

Spin-isospin ordered shell model states in nuclei

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Several shell model configurations entitled to describe a spin-isospin phase in nuclei are analyzed at different nuclear densities and deformations in the region $A = 12-36$. Shell effects are found to be dominant. As a result, the spin-isospin alignment ($\sigma_3\tau_3 = +1$ or $\sigma_3\tau_3 = -1$) is most favored in nuclei around $A = 28$ because of their distinctive shell model structure. Moreover, in a given nucleus an energy gain over the normal and other possible phases can be obtained at reasonably low densities, if a fraction rather than the totality of nucleons have the same spin-isospin alignment. The analysis of the results strongly suggests that the ordered phase should be described by a correlated shell model state composed of a suitable number of these partially ordered configurations. The relevance of the results to possible collective properties of properly chosen nuclei at ordinary density is pointed out.

[NUCLEAR STRUCTURE Spin-isospin phases in nuclei. Shell model. Totally
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I. INTRODUCTION

The spin-isospin ($\sigma\tau$) phase in nuclear matter,¹ or the neutral pion condensate,² is a state of matter in which layers of neutrons with spin up and protons with spin down, hereafter called $\sigma\tau$ -up nucleons, alternate with layers of $\sigma\tau$ -down nucleons (neutrons with spin down and protons spin up).

Such a $\sigma\tau$ -ordered structure is strongly favored with respect to the normal homogeneous phase by the one-pion-exchange (OPE) potential, whose contribution, already at normal density ρ_0 , more than compensates for the kinetic energy increase caused by a certain degree of localization of the nucleons.³

It is expected, however, that, because of the competing effect of the repulsive short range components of the $\sigma\tau$ -dependent nucleon-nucleon interaction, the transition takes place, if at all, at a density higher than ρ_0 . One can only hope that such a critical density ρ_c is not too high, so that it can be reached during the collision between ions. Unfortunately its estimate is quite uncertain, depending critically on the short range correlations, which are very hard to treat, and on the mechanism through which the transition takes place.

It is generally assumed that the $\sigma\tau$ phase is reached as soon as the excitation energy of a collective excitation mode with the quantum number of the pion goes to zero. In all calculations based on this assumption,⁴ the interaction enters mainly in the particle-hole (p-h) channel, and the p-h matrix elements of its short range part are simply parametrized by a constant term. Such a parametrization seems to be justified on the grounds of recent microscopic calculations⁵ using a Brueckner reaction matrix G derived from realistic interactions. The critical value ρ_c was estimated to be more than twice the normal density ρ_0 and was destined to become much higher if screening effects were taken into account.

Without going into the merits of how conclusive those calculations can be considered, we would like to point out

that ρ_c can also be determined, as in Refs. 1 and 3, by a direct comparison between different phases. This method does not depend on the way the transition might take place, but rather on the picture one makes of the $\sigma\tau$ phase. The role of the p-h channel is not necessarily crucial in determining ρ_c , which might therefore result in it being quite different. Unfortunately, not much progress has been made in estimating ρ_c using this approach, since the $\sigma\tau$ -density modulation of the ordered phase makes the treatment of the short-range correlations more involved than in the other case.

In the absence of conclusive calculations one cannot exclude the possibility that the $\sigma\tau$ phase might be accessible to nuclei, though in a transient way. With this perspective we intend to analyze the model of $\sigma\tau$ phase proposed for nuclei by Do Dang,⁶ to be referred to as I.

In this model the phase can exist only if nuclei become very oblate, and it is simply described by a single shell model (sm) configuration, hereafter called a totally ordered (TO) configuration, whose lowest single particle (sp) orbits are filled with one kind of nucleon pairs only, either $\sigma\tau$ -up or $\sigma\tau$ -down nucleons (not both as in the normal phase). A calculation using the OPE potential, made regular at short distances by a cutoff mass term, was carried out for ^{12}C . It was found that ρ_c was above $3\rho_0$. Apparently the price paid in energy, necessary to $\sigma\tau$ align all nucleons, was too high.

Subsequent calculations⁷ showed that the value of ρ_c is drastically reduced if the ρ -exchange potential is included, and especially if admixtures of the Δ isobar are taken into account. An opposite effect is to be expected, however, from the short-range components, especially if included in a less schematic sm calculation, which takes the p-h channel into account.

