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Parametrization of the total cross section for $\pi d \rightarrow pp$ below 330 MeV

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The parametrizations developed by Spuller and Measday for the energy dependence of the total cross section for $\pi d \rightarrow pp$, which have been used to normalize some $\pi d \rightarrow pp$ angular distributions, are reviewed in light of high accuracy data taken since 1975. Slight modifications of the parameters of the earlier analysis yield significant improvement in the agreement with the newer data. A simpler parametrization which fits the data equally well is suggested for absolute normalizations of relative angular distributions for the process.

[NUCLEAR REACTIONS $d(\pi^+,p)p, T_\pi < 330$ MeV; parametrization of $\sigma(E)$ using phenomenological formulas; discussion of previous work; comparison with data; simple formula suggested for absolute normalization of measured relative $d\sigma/d\Omega$.]

In 1975, Spuller and Measday[1] (SM) examined the then existing data on the reactions $\pi d \leftrightarrow NN$ in order to obtain information on s-wave pion production at threshold. The SM work used two forms to express the energy dependence of the total cross section:

$$\sigma = \frac{2}{3}\frac{P_p^2}{\eta^2}(\alpha_0\eta + \alpha_1\eta^2 + \alpha_2\eta^3 + \alpha_3\eta^4) + \frac{G(s)\pi \lambda\!\!\!^-{}_\pi^2 \Gamma_{el}\Gamma_r}{(E-E_R)^2 + \Gamma_T^2/4} , \quad (1)$$

where

$$\Gamma_{el} = \gamma_{el}[(\eta R)^3/(1+(\eta R)^2)] ,$$

$$\Gamma_r = k\gamma_T(RP_p) ,$$

$$\Gamma_T = \gamma_T\{(\eta R)^3/[1+(\eta R)^2] + k(RP_p)\} ,$$

R is the channel radius in units of $h/m_\pi c$, P_p is the c.m. momentum of proton in units of $m_\pi c$, η is the c.m. momentum of pion in units of $m_\pi c$, E_R is the resonant energy,

$$G(s) = \frac{2J+1}{(2S_\pi+1)(2S_d+1)} = \frac{5}{3} ,$$

k is the fraction of total width from $\pi^+ + d \rightarrow p + p$ channel $= 0.05$ for $\eta < 2.8$; and

$$\sigma = \frac{2}{3}\left(\frac{P_p}{\eta}\right)^2 \sum_{i=0}^{4} \alpha_i \eta^{i+1} \quad (2)$$

for $\eta \leq 1.55$, where α_1 was set equal to -0.2 as in Ref. 1 based on the calculations of Afnan and Thomas.[2] Because this formalism provided a means of obtaining predictions of the total cross section for any energy below 330 MeV, several experiments[3-6] were normalized using the results of the SM parametrizations. After that analysis was performed in 1975, several experiments[7-9] greatly improved the experimental knowledge below 140 MeV. A review of the experimental situation up to 1981 may be found in Ref. 10. The use of the SM analysis for absolute normalization raises the question as to whether the parameters arrived at in the SM analysis are still valid. This Brief Report addresses that question and suggests a simpler parametrization for absolute normalization.

In doing the refitting, a data base has been chosen[5,7-9,11-13] which emphasizes the newer data. The addition of the remaining data sets would not alter the results to any great extent owing to the large uncertainties in the older data. The Rose measurements[11] are included in the data base, even though they possess relatively large errors (> 8%), since they still provide the only data below 20 MeV. Coulomb corrections[14] were applied to the data before fitting, as in the SM analysis.

The results of fitting Eq. (1) to the restricted data base are given in Table I. Also given for comparison are the SM values (their "fit F"), and both sets of results for σ are shown in Fig. 1. The parameter R was set equal to 1.2 as in the SM analysis. Though the values for all parameters agree within error, the newer parameter set yields a significant improvement in agreement with the data. For example, the SM parameter set predicts 8.93 mb at 80 MeV (with Coulomb corrections removed) which compares with the experimental value[9] of 8.64 ± 0.15 mb; the new parameter set given in Table I predicts 8.65 mb (with Coulomb corrections removed), substantially better in agreement.

As for Eq. (2), the results are given in Table II, along with the SM results (their "fit C"). The improvement in

TABLE I. Least squares fit results obtained for Eq. (1) in this work and in the Spuller-Measday analysis (Ref. 1). χ^2/ν_D is based on the data base (54 data points) discussed in the text. All cross section data have Coulomb corrections (Ref. 12) applied. Errors shown represent one standard deviation.

	Present work	Spuller and Measday
α_0(mb)	0.25 ± 0.04	0.27 ± 0.04
α_1(mb)	−(0.42 ± 0.19)	−(0.5 ± 0.2)
α_2(mb)	−(0.11 ± 0.15)	0.05 ± 0.01
α_3(mb)	0.08 ± 0.06	0.03 ± 0.03
$\gamma_e(m_\pi c^2)$	0.60 ± 0.11	0.60 ± 0.15
$\gamma_t(m_\pi c^2)$	0.70 ± 0.04	0.71 ± 0.06
E_R(MeV)	2182 ± 7	2183 ± 8
χ^2/ν_D	1.7	3.9

