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Two body residual interaction in the statistical multistep compound and direct theories

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It is shown that within the multistep compound theory the use of matrix elements calculated with a density dependent two-body residual interaction of the Yukawa form makes possible reproduction of the experimental data of multistep compound reactions with an interaction strength V_0 which turns out to be consistent with the one used in the analysis of multistep direct processes.

[NUCLEAR REACTIONS Statistical multistep compound and direct analysis, deduced two-body residual interaction strength.]

I. INTRODUCTION

The statistical theories of multistep compound and direct reactions, developed in 1980 by Feshbach, Kerman, and Koonin,[1] represent an important step toward a unified description of nuclear reaction mechanisms. Starting with a common set of assumptions based on the physics underlying the mechanisms being studied, these two quantistic theories are concerned mainly with the description of particles emitted during the so-called precompound (or preequilibrium) phase.

These two theories are quite different in nature: While the statistical multistep compound emission (SMCE) theory describes the long-lasting, quasiequilibrium compound-like processes, the statistical multistep direct emission (SMDE) theory describes the fast peripheral direct-like processes. The very general ideas on which both theories are based, however, make it possible to include in the same framework the other two fundamental nuclear reaction mechanisms as limiting cases; the single-step direct effect, (DE), widely used to describe transitions to low-lying discrete states, takes place in the first less "complex" step of the SMDE chain, while the compound nucleus (CN) picture can be closely associated with the closed states of the last and most "complex" step of the SMCE chain.

A very important point is that, due to the quantum-mechanical formulation of both processes, the matrix elements of the residual interaction can be explicitly calculated *a priori* from basic assumptions on the various ingredients entering in their definition, as is usually done, for example, in any DWBA calculation. Once these matrix elements are known, as is the number of levels involved at each step of the chain, it is possible to calculate both the contimuum transition probabilities present in the SMDE cross section and the bound-to-bound and bound-to-continuum n-exciton widths ($\Gamma_n^{\downarrow}$ and $\Gamma_n^{\uparrow}$, respectively) on which the SMCE cross section is based. This improvement over the early semiclassical preequilibrium models,[2] particularly the exciton model,[3] where the intranuclear and the continuum decay rates were calculated starting with rather drastic assumptions, i.e., the free nucleon-nucleon cross section for the former and the inverse cross section for the latter, is rather important. Indeed, once the above matrix elements are explicitly microscopically calculated, it will be possible to compare on an equivalent basis the residual interactions needed from the different theories (SMCE, CN, SMDE, and DE) to fit the appropriate experimental data.

Since the residual two-body interaction is known to be energy and density dependent but should not be theory dependent, the above comparison can be considered as an important check on consistency.

The purpose of this paper is to show that by using a realistic form for the two-body interaction, this consistency, found previously to exist between the SMDE and DE results, can be extended to the SMCE and CN processes.

After a brief history of the applications of the SMCE and SMDE theories to experimental data (Sec. II), the details of the new SMCE calculations are presented (Sec. III) and some comments are made (Sec. IV).

II. APPLICATION TO EXPERIMENTAL DATA

The first applications of the SMCE theory to experimental data were made by Feshbach *et al.*[1] and the Milan group.[4–6] This work was concerned mainly with the calculation of differential cross sections and total widths of the different stages of the SMCE chain, including the equilibrium or so-called r stage. The reactions considered, which were either nucleon induced or ^{3}He induced, brought the composite system to an excitation energy of 20–30 MeV with an angular distribution symmetric to 90°, as predicted by the SMCE theory. The numerical calculations were carried out initially by using all the approximations worked out by Feshbach *et al.*[1]:

(a) The radial wave functions associated with the interacting particles were all assumed to be l independent and constant within the nucleus.

(b) The two-body interaction was assumed to be of the zero-range form

$$V_{\rm res}(\vec{r}_1,\vec{r}_2)=V_0\tfrac{4}{3}\pi r_0^3\delta(\vec{r}_1-\vec{r}_2)\ . \tag{2.1}$$

With these assumptions the radial overlap integrals for

both bound-to-bound and bound-to-continuum transitions can be expressed analytically as follows[1]:

$$\vartheta_b = V_0 \tfrac{4}{3}\pi r_0^3 \frac{1}{4\pi} \int_0^\infty u_{j_1} u_{j_2} u'_{j_1} u'_{j_2} \frac{dr}{r^2} = \frac{V_0}{A} , \tag{2.2}$$

$$\vartheta_l = V_0 \tfrac{4}{3}\pi r_0^3 \frac{1}{4\pi} \int_0^\infty u_{j_1} u_{j_2} u'_{j_1} u_l \frac{dr}{r^2}$$

$$= \left[\frac{4}{3} \frac{V_0^2 r_0^3 K m T_l}{A} \right]^{1/2} \Big/ 2\pi . \tag{2.3}$$