It is therefore desirable to look for different, less energy demanding mechanisms, through which the phase can be formed. As shown in Sec. II, alternative TO states, which can be described by a single sm configuration, indeed exist. These are, however, unlikely to be favored over the

phase proposed in I. For this reason our analysis took the following directions. In the first place, it might well be that nuclei other than ^{12}C might reach the phase at accessible densities. For this purpose we explored (Sec. III) nuclei in the region between $A=12$ and 36 and found that the shell structure makes some nuclei, particularly ^{28}Si , much more promising. Given the importance of shell effects and the flexibility of the shell model, moreover, we found that it was worth examining partially ordered (PO) configurations, in which only a fraction of nucleons is $\sigma\tau$ aligned, while the others fill the lowest orbits forming an inert core. Such a mechanism was suggested by the present authors in a short paper,⁸ where it was shown that some PO configurations are favored over the TO ones of I by the OPE potential.

In Sec. III of this work a thorough and critical analysis of this PO scheme is presented, hoping that, in spite of its schematic character, the model might provide useful suggestions in view of a more realistic sm study of the ordered phase. For the sake of simplicity, an interaction which includes the OPE plus the ρ exchange is used throughout the analysis.

II. ORDERED CONFIGURATIONS IN NUCLEI

The neutral pion condensate is characterized by a non-vanishing mean value of the pion field $\langle\phi_0\rangle$.

In the static limit $\langle\phi_0\rangle$ is related to the nuclear structure through

$$(\nabla^2 - m_\pi^2)\langle\phi_0\rangle = \vec{\nabla} \cdot \vec{\mathcal{S}}, \quad (2.1)$$

where the pion source function $\vec{\mathcal{S}}$ is the $\sigma\tau$ -nuclear density:

$$\vec{\mathcal{S}}(\vec{r}) = \frac{f_\pi}{m_\pi} \langle \Phi | \psi^\dagger(\xi, 0) \tau_3 \vec{\sigma} \psi(\xi, 0) | \Phi \rangle. \quad (2.2)$$

$|\Phi\rangle$ is the ground state of the nuclear system, $\psi(\xi, t)$ is the nucleon field operator, ξ stands for spatial and $\sigma\tau$ coordinates as well, m_π is the pion mass, and f_π is the pion-nucleon coupling constant.

If $|\Phi\rangle$ is a Slater determinant of sp wave functions factorized in their orbital and $\sigma\tau$ components, the $\sigma\tau$ -nuclear density assumes the expression

$$\vec{\mathcal{S}}(\vec{r}) = \hat{z} \frac{f_\pi}{m_\pi} \sum_{a\sigma_3\tau_3} (\sigma_3\tau_3)_a \rho_a(\vec{r}), \quad (2.3)$$

where \hat{z} is the spin quantization axis, and $\rho_a(\vec{r})$ the nucleon density in the sp state with orbital quantum numbers a .

Apparently $\vec{\mathcal{S}}(\vec{r})=0$, if each orbit is filled with both $\sigma\tau$ -up and $\sigma\tau$ -down nucleons, as in the normal or $\sigma\tau$ -disordered phase.

In order to create a $\sigma\tau$ -ordered structure it is necessary to selectively fill the sp orbits. In nuclear matter this was done by imposing that $\sigma\tau$ -up and $\sigma\tau$ -down nucleons are alternatively localized around layers perpendicular to the spin quantization axis. The corresponding value of $\vec{\mathcal{S}}(\vec{r})$ is

$$\vec{\mathcal{S}}(\vec{r}) = \hat{z} \frac{f_\pi}{m_\pi} \rho_\perp \sum_l |\phi_l(z)|^2 (\sigma_3\tau_3)_l, \quad (2.4)$$

where $\phi_l(z)$ describes the localizations of nucleons, all having the value $(\sigma_3\tau_3) = -(-)^l$, around the l th layer, while

ρ_\perp is the constant two-dimensional nuclear density in the layer.

In the nuclei, it is appropriate to use an axially symmetric deformed sp basis. This for simplicity is assumed to be a complete set of eigenfunctions $\psi_a(r)$ of an anisotropic oscillator Hamiltonian h . In cylindrical coordinates,

$$h = \frac{p^2}{2m} + \frac{1}{2} m \omega_z^2 z^2 + \frac{1}{2} m \omega_\perp^2 r_\perp^2, \quad (2.5)$$

$$\psi_a(\vec{r}) = \phi_{nm}(r_\perp, \phi) R_{n_z}(z), \quad (2.6)$$

where $R_{n_z}(z)$ describes oscillations of frequency ω_z along the symmetry axis, coincident with the spin quantization axis, and $\phi_{nm}(r_\perp, \phi)$ describes the oscillations of frequency ω_\perp in the perpendicular plane.

