Off-shell and nuclear structure effects in quasifree electron scattering

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The quasifree scattering process $^{12}C(e,e'p)^{11}B$ is investigated in the distorted wave impulse approximation using the effective Hamiltonian of McVoy and Van Hove. The off-energy-shell effects are correctly taken into account by replacing the momentum of the proton by the gradient operator $\vec{\nabla}$. The influence of the spin-orbit coupling in the final state interaction and the nuclear deformation has also been studied. The results indicate that the effect of the gradient operator is to decrease the cross section over a wide range of momentum and to shift the peak of the cross section towards the higher momentum region. It is found that the spin-orbit interaction affects only the maxima and minima of the cross section. The deformed oblate potential of Nilsson with N and $N+2$ coupling for the bound state wave function is found to improve the agreement of the cross section with the experimental data.

[NUCLEAR REACTIONS Quasifree electron scattering from ^{12}C, distorted wave impulse approximation, off-shell effects, gradient operator for the proton, nuclear deformation, oblate deformed Nilsson wave function.]

I. INTRODUCTION

The quasifree knockout process has been widely used with various particles as a probe for the investigation of nuclear structure. The reaction is conceptually simple in its formalism and yields reliable information about the single particle aspects in light nuclei.[1] More than a decade ago, the quasifree $(e,e'p)$ reaction was studied by one of us[2] in the plane wave impulse approximation (PWIA) using the effective Hamiltonian of McVoy and Van Hove[3] for the electron-nucleon interaction, and subsequently the treatment has been greatly improved by Epp and Griffy[4] and Radhakant[5] by including the final state interaction for the process. In recent years, Mougey *et al.*[6] and Boffi *et al.*[7,8] have contributed substantially to the study of this process and dexterously extracted the single particle features in nuclei using a factorizable form for the coincidence cross section. The aim of the present investigation was to carefully assess the off-energy-shell effects in the quasifree process. Our results indicate that the off-shell effects reduce the coincidence cross section in ^{12}C by about 50%, and this is in accordance with the experimental observation of Mougey *et al.*[6] who reported that the absolute cross section is 40% lower than expected from the theory[7] which neglects the off-shell effects.

The process of quasifree scattering of electrons involves the study of the following four essential ingredients, viz.,

(1) the effective Hamiltonian for the basic electron-proton interaction,
(2) off-energy-shell effects which inevitably occur in the nuclear problem,
(3) the bound state wave function of the nucleon which is knocked out, and
(4) the final state interaction of the outgoing proton with the residual nucleus.

For the effective electron-proton interaction, we use the McVoy and Van Hove Hamiltonian which reproduces the free electron-proton cross section admirably well. Also this Hamiltonian allows the inclusion of off-energy-shell effects in the case of the electron-nucleus interaction. The off-energy-shell effects are taken into account correctly by replacing the momentum of the proton by the gradient operator $\vec{\nabla}$. The Saxon-Woods form of Elton and Swift[9] and Giannini and Ricco[10] potentials are used to generate the bound state wave functions. These potentials correctly yield the separation energy of the protons in different shells. Since it is known that the ground state of the ^{12}C nucleus is deformed, the deformed oblate potential of Nilsson is also used in the study. The optical potentials of Glassgold and Kellogg,[11] Giannini and Ricco,[10] and Jackson and Abdul-Jalil[12] are used for the final state interaction of the proton with the residual nucleus, and these potentials yield the elastic proton-nucleus scattering data quite well.