It was therefore quite simple to develop a computer code[7] to compare the predictions of the SMCE theory with experiments. The first guess for V_0 in expression (2.1) turned out to be 0.7 MeV and was obtained by reproducing at the same time the precompound five-exciton width Γ_5 and the CN r-stage width Γ_r extracted from the ^{27}Al(^{3}He,p) reaction by means of a multiclass fluctuation analysis.[8,9]

Meanwhile, the Milan group also worked out the application of the SMDE theory to reactions induced by higher energy projectiles, thus feeding the unbound states and giving rise to forward peaked angular distributions. The ^{120}Sn(p,n) reaction at 45 MeV was studied first,[10] and the analysis was subsequently extended to other similar reactions.[11].

This work was simplified by the availablilty of very well tested DWBA computer codes, which were able to supply the matrix elements needed to build the SMDE cross section. The codes used[12,13] employ a relatively sophisticated formulation of the inelastic scattering problem, refined after years of application to the calculation of DE transitions: The matrix elements of $V_{\rm res}$ are calculated with realistic ingredients such as Woods-Saxon or harmonic oscillator wave functions and a Yukawa shape for $V_{\rm res}$. This work shows that a good fit of the (p,n) differential cross section can be achieved with a V_0 value, averaged over the p-n and the n-n interactions, of 15—17 MeV. This value, in turn, was shown to be consistent with the one extracted from the usual DWBA DE analysis with a microscopic model,[11] but was clearly much larger than the one obtained from the SMCE work, even when this comparison was based on the volume integrals of the zero-range and Yukawa interactions.

Efforts were thus made to improve the SMCE formulation, releasing the approximations quoted above one by one.

In Ref. 6 harmonic oscillator wave functions and distorted waves were used for the bound and continuum states, respectively. The radial overlap integrals were thus computed numerically. More realistic n-exciton level densities were also used. The V_0 value extracted from a wide comparison with (p,n) and (n,p) cross sections and (^{3}He,p) five-exciton and r-stage widths was 5 MeV.

III. CALCULATION OF MATRIX ELEMENTS OF A REALISTIC INTERACTION IN SMCE PROCESSES

The next step in this work is obviously to replace the zero-range form for $V_{\rm res}$ with a more realistic interaction. In order to facilitate the comparison with the SMDE results we used a Yukawa form. An advantage of this form is that its multipole expansion

$$V_{\rm res}(r_i r_f) = 4\pi V_0 \sum_{LM} g_L(r_i, r_f) Y_L^M(\vartheta_i \varphi_i) Y_L^M(\vartheta_f \varphi_f)^* \tag{3.1}$$

can be analytically expressed, through the Bessel functions K and I, by the simple formulas[14]

$$g_L(r_i,r_f) = \mu^{-1}(r_i r_f)^{-1/2} K_{L+(1/2)}(\mu r_i) I_{L+(1/2)}(\mu r_f) \quad (r_i \geq r_f) ,$$
$$g_L(r_i,r_f) = \mu^{-1}(r_i r_f)^{-1/2} K_{L+(1/2)}(\mu r_f) I_{L+(1/2)}(\mu r_i) \quad (r_i < r_f) \tag{3.2}$$

where μ is the range of the interaction, assumed to be 1 fm, and L is the angular momentum transferred.

With this form the overlap integrals (2.2) and (2.3) become double integrals that can be expressed as

$$\vartheta_b = V_0 \int_0^\infty \int_0^\infty u_{j_1}(r_i) u_{j_2}(r_i) u'_{j_1}(r_f) u'_{j_2}(r_f) g_L(r_i,r_f) r_i^2 r_f^2 dr_i dr_f \tag{3.3}$$

(the same holds true for ϑ_l). After the $g_L(r_i,r_f)$ functions for the possible values of L have been calculated, these integrals are numerically computed by means of the Simpson rule for each set of values of the angular momenta of the initial (i) and final (f) states and are inserted in the SMCE formulas. Comparison with the experimental results of Γ_5 and Γ_r (Refs. 4 and 5) now gives $V_0 = 13$ MeV.

A last refinement was made by taking into account the density dependence of $V_{\rm res}$. This effect, which causes attenuation of the interaction with increasing nuclear density, is due to the saturating properties of nuclear forces and was thoroughly studied recently, particularly in Brueckner-Hartree-Fock calculations of the optical model potential.

The introduction of this effect into the SMCE calculations was motivated by the fact that the SMCE processes, due to their compound-like character and the importance of low l values in the partial wave expansions, are more likely to involve the highly dense inner part of the nucleus than the surface-localized DE and SMDE reactions. Recent DWBA calculations on (d,p) reactions, made by Kosugi and Kosugi,[15] have shown indeed the insensitivity of the absolute value of cross sections to the density dependence of $V_{\rm res}$.

