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Deformed bosons in the Nilsson scheme

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Dyson boson mapping is used to derive the deformed boson from the Nilsson Hamiltonian. Single j-shell Nilsson model with pairing interaction is treated as an example. A general method of reconstructing bosons with definite angular momentum is outlined.

[NUCLEAR STRUCTURE Deformed boson mapping, interacting boson model.]

The interacting boson model (IBM) (Ref. 1) has been very successful in providing a simple description of nuclear collective spectra. The s and d bosons of angular momentum tensor rank 0 and 2 are involved in the collective Hamiltonian. In the limiting cases, the phenomenological IBM Hamiltonian can be solved analytically giving rise to spectra with vibrational [SU(5)] and rotational [SU(3) and O(6)] characters.

Some microscopic interpretations of IBM are based essentially on various methods of boson mapping[2] in trying to derive the phenomenological Hamiltonian from a shell-model picture. The collective bosons in the mapping process are usually taken to be those of the Tamm-Dancoff (TDA) and/or random phase approximations. These collective bosons, which are generally considered to be the fundamental basis of the IBM, are derived from spherical shell-model Hamiltonians. Therefore, one may find difficulties in this method for nuclei far from the vibrational region.

From recent studies[3] on the IBM intrinsic states and studies[4] of the correspondence between the IBM and the geometrical model of Bohr and Mottelson, one finds that the introduction of deformed bosons may be useful. The purpose of this Rapid Communication is to report results of our effort in deriving these deformed bosons, using the Dyson mapping,[5] from a deformed shell-model Hamiltonian.

The resulting boson Hamiltonian and its corresponding wave functions may be used to analyze the effect of g and i bosons in the well-deformed nuclei. Such a deformed boson is expected to carry the essence of the collective deformation degrees of freedom as do those of the Nilsson scheme. As a parallel study to the work of Ref. 6, a Nilsson-type Hamiltonian with pairing interaction is solved exactly in the present study.

The Dyson boson mapping has been extensively discussed in the literature.[2] The bifermion operators are mapped onto the ideal boson operators, which retain the commutator algebra of the bifermion operators, i.e.,

$$\begin{aligned} a^\alpha a^\beta &\rightarrow R^{\alpha\beta}=B^{\alpha\beta}-B^{\alpha\gamma}B^{\beta\delta}B_{\gamma\delta} \ , \\ a_\beta a_\alpha &\rightarrow R_{\alpha\beta}=B_{\alpha\beta} \ , \\ a^\alpha a_\beta &\rightarrow U^\alpha{}_\beta=B^{\alpha\gamma}B_{\beta\gamma} \ , \end{aligned} \tag{1}$$

where $\alpha,\beta,\gamma,\ldots$ stand for the quantum numbers necessary to specify a fermion space, and $\{a_\alpha,a^\beta\}=\delta_{\alpha\beta}$. Summation over repeated indices is assumed throughout the paper. a^α and a_α stand, respectively, for creation and annihilation operators for the state α. Application of Eq. (1) to the spherical region with some realistic interactions has previously been reported.[2]

In the deformed region, the deformed field approximation with an axial symmetry leads to single-particle orbitals of good angular momentum projection quantum number, K. Omitting all other somewhat irrelevant quantum numbers, the Nilsson Hamiltonian with pairing interaction is given by

$$H=\sum \epsilon_k a^k a_k - G \sum_{kk'>0} a^k a^{\bar{k}} a_{\bar{k}'} a_{k'} \ , \tag{2}$$

which can be mapped into the boson Hamiltonian as follows:

$$H_B=\sum \epsilon_k U^k{}_k - G\sum R^{k\bar{k}} R_{k'\bar{k}'} \tag{3a}$$

or

$$H_B=\sum \epsilon_k B^{kk'}B_{kk'} - G\sum (B^{k\bar{k}}-B^{kk_1}B^{\bar{k}k_2}B_{k_1k_2})B_{k'\bar{k}'} \ . \tag{3b}$$

It is trivial to find that the one-boson solution to the Hamiltonian in Eq. (3) is given by $b^\dagger(E_i)|0)$, where $|0)$ is the vacuum of the ideal boson, with

$$b^\dagger(E)=\sum_k \frac{1}{2\epsilon_k-E} R^{k\bar{k}} \ , \tag{4}$$

and the eigenvalue E_i is a solution of the TDA equation:

$$\frac{1}{G}=\sum_k \frac{1}{2\epsilon_k-E} \ . \tag{5}$$

Since the Dyson images of the bifermion operators satisfy the same commutative algebra as that of the bifermion operators, the technique of the commutator algebra of Ref. 6 can be used to show that the N-boson eigenfunctions of the Hamiltonian in Eq. (3) are given by

$$\Phi=\prod_{i=1}^{N} b^\dagger(E_i)|0) \ , \tag{6}$$

with energy $E=\sum E_i$, provided that the pair energy E_i satisfies the N coupled equations of motion:

$$\frac{1}{G}+\sum_{j\neq i}\frac{2}{E_j-E_i}=\sum_k \frac{1}{2\epsilon_k-E_i}, \quad i=1,2,\ldots,N \ . \tag{7}$$

