^{-2}{}_{11/2}$ configuration.[34] This has been done through a study of their g factors, which are sensitive tools for studying configuration mixing. The yrast 10^+ (3.433 MeV) in ^{144}Gd, because of its positive g factor, has been suggested[26] to have the main configuration of

$$(\pi d^{-2}{}_{5/2}\, h^{2}{}_{11/2})10(\nu h^{-2}{}_{11/2})0 \; .$$

This suggests that perhaps the true yrast 10^+ state in these nuclei has not yet been identified. This point is strengthened by the fact that the yrast 8^+ has not yet been located in any of these nuclei. In fact, the 6^+ state has not surfaced yet in ^{144}Gd. The nuclei in this region are currently under active investigation. The excited states of many nuclei in this region have not been located, (e.g., ^{146}Dy and ^{150}Yb). New data are still coming in. Many of the level assignments should be regarded as tentative. This statement should be viewed in light of the fact that in ^{146}Gd only recently a whole set of positive parity states had their angular momentum and parity changed under more careful scrutiny.[1] So, the extraction of the $\nu h^{-2}{}_{11/2}$ spectrum from ^{146}Gd will have to wait until some future time. When data on it become available, the best nucleus for this purpose would be ^{150}Yb. One can, however, predict the positive parity yrast states of these nuclei ($N=80$, even $Z=58$—70) to be close to those of ^{148}Er.

The matrix elements for an identical particle j^n configuration can be calculated in terms of the corresponding j^2 values[28] by using a two particle coefficient of fractional parentage (cfp). The coefficients needed are tabulated in Ref. 28. The matrix elements from the spectrum of ^{148}Er have been taken to predict the spectrum of $N=78$ isotones treating them as having the simple configuration of $\nu h^{-4}{}_{11/2}$ The results are given in Table I. Interestingly, this interaction mixes seniority. The positive parity yrast states indicated are the ones after diagonalization in seniority space. Note that for 2^+ and 10^+ good agreement occurs between the shell model predicted and the known positive parity yrast states in ^{136}Ce, ^{138}Nd, ^{140}Sm, and ^{142}Gd. The identity persists in these nuclei even as far away as four nucleon holes in the $N=82$ closed shell. As such, 2^+ and 10^+ states appear to have a simple dominant neutron configuration of $\nu^{-4}{}_{11/2}$. It has already been argued[29] that the only known 10^+ (3.0957 MeV) among these isotones has a $\nu h^{-2}{}_{11/2}$ configuration. Also the 8^+ state in ^{136} Ce has been suggested to have a $\nu h^{-2}{}_{11/2}$ configuration.[29] In addition to supporting the picture presented here, it shows the veracity of the two body matrix elements used.

The $N=78$ isotones ^{142}Gd and ^{140}Sm were studied by Mariscotti *et al.*[30] When applying the VMI model to explain the level scheme therein, the yrast 2^+ and 4^+ states were used to determine the VMI parameters, the ground state moment of inertia $\mathscr{I}_0$, and the softness σ. The

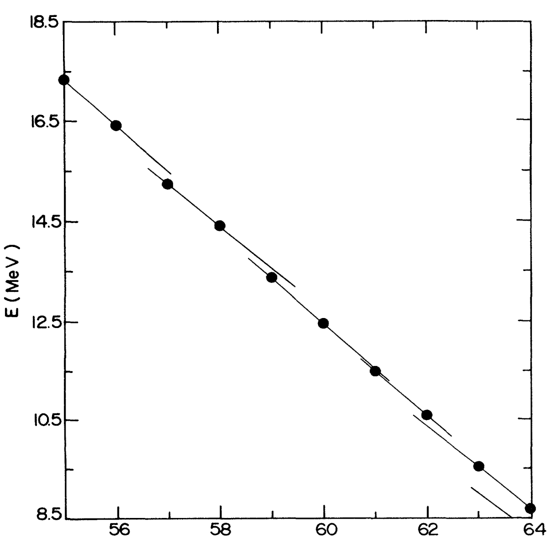

FIG. 8. Two proton separation energies (y axis) vs Z number of some $N=82$ isotones (Refs. 5, 6, and 38).

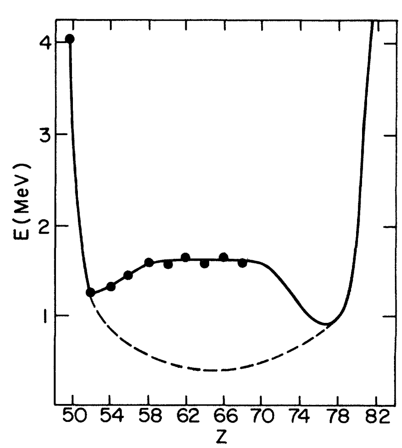

FIG. 9. Energy of the lowest excited state over the ground state for Z even, $N=82$ isotones. Yrast 3^- is plotted for ^{146}Gd.