The purpose of this paper was to make a systematic and detailed study of the various factors that influence the quasifree scattering process and to investigate the effect of the gradient operator $\vec{\nabla}$ and the nuclear deformation on the coincidence cross section. The approach is rigorous and does not aim at the factorization of the cross section into the free electron-proton cross section and the nuclear spectral function. The plan of the paper is outlined below. The general formalism for the reaction process is presented in Sec. II. The evaluation of nuclear matrix elements in the distorted wave impulse approximation (DWIA) is detailed in Sec. III. In contrast to the work of Radhakant,[5] we have obtained a simple and compact expression for the entire matrix element of the process in DWIA with a view to investigate the effect of the gradient operator $\vec{\nabla}$ and the spin-orbit coupling for the outgoing proton. The evaluation of the distorted spectral density function from one of the terms of the matrix element has also been outlined in that section. The deformation of the nucleus also influences the cross section along with the final state interaction. In Sec. IV the effect of the nuclear deformation on the cross section is discussed using the nuclear wave functions obtained by extending the Nilsson model to include the single particle orbitals and contributions from higher major shells. In Sec. V the results obtained for the reaction $^{12}C(e,e'p)^{11}B$ are compared with the available experimental data and earlier calculations.[4–8]

II. GENERAL FORMALISM

The quasifree knockout of a proton by an electron is illustrated in Fig. 1. An incident electron (energy

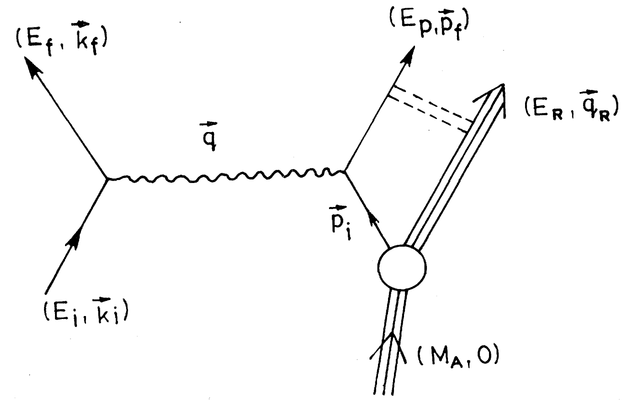

FIG. 1. Quasifree knockout process with final state interaction.

E_i and momentum $\vec{k}_i$) is scattered by a proton with momentum $\vec{p}_i$ and separation energy E_s bound inside a nucleus. In the PWIA, the rest of the nucleus is assumed to be not affected but moving with momentum $\vec{q}_R = -\vec{p}_i$, where $\vec{p}_i$ is the momentum of the struck proton in the nucleus. The conservation laws of momentum and energy require that

$$\vec{q}_R = -\vec{p}_i = \vec{k}_i - \vec{k}_f - \vec{p}_f \ , \tag{1}$$

$$E_i - E_f = E_s + E_p + \frac{{q_R}^2}{2(A-1)M} \ , \tag{2}$$

where E_f and $\vec{k}_f$ are the final energy and momentum of the electron and $\vec{p}_f$ is the momentum of the emitted proton with kinetic energy E_p. The interaction between the electron and the proton is represented by the McVoy and Van Hove Hamiltonian.

The matrix element for the reaction with the above Hamiltonian can be written in a compact form[2] as given below.

$$M = -\left\{\frac{4\pi e^2}{{q_\mu}^2}\right\} \langle u_f \mid Q - \vec{\alpha}\cdot\vec{J} \mid u_i \rangle F({q_\mu}^2)\delta\left[E_i - E_s - E_f - \frac{{p_f}^2}{2M} - \frac{{q_R}^2}{2(A-1)M}\right] . \tag{3}$$

The quantities involved in Eq. (3) are defined in our earlier work.[2] Squaring the matrix element, summing, and averaging over electron spins,[13] we obtain

$$\mid M \mid^2 = \left\{\frac{4\pi e^2}{{q_\mu}^2}\right\}^2 \mid F({q_\mu}^2) \mid^2 [\,(4E_iE_f + {q_\mu}^2)QQ^* - {q_\mu}^2 \vec{J}\cdot\vec{J}^{\,*} + 2(\vec{k}_f\cdot\vec{J})(\vec{k}_i\cdot\vec{J}^{\,*}) + 2(\vec{k}_f\cdot\vec{J}^{\,*})(\vec{k}_i\cdot\vec{J})$$
$$-2E_f\{(\vec{k}_i\cdot\vec{J})Q^* + (\vec{k}_i\cdot\vec{J}^{\,*})Q\} - 2E_i\{(\vec{k}_f\cdot\vec{J})Q^* + (\vec{k}_f\cdot\vec{J}^{\,*})Q\}\,] . \tag{4}$$

The coincidence cross section is given by

$$\frac{d^3\sigma}{dE_f d\Omega_e d\Omega_p}=\frac{2\pi}{4E_iE_f}|M|^2\rho_f \ , \tag{5}$$

where ρ_f is the three-particle density of states given in Ref. 2.