The solution of Eq. (6) is identical to that given in Ref. 6. This indicates only that the transformation from the fermion space to the boson space is a faithful mapping. Thus Eqs. (2) and (3) possess equivalent physical solutions. It is worthwhile to point out that if one neglects the two-boson interaction term in Eq. (3b), then the equation of motion of Eq. (7) becomes the TDA equation of Eq. (5). Thus one can identify the two-body boson interaction as the source of the pair interaction term in Eq. (7), which is generally understood as due to the Pauli principle.

Once the intrinsic boson wave functions are obtained, one can then use the projection technique to reach the eigenvalues of various angular momentum eigenstates. Alternatively, one can also construct the ideal bosons with definite angular momentum by using transformation coefficients between the Nilsson orbitals and the spherical shell-model orbitals, i.e.,

$$a^k=\sum_j C_{jk}a^{jk} \ , \tag{8}$$

where C_{jk} can be obtained from the Nilsson model calculation. Bifermion operators are then transformed according to

$$a^{k_1}a^{k_2}=\sum_{jj'} C_{jk_1}C_{j'k_2}a^{jk_1}a^{j'k_2} \ . \tag{9}$$

The image of these bifermion operators is then related by

$$\begin{aligned} R^{k_1k_2} &= \sum_{jj'} C_{jk_1}C_{j'k_2}R^{jk_1j'k_2} \\ &= \sum_{jj'J} C_{jk_1}C_{j'k_2}\langle jk,j'k|JK\rangle R^{(jj')JK} \\ &= \sum_{jj'J} X^{k_1k_2}_{jj',JK}R^{(jj')JK} \ , \end{aligned} \tag{10}$$

where the coefficients $X^{kk}_{jj',JK}$ transform the axially symmetric bosons to the ideal spherical bosons. Using Eq. (10), one can then analyze the boson content in the deformed intrinsic wave function for any realistic Nilsson-type wave function and Hamiltonian.

As an illustration, we present a single j-shell (SJS) Nilsson model, where $C_{jk}=1$. Thus

$$R^{k_1k_2}=\sum_{JK}\langle jk_1jk_2|JK\rangle R^{JK} \ . \tag{11}$$

The one-body Hamiltonian of Eq. (3b) becomes

$$\begin{aligned} H_B &= \sum_{k_1k_2JK}\epsilon_{k_1}\langle jk_1jk_2|JK\rangle^2 B^{JK}B_{JK}-G\,\Omega\,B^{00}B_{00} \\ &= \sum_{JK}\hat{\epsilon}_{JK}B^{JK}B_{JK} \ , \end{aligned} \tag{12}$$

with $\Omega=(2j+1)/2$ and

$$\begin{aligned} \tilde{\epsilon}_{JK} &= \sum_{k_1k_2}\epsilon_{k_1}\langle jk_1jk_2|JK\rangle^2 \ , \\ \hat{\epsilon}_{JK} &= \tilde{\epsilon}_{JK}-G\,\Omega\,\delta_{J0}\delta_{K0} \ . \end{aligned} \tag{13}$$

One observes that the one-body part of the pairing interaction gives rise to a collective s boson single-particle energy. The Nilsson deformed single-particle energies induce further splitting of $\tilde{\epsilon}_{JK}$. The analysis of the two-body boson interaction term is identical to that of Ref. 2. For a degenerate ϵ_K, one recovers the result obtained in Ref. 2 and the wave function of Eq. (6) becomes the seniority wave function for SJS.

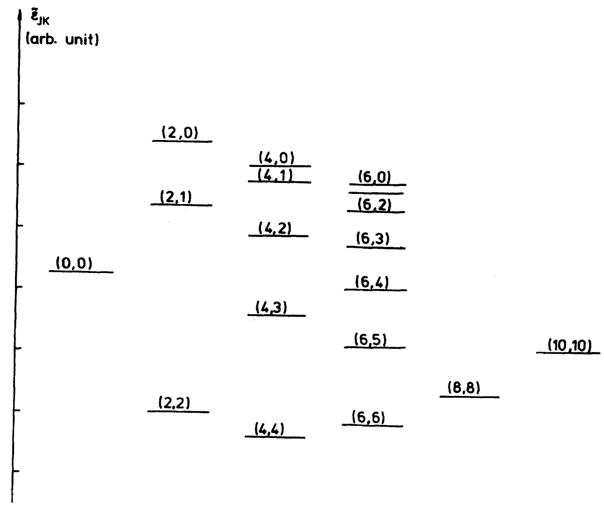

FIG. 1. The single-particle energy of the ideal deformed boson in the single j-shell scheme, where the Nilsson single-particle energy is assumed to be $\epsilon_{k_1}\propto k_1^2$.