In order to establish a close analogy with the nuclear matter case, one must fill the oscillator orbits according to a selective order dependent on the quantum number n_z only. The $\sigma\tau$ -nuclear density can then be written

$$\vec{\mathcal{S}}(\vec{r}) = \hat{z} \frac{f_\pi}{m_\pi} \sum_{n_z} (\sigma_3\tau_3)_{n_z} \rho_{\perp n_z} R_{n_z}^2, \quad (2.7)$$

where

$$\rho_{\perp n_z} = 2 \sum_{nm} |\phi_{nm}(r_\perp, \phi)|^2. \quad (2.8)$$

In the TO configuration of I $(\sigma_3\tau_3)_{n_z} = +1$ [or $(\sigma_3\tau_3)_{n_z} = -1$] for all n_z , namely all nucleons have the same $\sigma\tau$ alignment, so that each of the lowest orbits contains only a pair of, say, $\sigma\tau$ -up nucleons.

An alternative TO scheme should be obtained by filling the orbits of a given n_z value with pairs of nucleons having $(\sigma_3\tau_3)_{n_z} = -(-)^{n_z}$. This is the closest analog to the nuclear matter case. A complete analogy is practically impossible though. In nuclear matter the different layers are equally and uniformly filled. In nuclei, because of the shell structure, the number of filled orbits with even n_z is, in general, different from those with odd n_z . This is more so in light nuclei at large oblate deformations, since almost all their lowest orbits have $n_z=0$, as can be seen from Fig. 1. Therefore, in these nuclei this new scheme differs very little from the TO scheme of I. The two schemes are actually identical in nuclei like ^{12}C . We will see, however, that where even small differences exist, this new scheme is strongly unfavored.

PO configurations are easily obtained by imposing that only a fraction of nucleon pairs selectively fill the external orbits, while the others completely fill the inner orbits forming an inert core.

The energy of the different $\sigma\tau$ phases at different densities and deformations is estimated using the Hamiltonian

$$H = \sum_{i=1}^A h(i) + \sum_{i<j}^A V_{\sigma\tau}(i,j), \quad (2.9)$$

where h is given in (2.5), and

$$V_{\sigma\tau}(1,2) = V_\pi(1,2) + V_\rho(1,2). \quad (2.10)$$

In momentum space, V_π and V_ρ have the expressions

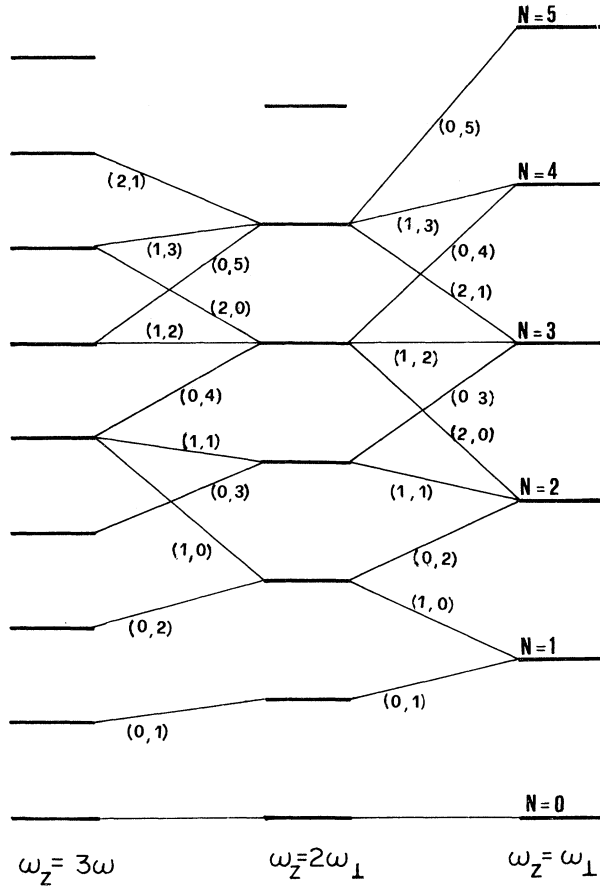


FIG. 1. Level scheme for various deformations.