TABLE I. $\nu h^2_{11/2}$ matrix elements and the predictions for $N=78$ isotones with the Z shell closed. All energies are in MeV.

State	$\nu h^{-2}_{11/2}$ matrix elements	$\nu h^{-4}_{11/2}$ prediction	Positive parity yrast states ^{136}Ce	^{138}Na	^{140}Sm	^{142}Gd
0^+	0	0	0	0	0	0
2^+	0.6466	0.535	0.5520	0.5209	0.5308	0.5260
4^+	1.524	1.142	1.3143	1.2499	1.2456	1.2483
6^+	2.5269	1.825	2.3667	2.1321	2.0816	2.0806
8^+	2.7844	2.261	2.9901		2.9688	2.9271
10^+	2.9155	3.158	3.0957			

values of these parameters for ^{142}Gd and ^{140}Sm were identical to and close to the corresponding values for the other $N=78$ isotones ^{13}Nd and ^{136}Ce. Also, these values showed that there was only small deformation in this region. This is another confirmation of the fact that there is identity amongst these nuclei and that these nuclei are close to a spherical nucleus.

A useful way of indicating a possible vibrational, rotational, or magic character of a nuclei is to plot $E_1(4^+)/E_1(2^+)$. This is done in Ref. 39. The deformed region, spherical region, and magic region have this ratio going as ~ 2.25 to 3.3, ~ 1.75 to 2.25, and 1 to 1.75, respectively. For the nuclei under consideration ($N=82$, $Z=58$ to 70) this ratio is ~ 1.3. This puts all these nuclei into the category of "magic" nuclei.

The fact that nuclei with $N=82$ and $Z=58$, 60, 62, 64, 68, and 70 form a closed shell and that they are identical in many ways would have implications for the interacting boson approximation (IBA) models. To fit the experimental data for heavy and medium-heavy nuclei, the IBA model assumes shell closure at 50, 82, and 126.[31] The total number of bosons is taken as the sum of proton and neutron bosons.[32] So, for ^{148}Gd the number of proton bosons is taken as $(64-50)/2=7$ and the number of neutron bosons as $(84-82)/2=1$. Now, due to the fact that $Z=58$, 60, 62, 64, 68, and 70 are closed shells, there are no proton bosons to be considered for these nuclei. This would require redoing calculations for these nuclei in the IBA model. In fact, some work along this line has been done recently[40] for $Z=64$ shell closure.

Another experimental confirmation of ideas presented in this paper has come from a different corner. Schery *et al.*[33] did (p,n) quasi-elastic scattering on ^{138}Ba, ^{142}Nd, and ^{144}Sm to analyze the neutron distribution in these nuclei. The (p,n) reaction is very sensitive to the ratio of neutron to proton rms radii, r_n/r_p. In ^{208}Pb and tin isotopes, r_n/r_p turns out to be greater than one and tends to increase with neutron excess.[33] Interestingly enough, they[33] found that, for the $N=82$ isotone studied, the ratio r_n/r_p is close to unity and retains this value as the neutron excess is changed. This is a beautiful confirmation of the ideas presented in this paper. The fact that in the nuclei studied, the ratio r_n/r_p remains the same indicates that these nuclei are identical in terms of matter distribution; and this ratio being close to one in each case indicates that all these are spherical nuclei. The latter conclusion follows from the fact that deformation is known to be a function of excess nucleons outside a closed core. The fact, that here, as Z is changed by six units the matter distribution is not affected, leads to the conclusion that the nuclear shape has to be spherical. This shows that for $N=82$ and even Z of these nuclei, there are no excess nucleons; all these are indeed closed shells. What we observe in low energy nuclear spectroscopy is manifesting itself in the study of the distribution of matter.

Many theoretical studies have been done[41] for nuclei in this region. However, the point that has to be borne in mind in further studies is that the identical particle interaction may mix seniority, as shown here.

The question of why or how this unique identity and inertia comes about needs further investigation. A study of transition rates for various multipolarities for nuclei in this region is important too.

Further evidence of stability is provided in Fig. 9. If there were no extra stability involved with $Z=58$ to 70, one would have obtained the lowest excited state going by the dashed line. One immediately notices the "plateau" of stability for even $Z=58$ to 70. This consolidates the argument of "magicity" of these nuclei. The solid line beyond $Z=68$ is the qualitative prediction.

The qualitative prediction is made with the assumption that beyond $Z=70$ up to $Z=82$ there will be a large region of open shell and the lowest excited state should come down. The lowest point would occur around the middle ($Z\sim 76$) and the magicity at $Z=82$ would then assert itself by kicking the lowest excited state higher up.

Before closing, let me point to another interesting coincidence. The atoms with $Z=58$ to 71 are referred to as lanthanides or rare earths. Chemically all these elements are almost identical. The study of the corresponding nuclei in this paper indicates unique shell closure and similarity of these very nuclei.

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