III. EVALUATION OF NUCLEAR MATRIX ELEMENTS IN DWIA

The complexity of the problem increases when the plane wave approximation is replaced by the distorted wave formalism which includes the final state interaction between the residual nucleus and the emitted proton. However, in the distorted wave theory of Epp and Griffy[4] the momentum operator $\vec{p}$ has been replaced by the asymptotic momentum $\vec{p}_f$ which is valid only for plane waves. The procedure of Radhakant[5] is cumbersome and numerically difficult for evaluation since it involves a large number of terms in the matrix element. Thus, the earlier works are approximate and incomplete. Here, we present a complete and rigorous method of evaluating the matrix element in DWIA including the gradient operator $\vec{\nabla}$ and the spin-orbit coupling for the knocked-out proton. The quantities Q and $\vec{J}$ in Eq. (3) assume simple forms if we restrict our consideration to nuclei with closed shells or subshells.

$$Q=\langle \vec{p}_f,\tfrac{1}{2}m_s \left| \left[1+\frac{{q_\mu}^2}{8M^2}(1+2K)\right] e^{i\vec{q}\cdot\vec{r}} \right| u_{nLJ}(r),L\tfrac{1}{2}JM\rangle \ , \tag{6}$$

$$\begin{aligned}\vec{J}&=\frac{1}{2M}\langle \vec{p}_f,\tfrac{1}{2}m_s\,|\,\vec{p}e^{i\vec{q}\cdot\vec{r}}+e^{i\vec{q}\cdot\vec{r}}\vec{p}+i(1+K)(\vec{\sigma}\times\vec{q})e^{i\vec{q}\cdot\vec{r}}\,|\,u_{nLJ}(r),L\tfrac{1}{2}JM\rangle\\&=\frac{1}{2M}\langle \vec{p}_f,\tfrac{1}{2}m_s\,|\,e^{i\vec{q}\cdot\vec{r}}\{(\vec{q}+2\vec{p})+i(1+K)(\vec{\sigma}\times\vec{q})\}\,|\,u_{nLJ}(r),L\tfrac{1}{2}JM\rangle \ .\end{aligned} \tag{7}$$

They depend upon the shell from which the proton is knocked out and $|u_{nLJ}(r),L\frac{1}{2}JM\rangle$ represents the single particle shell model wave function. Replacing the proton momentum $\vec{p}$ by the gradient operator $\vec{\nabla}$, we have

$$\vec{J}=\left[\frac{1}{2M}\right]\langle \vec{p}_f,\tfrac{1}{2}m_s\,|\,e^{i\vec{q}\cdot\vec{r}}\sum_{n,N=0,1}\sum_{\mu,\lambda,\nu}\nabla_n^\mu\sigma_N^\lambda\vec{A}_{nN}(\mu,\lambda,\nu)K_N^\nu\,|\,u_{nLJ}(r),L\tfrac{1}{2}JM\rangle \ , \tag{8}$$

where

$$\vec{A}_{00}=\vec{q},\ \ \vec{A}_{01}=\hat{\epsilon}_1^{\lambda *}\times\hat{\epsilon}_1^{\nu *},\ \ \vec{A}_{10}=-2i\hat{\epsilon}_1^{\mu *},\ \ \vec{A}_{11}=\vec{0},\ \ K_0^0=1,\ \ K_1^\nu=i(1+K)q_1^\nu \ . \tag{9}$$