$$V_T(\vec{q}) = -\frac{1}{(2\pi)^3} \frac{f_\pi^2}{m_\pi^2} \Gamma_\pi^2(q^2) \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{q^2 + m_\pi^2}, \quad (2.11)$$

$$V_\rho(\vec{q}) = -\frac{1}{(2\pi)^3} \frac{f_\rho}{m_\rho} \Gamma_\rho^2(q^2) \frac{(\vec{\sigma}_1 \times \vec{q})(\vec{\sigma}_2 \times \vec{q})}{q^2 + m_\rho^2}, \quad (2.12)$$

with the form factors Γ_π and Γ_ρ given by the expressions

$$\Gamma_\pi(q^2) = \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 + m_\pi^2}; \quad \Gamma_\rho(q^2) = \frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 + m_\rho^2}, \quad (2.13)$$

where Λ_π and Λ_ρ are cutoff mass parameters.

Consistent with the mean field approximation we calculated only the direct part of the expectation values. These are

$$\langle V_\pi \rangle = -\frac{1}{2} \int d\vec{q} \frac{\Gamma_\pi^2(q^2)}{q^2 + m_\pi^2} |\vec{q} \cdot \vec{S}(\vec{q})|^2, \quad (2.14)$$

$$\langle V_\rho \rangle = -\frac{1}{2} \frac{(f_\rho/m_\rho)^2}{(f_\pi/m_\pi)^2} \int d\vec{q} \frac{(q^2)}{q^2 + m_\rho^2} |\vec{q} \times \vec{S}(\vec{q})|^2, \quad (2.15)$$

where $S(q)$ is the Fourier transform of $S(r)$:

$$S(\vec{q}) = \hat{z} \frac{f_\pi}{m_\pi} \sum_{n_z} (\sigma_3 \tau_3)_{n_z} \rho_{n_z}(q_z) \rho_{n_z}(q_\perp) \quad (2.16)$$

and

$$\rho_{n_z}(q_\perp) = 2 \sum_{mm} \rho_{nm}(q_\perp), \quad (2.17)$$

$\rho_{n_z}(q_z)$ and $\rho_{nm}(q_\perp)$ are the Fourier transforms of $R_{n_z}^2(z)$ and $\rho_{nm}(r_\perp)$. The total energy is given by

$$E_{\sigma\tau} = \langle V_{\sigma\tau} \rangle + \sum_{a\sigma_3\tau_3} [(n+1)\hbar\omega_1 + (n_z + \frac{1}{2})\hbar\omega_z] N_{a\sigma_3\tau_3}. \quad (2.18)$$

The values of $E_{\sigma\tau}$ of the different phases are to be compared among one another and with the normal phase, for which $\langle V_{\sigma\tau} \rangle = 0$. For this latter phase a spherical potential is assumed, so its energy results to be

$$E_{os} = 4 \sum_a (\mathcal{N}_a + \frac{3}{2}) \hbar\omega_s, \quad (2.19)$$

where \mathcal{N}_a is the total oscillator quantum number and

$$\omega_s^3 = \omega_z^2 \omega_\perp, \quad (2.20)$$

which ensures that the volume of the system in different phases is conserved. The density of the system is related to the oscillator frequency ω_s by

