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Odd Ni isotopes in the Lipkin-Nogami approach

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The modified Tamm-Dancoff approximation is applied to odd Ni isotopes after incorporating the Lipkin-Nogami procedure to improve the nonconservation of number of particles. The numerical calculations are performed employing the Kuo-Brown effective interaction. The Lipkin-Nogami procedure is found to give a general improvement in the agreement with the data.

[NUCLEAR STRUCTURE MTDA method using Lipkin-Nogami approach, odd Ni isotopes. Kuo-Brown interaction.]

Two different approaches[1–7] have been suggested in the literature to improve upon the problem related to the nonconservation of the number of particles associated with the quasiparticle modified Tamm-Dancoff approximation (MTDA) method for the description of low-lying states of spherical vibrational nuclei.[8–10] The first approach[1,2] makes use of the particle number projection method while in the second approach, suggested by Lipkin[3] and Nogami and collaborators,[4,7] a term $\lambda_2 N^2$ is added in the nuclear Hamiltonian in addition to the usual term $\lambda_1 N$ included in the MTDA method. The parameter λ_2 can be determined in different ways as described by Ng and Castel.[11] Recently the Lipkin-Nogami (LN) method has been applied by Sandhu and Rustgi to f-p shell nuclei and by Raj and Rustgi[12] to detailed structure calculations of even Ni isotopes. In this Brief Report, we have extended our work to odd nuclei and applied it to study the detailed nuclear properties of odd Ni isotopes.

The procedure for incorporating the Lipkin-Nogami modification in the MTDA method for the description of spherical odd nuclei is already described by us in an earlier paper[13] and will not be repeated here. Only relevant formulas are given. The results are applied to odd Ni isotopes assuming a ^{56}Ni core. The Kuo-Brown effective interaction is used.[14] A comparison of the results with the earlier MTDA calculations[15] is also presented.

The low-lying states of spherical odd nuclei with identical valence nucleons are described as a superposition of one- and three-quasiparticle orthonormal states. The one-quasiparticle state in the notation of Ref. 13 is represented as

$$|\alpha\rangle = a_\alpha^\dagger |0\rangle \quad . \tag{1}$$

The three-quasiparticle states are classified according to the shell-model seniority scheme and can have the following types of configurations:

$$\text{(i)}\quad |a^3 J\nu\rangle \ ,$$
$$\text{(ii)}\quad |a_1^2 J_1, a_2; J\rangle \ , \tag{2}$$
$$\text{(iii)}\quad |a_1 a_2 J_1, a_3; J\rangle \ .$$

Here J is the total angular momentum and ν is the seniority. The superscript on a denotes the number of quasiparticles present in that state. The second quantized version of the three-quasiparticles states of the types (ii) and (iii) of Eq. (2) can be represented in the notation of Refs. 9 and 13 by

$$[\mathcal{A}^\dagger(a_1 a_2 J_1) a_{a3}^\dagger]^J |0\rangle \quad . \tag{3}$$

The expression for the type (i) configuration of Eq. (2), in the second quantized form in terms of fractional parentage coefficient, is given below:

$$|a^3 J\nu\rangle = [\sqrt{3!}\langle a^2(\bar{J}_1); a, J|\,|a^3 J\nu\rangle]^{-1} \times [A^\dagger(a a \bar{J}_1), a_a^\dagger]^J |0\rangle \quad . \tag{4}$$

It can be seen that all permissible values of $\bar{J}_1$ for a given J on the right hand side of Eq. (19) are equivalent to the single state on the left hand side.

The matrix elements of H_{22} between three-quasiparticle states and that of H_{31} (H_{13}) connecting one- and three-quasiparticle states of the types discussed above are contained in Ref. 9. Only the matrix elements of H'_{nm} of Eq. (2.12) of Ref. 13 between different quasiparticle states are required, which may be evaluated by the method discussed in Ref. 9.