In Eq. (8), ∇_0 and σ_0 denote unit operators whereas ∇_1 and σ_1 denote the gradient operator and the Pauli spin operator, respectively. With a view to include the effect of the spin-orbit coupling[14,15] in the final state interaction, the outgoing proton wave function is expanded in terms of the definite final angular momentum states J_f. We arrive at the following expression using the angular momentum algebra:

$$\begin{aligned}\vec{J}=\frac{(4\pi)^2}{2M}\sum_{\substack{l_f,l_q,J_f,L,\mathscr{L}\\ n,N,m_f,m_q,\mu,\lambda}}&(i)^{l_q-l_f}Y_{l_f}^{m_f}(\hat{p}_f)Y_{l_q}^{m_q *}(\hat{q})\vec{A}_{nN}(\mu,\lambda,\nu)K_N^\nu\\&\times C(l_f\tfrac{1}{2}J_f,m_fm_sM_f)C(l_qnL,m_q\mu m_L)C(LN\mathscr{L},m_L\lambda m_{\mathscr{L}})\\&\times C(J_i\mathscr{L}J_f,M_im_{\mathscr{L}}M_f)\langle g_{l_fJ_f}(p_fr),l_f\tfrac{1}{2}J_f||j_{l_q}(qr)\\&\times\{[Y_{l_q}(\hat{r})\times\nabla_n]_L\times\sigma_N\}_{\mathscr{L}}||u_{n_il_ij_i}(r),l_i\tfrac{1}{2}J_i\rangle \ .\end{aligned} \tag{10}$$

When the momentum transfer $\vec{q}$ is chosen along Z direction,

$$Y_{l_q}^{m_q}(\hat{q})=\frac{[l_q]}{(4\pi)^{1/2}}\delta_{m_q,0} \ ,$$

and ν becomes zero.

The radial part of the proton wave function $g_{l_fJ_f}(p_fr)$ is obtained by solving the Schrödinger wave equation with suitable proton-nucleus optical potential that can reproduce the elastic scattering data. In the limit of the plane wave approximation, the radial wave function $g_{l_f J_f}(p_f r)$ reduces[16] to the spherical Bessel function $j_{l_f J_f}(p_f r)$. The reduced matrix element in Eq. (10) has to be evaluated only for the values (a) $n=0$ and $N=0$, (b) $n=1$ and $N=0$, and (c) $n=0$ and $N=1$. The evaluation of $\vec{J}$ involves the calculation of the following radial integrals which include the derivatives of the bound state wave function:

$$F=\int g^*_{l_f J_f}(p_f r) j_{l_q}(qr) u_{n_i l_i J_i}(r) r^2 dr \ , \tag{11}$$

$$F_- = \int g^*_{l_f J_f}(p_f r) j_{l_q}(qr) D_- u_{n_i l_i J_i}(r) r^2 dr \ , \tag{12}$$

$$F_+ = \int g^*_{l_f J_f}(p_f r) j_{l_q}(qr) D_+ u_{n_i l_i J_i}(r) r^2 dr \ , \tag{13}$$

where

$$D_- = \left[\frac{d}{dr} - \frac{L}{r} \right]$$

and

$$D_+ = \left[\frac{d}{dr} + \frac{L+1}{r} \right] . \tag{14}$$

For angular momentum coefficients and reduced matrix elements, we follow the notations and conventions of Rose.[17] Throughout, the symbol $[l]$ denotes $(2l+1)^{1/2}$. The advantage of this method is that the various terms in Eq. (4) are easily calculated once the value of $\vec{J}$ is obtained in terms of its spherical components. The evaluation of one such term is

$$(\vec{k}_i \cdot \vec{J})(\vec{k}_f \cdot \vec{J}^*) = \sum_{m_s, M_i, \mu, \mu'} k_i^\mu J^{\mu *} k_f^{\mu'} J^{\mu'} \ . \tag{15}$$