$$\rho/\rho_0 = (\omega_s/\omega_0)^{3/2}. \quad (2.21)$$

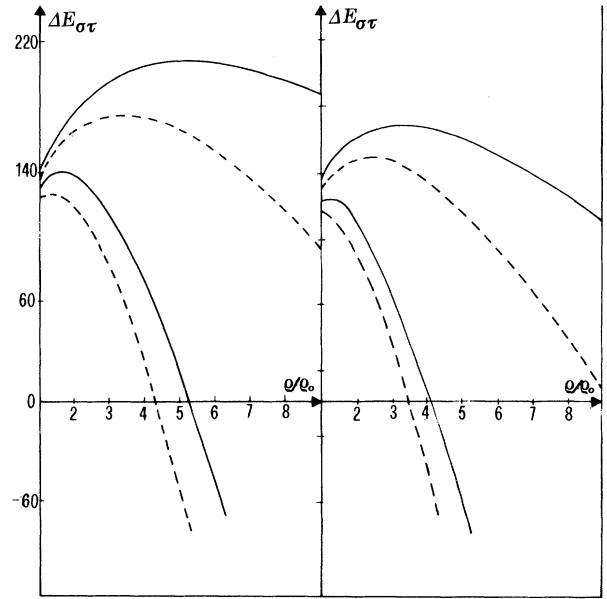


FIG. 2. $\Delta E_{\sigma\tau} = E_{\sigma\tau} - E_{os}$ (in MeV) vs ρ/ρ_0 for the $(\sigma_3\tau_3)_{n_z} = -1$ TO scheme (highest curves) and for the $(\sigma_3\tau_3)_{n_z} = 1$ [or $(\sigma_3\tau_3)_{n_z} = -1$] TO scheme (lowest curves) in ^{28}Si , at small (left-hand side) and large (right-hand side) deformations. The solid line refer to the V_π interaction, the dashed lines to the $V_\pi + V_\rho$ interaction.

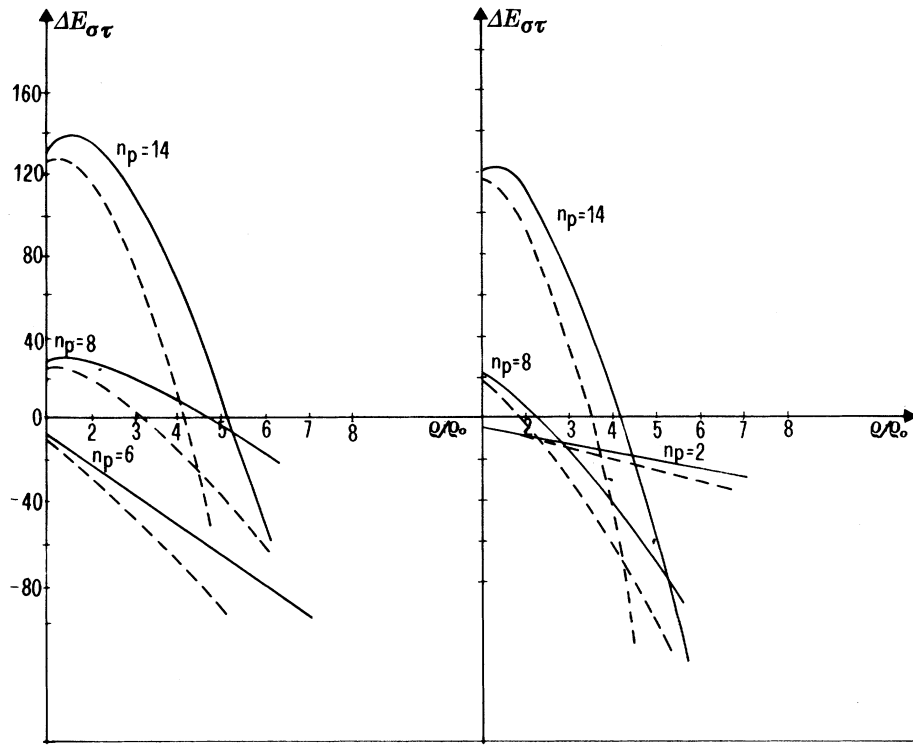


FIG. 3. $\Delta E_{\sigma\tau} = E_{\sigma\tau} - E_{os}$ (in MeV) vs ρ/ρ_0 of TO and some PO configurations in ^{28}Si for small (left-hand side) and large deformations (right-hand side). n_p is the number of the $\sigma\tau$ -ordered pairs. Solid and dashed lines as in Fig. 2.

At $\omega_z \cong 2\omega_1$ the energy of the PO configurations containing $n_p = 2$ $\sigma\tau$ -aligned pairs is lowest at normal density. This is a joint effect due to deformation and shell structure, not directly dependent on the interaction $V_{\sigma\tau}$. Already in the disordered phase, in fact, the system would gain in energy if the nucleons moved from the lowest spherical sp orbits to the $\omega_z \cong 2\omega_1$ deformed ones. This can be seen by inspecting Fig. 1. The increment of energy required to align a nucleon pair is only partially compensated by turning on the attractive interaction through the alignment. As the number of ordered pairs is more than two, ρ_c jumps. The subsequent fluctuations versus the number of the $\sigma\tau$ -aligned pairs are the result of a balance between the energy nucleons might need in order to jump the shells and the increased attractive contribution of $V_{\sigma\tau}$ induced by the $\sigma\tau$ alignment. The shell structure favors the $\sigma\tau$ configuration with $n_p = 8$ ordered pairs at a reasonably low density ($\rho_c^\pi = 2.27$). The behavior of some typical configurations versus ρ/ρ_0 is shown in Fig. 3. It is worthwhile to observe the increasing importance of V_ρ as the density increases. This is to be expected, being that V_ρ is short ranged. For the same reason, on the other hand, the inclusion of such a force in an uncorrelated sm scheme, like the present one, is unlikely to be justified, especially at high density.