Figure 1 displays the boson single-particle energy $\tilde{\epsilon}_{JK}$ for the single j-shell Nilsson orbital ($j=\frac{21}{2}$) with $\epsilon_{k_1}\propto k_1^2$.

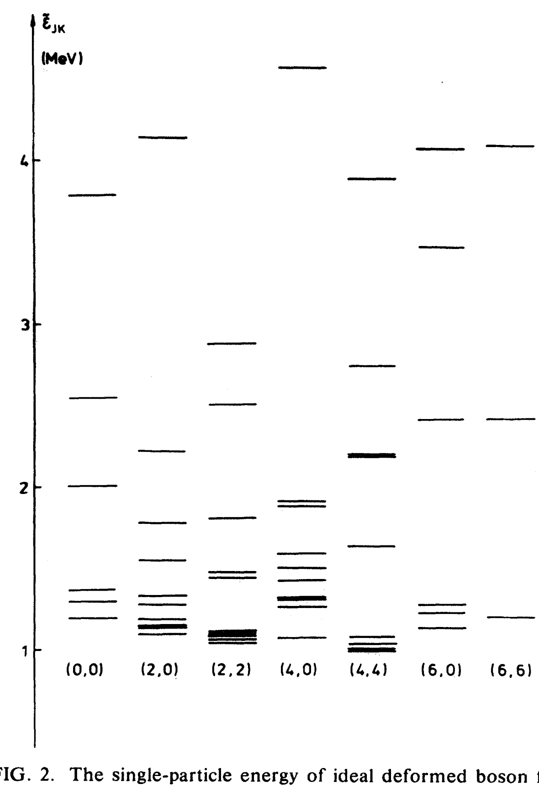

FIG. 2. The single-particle energy of ideal deformed boson from the multishell Nilsson orbitals.

TABLE I. Coefficients of the Nilsson wave function used in the present model calculation.

k	ϵ_k	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$g_{7/2}$	$g_{9/2}$
$\frac{1}{2}^+$[431]	4.6716	−0.260 24	0.518 45	−0.198 19	0.732 88	0.295 11
$\frac{7}{2}^+$[413]	4.7895	0	0	0	−0.129 61	0.991 56
$\frac{3}{2}^+$[422]	4.9017	0	0.302 31	−0.212 91	0.893 61	0.254 46
$\frac{5}{2}^+$[422]	5.5624	0	0	0.194 95	−0.163 00	0.967 17
$\frac{1}{2}^+$[420]	4.9283	0.423 20	−0.055 35	0.652 28	0.511 46	−0.361 64
$\frac{9}{2}^+$[404]	5.0535	0	0	0	0	1

One observes that the *d*, *g*, and *i* bosons are equally important with the two-body interaction turned off. As already pointed out by Geyer and Lee,[2] it is not very good to truncate the pair bosons of the system. The physical *s* or *d* boson should rather be associated with some paired combination of these ideal bosons. In the SJS case, the essential feature of the physical *s* and *d* bosons in the deformed Nilsson scheme is much the same as that of the spherical basis.[2]

For a realistic Nilsson multi-j-shell orbital, the one-body single-boson energy is given by

$$\tilde{\epsilon}_{(jj')JK} = \sum_{k_1 k_2} \epsilon_{k_1} X^{k_1 k_2}_{(jj')JK} \quad .$$

To examine the property of the single-particle energy of the deformed bosons, we choose the Nilsson wave function obtained by Davidson[7] as given in Table I. The single-particle energies of the ideal deformed bosons $B_{(jj')JK}$ are displayed in Fig. 2 for $(J,K)=(0,0)$, $(2,0)$, $(2,2)$, $(4,0)$, $(4,4)$, $(6,0)$, and $(6,6)$, respectively. One notes that the low angular momentum states indeed become somewhat more important. The interaction between two nucleons is of a short-range nature. Thus the low angular momentum bosons shall be pushed downward relative to the high angular momentum bosons. It is, however, worth mentioning that the *g* boson is still important in the example discussed here.

In conclusion, we have applied the method of the Dyson boson mapping to map the deformed fermion pairs into deformed ideal bosons. A simple pairing model is used to demonstrate the methodology. We find that the pair interaction terms [Eq. (7)], which give rise to Pauli correction, can be associated with the two-body boson interactions. We also point out that a simple transformation exists between the deformed bosons and spherical bosons, with which one can analyze the important spherical tensor components in boson Hamiltonians derived from realistic shell-model Hamiltonians.

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