Since the momentum transfer $\vec{q}$ is chosen along the Z direction, the electron momentum vectors $\vec{k}_i$ and $\vec{k}_f$ in terms of the spherical components become

$$(k_i)_1{}^1 = (k_i)_1{}^{-1} = (k_f)_1{}^1 = (k_f)_1{}^{-1} = -i \frac{|\vec{k}_i| \sin\theta_{qk_i}}{2^{1/2}} \ , \tag{16}$$

$$(k_f)_1^0 = |\vec{k}_f| \cos\theta_{qk_f} \ , \tag{17}$$

$$(k_i)_1^0 = (q)_1^0 + (k_f)_1^0 \ , \tag{18}$$

where θ_{qk_i} and θ_{qk_f} are the angles made by the vectors $\vec{k}_i$ and $\vec{k}_f$ with the direction of $\vec{q}$, respectively.

In Eq. (4), if only the QQ^* term is retained omitting all other terms, the cross section becomes factorizable yielding the spectral function.[18] In a pure independent particle model the spectral distribution is given by[6]

$$S(\vec{p}, E) = \sum_\alpha N_\alpha \, |\Phi_\alpha(\vec{p})|^2 \delta(E - E_\alpha) \ , \tag{19}$$

where $\Phi_\alpha(\vec{p})$ is the Fourier transform of the single particle wave function in coordinate space with the eigenvalue E_α of the state α. N_α is the occupation number. The final state proton-nucleus interaction modifies the form of $\Phi_\alpha(\vec{p})$ as

$$\Phi^D_\alpha(\vec{p}_f, \vec{q}_R, E) = \frac{1}{(2\pi)^{3/2}} \int \chi^{(-)*}_{p_f}(r) \times e^{i \vec{q} \cdot \vec{r}} \phi_\alpha(r) d^3 r \ . \tag{20}$$

$\phi_\alpha(r)$ describes the initial bound state and $\chi^{(-)*}_{p_f}(r)$ the distorted outgoing proton wave function. Except for the constant factor, the form of $\Phi^D_\alpha(\vec{p}_f, \vec{q}_R, E)$ in DWIA is similar to Q in Eq. (6). Using the value of QQ^* for

$$|\Phi^D_\alpha(\vec{p}_f, \vec{q}_R, E)|^2 \ ,$$

we have calculated the momentum distributions for the $p_{3/2}$ state both in the perpendicular and parallel kinematics and compared them with recent experimental data.[19]

IV. NUCLEAR DEFORMATION

The deformation of the nucleus has influence on the coincidence cross section of the quasifree scattering process in the upper shell.[1] There is strong theoretical and experimental evidence for the deformation of ^{12}C, which has an oblate deformed intrinsic state, contrary to the conclusion arrived at by Friar and Negele.[20] The deformed structure is supported by a self-consistent Hartree-Fock calculation[21] using a density dependent effective interaction with a realistic spin-orbit component. The earlier works on ^{12}C also reveal that the ground state is the 0^+ state of a rotational band based on a deformed intrinsic state. There is considerable configuration mixing, as indicated by the work of Cohen and Kurath.[22] The explicit wave function given by Saayman *et al.*[23] for the ground state of ^{12}C has a large admixture of configurations $(p_{3/2})^8(p_{1/2})^0$ and $(p_{3/2})^6(p_{1/2})^2$, each of weight 40%. The recent calculation of longitudinal and transverse response functions[24] for quasifree electron scattering from ^{12}C also indicates similar results. The elastic and inelastic form factors have been calculated[25,26] for ^{12}C in the framework of the Nilsson model, and the results affirm other theoretical conclusions on the deformation of ^{12}C.