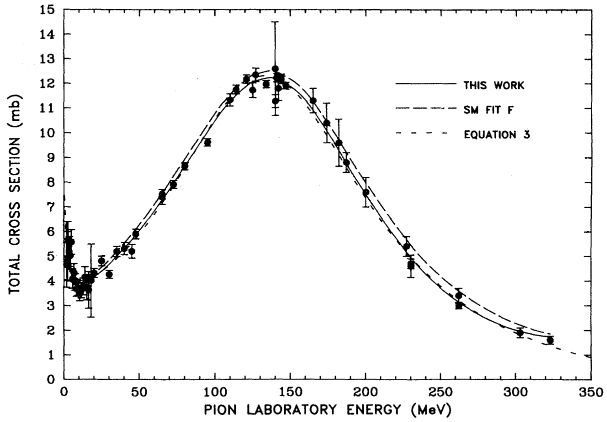

FIG. 1. Experimental data for $\pi d \rightarrow pp$ total cross section energy dependence compared with Eqs. (1) (parameters given in Table I) and (3) (parameters given in text). Experimental data are from Refs. 5, 7–9, and 11–13. Calculations and experimental data are shown with Coulomb corrections discussed in text *removed.*

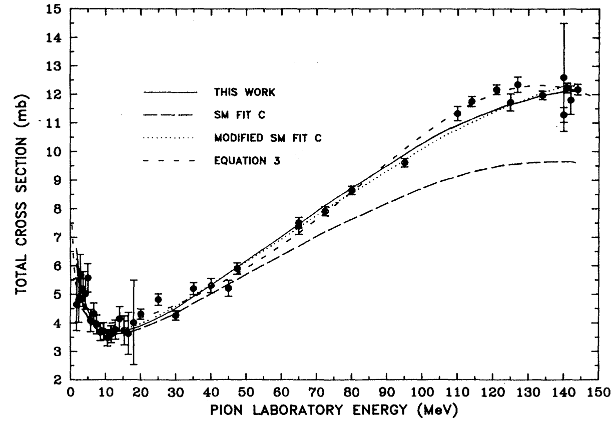

FIG. 2. Experimental data for $\pi d \rightarrow pp$ total cross section energy dependence compared with Eqs. (2) (using parameters given in Table II) and (3) (parameters given in text). Experimental data are from Refs. 5, 7–9, and 11–13. Calculations and experimental data are shown with Coulomb corrections discussed in text *removed.*

agreement is significant, but a slightly modified SM parameter set (within quoted errors; see Table II) yields almost identical agreement. These three sets yield the results shown in Fig. 2. Comparing the results obtained here (shown in Fig. 2) with those of SM (shown in SM Fig. 2) for the published SM fit C parameters, it is obvious that the two sets of results are quite different. The results obtained herein with the modified fit C parameters more closely resemble the results displayed in SM Fig. 2. Thus it may be concluded that the published SM fit C values were rounded from those actually used to generate SM Fig. 2. This discrepancy has important implications for experiments which used the published SM fit C parametrization for normalization. In all three parameter sets, the values agree within cited errors. However, Eq. (2), which includes terms up to η^5, emphasizes the small variations in the coefficients, and the small uncertainties in the data put stringent requirements on the values of α_4 and α_5.

Again, it should be pointed out that the intent of the SM work was primarily towards making some observations on s-wave pion production and not to provide experimentalists with an absolute normalization tool. It is not surprising, then, that the forms in Eqs. (1) and (2) seem somewhat cumbersome for simply normalizing data—they were never intended for that purpose.

TABLE II. Same as Table I, except for Eq. (2), using 41 data points below 150 MeV.

		Spuller and Measday	
	Present work	Published	Modified
α_0	0.267 ± 0.019	0.247 ± 0.017	0.247
α_2	0.25 ± 0.24	0.6 ± 0.3	0.6
α_3	1.66 ± 0.37	1.0 ± 0.5	1.05
α_4	-0.81 ± 0.14	-0.6 ± 0.2	-0.55
χ^2/ν_D	2.8	47	3.1

For the purpose of normalizing relative differential cross sections, a form much simpler than Eqs. (1) or (2) can be used which gives the total cross section directly (i.e., no Coulomb corrections):

$$\sigma(\text{mb}) = a + \frac{b}{\sqrt{T_\pi}} + \frac{c \times 10^4}{(E - E_R)^2 + d}; \quad T_\pi < 330 \text{ MeV} , \tag{3}$$

with $(a,b,c,d,$ and $E_R) = (-1.2 \pm 0.3$ mb, 3.5 ± 0.5 mb MeV$^{-1/2}$, 7.4 ± 0.7 mb MeV^{-2}, 5600 ± 400 MeV^2, and 2136 ± 1 MeV). Here T_π is the incident pion laboratory energy and $E = [(m_\pi + m_d)^2 + 2T_\pi m_d]^{1/2}$ is the invariant energy. This formula, using only T_π, is simpler in form than Eq. (1) and has a broader range of applicability than Eq. (2), making it more suitable for normalization purposes. With the values noted above, Eq. (3) yields $\chi^2/\nu_D = 1.7$, and the agreement with the data is shown in Figs. 1 and 2. At 80 MeV, Eq. (3) predicts $\sigma = 8.6$ mb, in exact agreement with the aforementioned measurement. The form of Eq. (3) works well throughout the region below 330 MeV since it simulates in a simple form the s-wave, p-wave, and higher contributions in its second, third, and first terms, respectively.

In summary, an analysis similar to that performed by Spuller and Measday indicates that the values arrived at for the parameters of Eqs. (1) and (2) need to be changed slightly to arrive at the best fit solutions for the more recent data. Though their conclusions on s-wave pion production at threshold are not altered significantly, the results obtained here are more appropriate to the high accuracy data now available on the process. More accurate data are still needed below 20 MeV to better determine α_0. Finally, for absolute normalization purposes, Eq. (3) offers a simpler parametrization of the total cross section energy dependence and is suggested for future use.

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