This effect was taken into account by replacing $V_{\rm res}$ with a density-dependent $V'_{\rm res}$

$$V'_{\rm res}(r_i,r_f) = V_{\rm res}(r_i,r_f) [1.4(1-2.03\rho^{2/3})] \tag{3.4}$$

as given in the theoretical work done by Jekeunne *et al.*[16] and Myers.[17] The number 1.4 was determined by the requirement that $V'_{\rm res}=V_{\rm res}$ for $\rho=\rho_0/3$, as suggested by Kobos *et al.*[18] and the expression chosen for ρ was the one given by Negele.[19]

The calculation of the experimental data of Refs. 4 and 5 now gives $V_0=16$ MeV.

IV. CONCLUSIONS

Table I summarizes the values for V_0 obtained from the various calculations, starting from the crude assumptions of Ref. 1 through the most refined calculations set forth in this paper. It is very gratifying to see that the V_0 value we obtained for the SMCE (and therefore for the CN) processes is consistent with the one used in the SMDE (and therefore in the DE) work. Clearly, due to the physical meaning of $\Gamma^{\downarrow}$ and $\Gamma^{\uparrow}$, for this comparison we are using the $\bar{V}_0$ values of Ref. 11, namely the ones averaged over unlike particles and like particles interactions. The answer to the problem raised in the Introduction is therefore given in quite a satisfactory way. Of course, this achievement must be paid for. The l-independent overlap integrals of the early work could even be calculated by hand. The refinements introduced now involve the calculation of $4^{j_{\max}}$ double integrals for each value of $\Gamma^{\downarrow}_n$ and $\Gamma^{\uparrow}_n(\epsilon)$. These integrals must be computed numerically. The only calculation of $\Gamma^{\downarrow}_n$ in SMCE takes about 5 min time in a medium-fast computer such as the Univac 1100/81. The cost of the whole business increases proportionally.

TABLE I. Values of the two-body residual interaction strength V_0 from different calculations of SMCE and SMDE processes. The range of the Yukawa interaction was always taken as 1 fm.

Calculation	V_0 (MeV) SMCE	SMDE
zero range, constant WF (Ref. 4)	0.7	
zero range, realistic WF (Ref. 6)	5	
Yukawa (Ref. 11 and present work)	13	15.5—17
density dependent Yukawa (present work)	16	

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[1]H. Feshbach, A. Kerman, and S. Koonin, Ann. Phys. (N.Y.) 125, 429 (1980).
[2]M. Blann, Annu. Rev. Nucl. Sci. 25, 123 (1975), and references quoted therein.
[3]G. M. Braga Marcazzan, E. Gadioli Erba, L. Milazzo Colli, and P. G. Sona, Phys. Rev. C 6, 1398 (1972).
[4]R. Bonetti, L. Colli Milazzo, A. De Rosa, G. Inglima, E. Perillo, M. Sandoli, and F. Shahin, Phys. Rev. C 21, 816 (1980).
[5]R. Bonetti, L. Colli Milazzo, M. Melanotte, A. De Rosa, G. Inglima, E. Perillo, M. Sandoli, V. Russo, N. Saunier, and F. Shahin, Phys. Rev. C 25, 717 (1982).
[6]R. Bonetti, L. Colli Milazzo, and M. Melanotte, Phys. Rev. C 27, 1003 (1983).
[7]R. Bonetti and L. Colli Milazzo, computer code MUCOM (unpublished).
[8]W. Friedman, M. S. Hussein, K. W. McVoy, and P. A. Mello, Phys. Lett. 99B, 179 (1981).
[9]R. Bonetti, L. Colli Milazzo, and A. Garegnani, Lett. Nuovo Cimento 29, 496 (1980).
[10]L. Avaldi, R. Bonetti, and L. Colli Milazzo, Phys. Lett. 94B, 463 (1980).
[11]R. Bonetti, M. Camnasio, L. Colli Milazzo, and P. R. Hodgson, Phys. Rev. C 24, 71 (1981).
[12]P. D. Kunz, computer code DWUCK4 (unpublished).
[13]R. Schaeffer and J. Raynal, computer code DWBA76 (unpublished).
[14]M. B. Johnson, L. W. Owen, and G. R. Satchler, Phys. Rev. 142, 748 (1966).
[15]S. Kosugi and T. Kosugi, Phys. Lett. 113B, 437 (1982).
[16]J. P. Jekeunne, A. Lejeune, and C. Mahaux, Phys. Rev. C 16, 80 (1977).
[17]W. D. Myers, Nucl. Phys. A 204, 465 (1973).
[18]A. M. Kobos, B. A. Brown, P. E. Hodgson, G. R. Satchler, and A. Budzanowski, Nucl. Phys. A 384, 65 (1982).
[19]J. W. Negele, Phys. Rev. C 1, 1260 (1970).