In the Nilsson model,[27] the nuclear single particle wave functions $\chi^{(i)}$ are the eigenstates of the Hamiltonian

$$H = H_0^0 + H_\delta + H_l \,, \tag{21}$$

where

$$H_0^0 = \frac{\omega_0}{2}(-\nabla^2 + r^2) \,, \tag{22}$$

$$H_\delta = -\frac{4}{3}\left[\frac{\pi}{5}\right]^{1/2} \omega_0 \delta r^2 Y_{20} \,, \tag{23}$$

and

$$H_l = C\,\vec{l}\cdot\vec{s} + Dl^2 \,. \tag{24}$$

The equipotential surfaces have a constant volume if

$$\omega_x \omega_y \omega_z = \text{constant} \,, \tag{25}$$

and therefore,

$$\omega_0(\delta) = \omega_0^0 (1 - \tfrac{4}{3}\delta^2 - \tfrac{16}{27}\delta^3)^{-1/6} \,. \tag{26}$$

The value $\omega_0(\delta)$ becomes ω_0^0 at $\delta = 0$. The deformation parameter δ is related to a new deformation parameter

$$\eta = \frac{\delta}{\mathscr{K}}(1 - \tfrac{4}{3}\delta^2 - \tfrac{16}{27}\delta^3)^{-1/6} \,. \tag{27}$$

In terms of η,

$$H_\delta + H_l = \omega_0^0 \left[-\eta \frac{4}{3}\left[\frac{\pi}{5}\right]^{1/2} r^2 Y_{20} - 2\,\vec{l}\cdot\vec{s} - \mu l^2 \right] , \tag{28}$$

with

$$\mathscr{K} = -\frac{C}{2\omega_0^0} \text{ and } \mu = \frac{2D}{C} \,. \tag{29}$$

The single particle wave function for the ith state $\chi^{(i)}$ is expanded as

$$\chi_\Omega^{(i)} = \sum_{Nl\Lambda\Sigma} a_{Nl\Lambda\Sigma}^{(i)} \,|\, Nl\Lambda\Sigma \rangle \,. \tag{30}$$

The states $|Nl\Lambda\Sigma\rangle$ are written in configuration space

$$\langle r \,|\, Nl\Lambda\Sigma \rangle = R_{Nl}(r) Y_{l\Lambda}(\hat{r}) \chi_{(1/2)\Sigma} \,, \tag{31}$$

where $R_{Nl}(r)Y_{l\Lambda}(\hat{r})$ is the spherical part of the solution. The Nilsson coefficients $a_{nlj}^{(i)}$ for a particular deformation η are obtained in the $|nlj\Omega\rangle$ basis with $n = \frac{1}{2}(N - l) + 1$ and $j = l + s$. The present calculation is done for the deformation parameter $\eta = -6$ and the single particle wave function[28] for the last filled Nilsson level of ^{12}C is obtained using the values $\omega_0^0 = 41A^{-1/3}$ MeV, $\mathscr{K} = 0.08$, and $\mu = 0$.

$$\chi_{1/2} = 0.766 \,|\, 1p_{3/2} \rangle + 0.642 \,|\, 1p_{1/2} \rangle \,. \tag{32}$$

The term $r^2 Y_{20}$ in H_δ mixes states with different N and l, whereas the spin-orbit term $\vec{l}\cdot\vec{s}$ in H_l mixes states with different Λ and Σ with the condition $\Omega = \Lambda + \Sigma$. If the coupling between N and $N+2$ states is taken into account, the Nilsson coefficients also include higher major shell contributions.

Essentially, the effect of the nuclear deformation is to modify the spherical bound state proton wave function into a linear combination of other possible spherical wave functions with corresponding Nilsson coefficients. We have calculated the matrix element in Eq. (4) using the Nilsson wave functions for the bound proton, and the effect of the deformed structure of the nucleus on the coincidence cross section is studied in the PWIA.