The nonvanishing matrix elements of H'_{22} between three-quasiparticle states are listed below:

$$\langle a_1^3 J\nu | H'_{22} | a_2^3 J'\nu'\rangle = -6\lambda_2 \delta_{a_1 a_2} \delta_{JJ'} (U_{a_2}{}^2 - V_{a_2}{}^2) - 4\lambda_2 [a_2] \delta_{\bar{J}_1 0} \delta_{a_2 J'} \delta_{a_1 a_2} \delta_{JJ'} U_{a_2}{}^2 V_{a_2}{}^2$$
$$+ 8\lambda_2 \sqrt{[\bar{J}_1]}\, \delta_{a_2 J'} \delta_{a_1 a_2} \delta_{JJ'} [\langle a_2^2(\bar{J}_1), a_2; J'|\,|a_2^3 J'\nu'\rangle]^{-1} \langle a_2^2(0), a_2; J'|\,|a_2^3 J'\nu'\rangle\, U_{a_2}{}^2 V_{a_2}{}^2 \ , \tag{5}$$

TABLE I. Calculated and experimental energy levels (MeV) of odd Ni isotopes. The label MTDA refers to the quasiparticle results with the mixing of different quasiparticle states. The label LN refers to the results with the Lipkin-Nogami modification.

A	J^π	$\frac{1}{2}^-_1$	$\frac{1}{2}^-_2$	$\frac{3}{2}^-_1$	$\frac{3}{2}^-_2$	$\frac{3}{2}^-_3$	$\frac{5}{2}^-_1$	$\frac{5}{2}^-_2$	$\frac{7}{2}^-_1$
59	Expt.	0.47	1.30	0.0	0.89		0.34	1.19	1.34
	MTDA	0.75	1.78	0.0	1.75	1.97	0.59	1.79	2.08
	LN	0.64	1.69	0.0	1.63	1.80	0.52	1.86	2.10
61	Expt.	0.28		0.0			0.07	0.91	
	MTDA	0.33	1.28	0.0	1.11	1.65	0.28	1.45	1.52
	LN	0.31	1.20	0.0	1.09	1.61	0.21	1.35	1.62
63	Expt.	0.0	1.01	0.16	0.53		0.09		
	MTDA	0.0	1.23	0.18	0.53	1.60	0.08	1.23	1.32
	LN	0.0	1.17	0.17	0.52	1.55	0.10	1.28	1.39
65	Expt.	0.06		0.32	0.70		0.0		
	MTDA	0.00	1.84	0.66	0.87	2.09	0.26	1.61	1.69
	LN	0.00	1.86	0.54	0.79	2.10	0.18	1.63	1.71

$$\langle a_1^3 J\nu | H'_{22} | [\mathcal{A}^\dagger(a_2a_3J_{23})a^\dagger_{a_4}]^{J'}|0\rangle = -2\sqrt{2}\lambda_2\sqrt{[a_1a_2]}\,\delta_{a_2a_3}\delta_{J_{23}0}\delta_{a_4J'}\delta_{a_4a_1}\delta_{JJ'}\sqrt{3!}$$

$$\times\langle a_1^2(0),a_1;J'||a_1^3J'\nu'\rangle U_{a_1}V_{a_1}U_{a_2}V_{a_2}\;, \tag{6}$$

$$\langle 0|[\mathcal{A}^\dagger(a_1a_2J_{12})a^\dagger_{a_3}]^{\dagger J}H'_{22}|\mathcal{A}^\dagger(a_4a_5J_{45})a^\dagger_{a_6}]^{J'}|0\rangle$$

$$= -\lambda_2 N(a_1a_2)N(a_4a_5)\bar{P}(a_1a_2J_{12})\bar{P}(a_4a_5J_{45})\delta_{JJ'}$$

$$\times[\{\delta_{a_1a_4}\delta_{a_2a_5}\delta_{a_3a_6}+U(a_3a_2J'a_1;J_{45}J_{12})\bar{P}(a_2a_3J_{23})(-1)^{a_1-J_{45}-J'}\delta_{a_1a_6}\delta_{a_2a_4}\delta_{a_3a_5}\}$$

$$\times(U_{a_4}{}^2-V_{a_4}{}^2)\{(U_{a_5}{}^2-V_{a_5}{}^2)+2(U_{a_6}{}^2-V_{a_6}{}^2)\}]$$

$$-4\lambda_2\sqrt{[a_1a_4]}\,\delta_{a_1a_2}\delta_{a_4a_5}\delta_{a_3a_6}\delta_{J_{12}0}\delta_{J_{45}0}\delta_{JJ'}\delta_{a_6J'}U_{a_1}V_{a_1}U_{a_4}V_{a_4}\;, \tag{7}$$

where

$$U(a_1a_2a_3a_4;J_{12}J_{23}) = \sqrt{[J_{12}J_{23}]}\,W(a_1a_2a_3a_4;J_{12}J_{23})$$

and W is the Racah's $6j$ coefficient. The operator $\bar{P}(a_1a_2J_{12}) = 1-(-1)^{a_1+a_2-J_{12}}P(a_1\leftrightarrow a_2)$; the operator

TABLE II. Magnetic moments of odd Ni isotopes in nuclear Bohr magneton μ_N.