Having shown that quite a few $\sigma\tau$ configurations are candidates to represent a possible $\sigma\tau$ phase in nuclei, it remains for us to ascertain whether these configurations

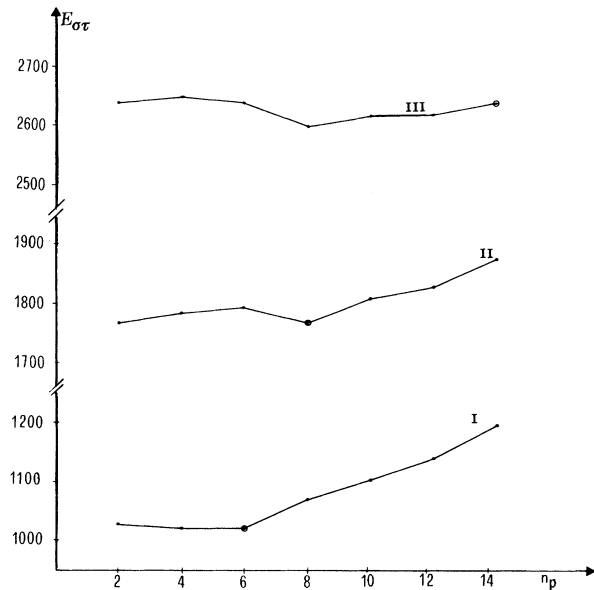


FIG. 4. $E_{\sigma\tau}$ (in MeV) versus the number n_p of the $\sigma\tau$ -ordered pairs in ^{28}Si at the critical density and deformation of the configurations indicated by a circle: (I) $n_p = 6$; $(\rho/\rho_0)_c = 1.$, $(\omega_z/\omega_1)_m = 1.$; (II) $n_p = 8$, $(\rho/\rho_0)_c = 2.27$, $(\omega_z/\omega_1)_m = 2.43$; (III) $n_p = 14$, $(\rho/\rho_0)_c = 4.13$, $(\omega_z/\omega_1)_m = 2.52$.

can be considered pure. Only in such a case are they entitled to represent the phase on their own. Figure 3 seems to indicate that this is not the case. This is more explicitly shown in Fig. 4. At a given critical density and corresponding deformations, configurations with different numbers of ordered pairs are quite close in energy, differing by a quantity which is of the same order of the two-body diagonal matrix elements of $V_{\sigma\tau}$. It is therefore to be expected that the $\sigma\tau$ order, if it exists at all, has a rather complicated shell structure, to be described by a linear combination of several $\sigma\tau$ -ordered configurations.

The result of the present analysis can be summarized as follows:

(i) Of the possible $\sigma\tau$ -ordered sm schemes, the one proposed in I is likely to be the scheme which energetically is most favored.

(ii) Shell effects are so dominant that, before the critical density of the TO configuration is reached, several PO configurations become lowest in energy.

(iii) All these configurations are close in energy and therefore should enter on equal footing in a full sm calcu-

lation. The possible PO $\sigma\tau$ phase would be therefore described by a correlated sm state.

(iv) This phase, if it exists at all, is more likely to be present in nuclei and for deformations such that the number of valence nucleons fill about half the number of quasidegenerate sp orbits. This is the case of ^{28}Si for $\omega_2 \cong \omega_1$ and $\omega_2 \cong 2\omega_1$. Heavier deformed nuclei, however, where the numbers of valence nucleons and sp orbits are considerably larger, might be better candidates.

The fact that, at least with the interaction considered here, the valence nucleons tend to get $\sigma\tau$ aligned already at normal density might get manifested through some collective behavior of special deformed heavy nuclei having the sm structure of the type just described for ^{28}Si . These collective properties should be quite different from those associated with precritical phenomena.⁹ They are rather suggestive of some kind of pairing correlations induced by the tensor force, somewhat similar to the one suggested in Ref. 10. An analysis aimed at a clear characterization of these collective properties is in progress.

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