	59		61		63		65	
J^π/A	MTDA	LN	MTDA	LN	MTDA	LN	MTDA	LN
$\frac{1}{2}^-$	0.67	0.69	0.68	0.69	0.66	0.68	0.65	0.67
$\frac{3}{2}^-$	−1.75	−1.76	−1.65	−1.66	−1.57	−1.59	−1.55	−1.57
$\frac{5}{2}^-$	1.34	1.36	1.37	1.38	1.39	1.40	1.38	1.40

TABLE III. Values of the reduced transition strength $B(M1)$ and $B(E2)$ for odd Ni isotopes. $B(M1)$ values are in units of μ_N^2 and $B(E2)$ values are in units of $e_{\text{eff}}^2/\alpha^2$ where e_{eff} is the effective charge of the neutron and $\alpha = M\omega/\hbar$ is the harmonic oscillator parameter.

Reduced transition strength	^{59}Ni MTDA	^{59}Ni LN	^{61}Ni MTDA	^{61}Ni LN	^{63}Ni MTDA	^{63}Ni LN	^{65}Ni MTDA	^{65}Ni LN
$B(M1\,\frac{1}{2}^- \to \frac{3}{2}^-)$	1.80	1.9	1.67	1.83	1.72	1.81	1.88	1.97
$B(M1\,\frac{3}{2}^- \to \frac{5}{2}^-)$	0.003	0.01	0.004	0.012	0.004	0.015	0.003	0.015
$B(E2\,\frac{1}{2}^- \to \frac{3}{2}^-)$	0.34	0.52	0.022	0.042	0.83	0.92	3.03	3.60
$B(E2\,\frac{3}{2}^- \to \frac{5}{2}^-)$	0.12	0.24	0.025	0.051	0.015	0.03	0.005	0.021

$P(a_1 \leftrightarrow a_2)$ interchanges a_1 and a_2.

The matrix elements, connecting one- and three-quasiparticle states, will arise through $H'_{31}(H'_{13})$ and are given below:

$$\langle a_1^3 J\nu | H'_{31} a_{a_2}^{\dagger} | 0\rangle = -2\sqrt{3!}\lambda_2 \langle a_1^2(0), a_1; J | | a_1^3 J\nu\rangle \sqrt{[a_1]}\, U_{a_1} V_{a_1} (U_{a_1}^2 - V_{a_1}^2)\delta_{a_1 a_2}\delta_{a_1 J} \;, \tag{8}$$

$$\langle 0 | [\alpha^{\dagger}(a_1 a_2 J_{12}) a_{a_3}^{\dagger}]^{\dagger J} H'_{31} a_{a_4} | 0\rangle = -2\sqrt{2}\lambda_2 \sqrt{[a_1]}\, U_{a_1} V_{a_1} (U_{a_3}^2 - V_{a_3}^2)\delta_{a_1 a_2}\delta_{a_3 a_4}\delta_{a_3 J}\delta_{J_{12} 0} \;. \tag{9}$$

The energy matrix in the space of one and three quasiparticles, for a given spin and parity, can now be set up and diagonalized for getting the eigenvalues and the eigenfunctions.

In order to carry out the calculations, ^{56}Ni is assumed to be an inert core and the active neutrons are distributed among the single-particle orbitals $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ orbitals, the unperturbed energies of which are taken to be the same as in Ref. 15. The Kuo-Brown[14] matrix elements are used. The procedure for calculating the quasiparticle parameters and removal of the spurious states is described in Refs. 9 and 15. The results of the calculations are listed in Tables I—III.

Table I indicates that the LN approach yields a slightly improved agreement with experiments for the first few low-lying levels. In general, the differences with the earlier MTDA calculations are slight but point to an improvement. The same is true for the calculated magnetic moments. However, the differences are quite large for the reduced transition strengths for $M1$ and $E2$ transitions listed in Table III. This is because amplitudes of two different states enter the calculation of transition strengths which are quite sensitive to even slight changes in the amplitudes. This does not apply to magnetic moments. It will be desirable to compare Table III with experiments as data become available.

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