V. RESULTS AND DISCUSSION

Detailed numerical calculations have been performed for the reaction $^{12}C(e,e'p)^{11}B$ for $s_{1/2}$ and $p_{3/2}$ states with incident electron energies of 635 and 500 MeV, respectively. The recoil momentum is varied by changing the outgoing proton angle and fixing the electron angle at 48.1° and 58° for the $s_{1/2}$ and $p_{3/2}$ states. The negative and positive values of $\vec{q}_R$ correspond to $\theta_p > \theta_q$ and $\theta_p < \theta_q$, respectively. The proton angle θ_p and the angle of momentum transfer θ_q are measured from the direction of the incident electron. The calculation is carried out in the perpendicular kinematics[19] for various choices of bound and scattering state potentials. The local potentials of Elton and Swift[9] (ES) and Giannini and Ricco[10] (GR) are used to calculate the bound state wave functions. The use of the GR potential is valid for both bound and scattering states and it takes care of the hole state effect and the final state interaction in a consistent way. One can introduce the effect of the decaying hole state[29] in this process by using the complex hole wave functions obtained with a suitable complex potential for the bound state. However, such calculations[30,31] do not produce any significant effect on the cross section. For the scattering state, the optical potentials of Glassgold and Kellogg[11] (GK) and Jackson and Abdul-Jalil[12] (JA) are used. Although the GK potential reproduces the elastic scattering data sufficiently, it does not contain any spin-orbit part in it. The use of the complex optical potential for the final state interaction is justified for an exclusive reaction in which only one final state of the target nucleus is specified, i.e., the one-hole state reached by DWIA.[32] Then the complex optical potential

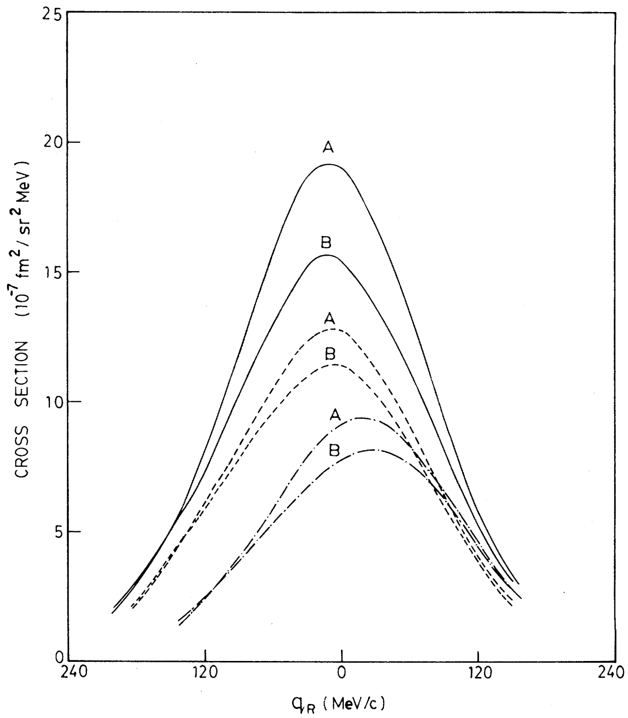

FIG. 2. Coincidence cross section versus the recoil momentum for the $s_{1/2}$ state. The continuous curve corresponds to the PWIA and the dashed and dotted-dashed curves correspond to DWIA calculations. The dashed curve is obtained using the approximation of Epp and Griffy and the dotted-dashed using the $\vec{\nabla}$ operator. Curves A and B are obtained with ES-GK and GR-GR potentials, respectively.

correctly takes into account the loss of flux from this final channel (the one-hole state) by further inelastic scattering of the outgoing struck nucleon. But the experimental cross section is inclusive since only the energy of excitation of the residual target is specified instead of a single final state. This may not affect the least-bound particles, namely, $1p$-shell particles in ^{12}C, but the DWIA knockout of the $1s$ particle will have to be treated as an inclusive $(e,e'p)$ reaction which might include the $1p$ hole followed by p-h excitation with the same kinematics.[33]

In Fig. 2 the coincidence cross section for the $s_{1/2}$ state is plotted as a function of the recoil momentum in PWIA and DWIA for the potentials ES-GK and GR-GR. As evident from the figure, the effect of the gradient operator $\vec{\nabla}$ is to decrease the cross section over a wide range of momentum values and to shift the peak towards the positive side of the recoil momentum. The calculation with the $\vec{\nabla}$ operator correctly takes into account the changes in proton momentum inside the nuclear medium, whereas in the approximation of Epp and Griffy[4] these are neglected and the proton operator is re-

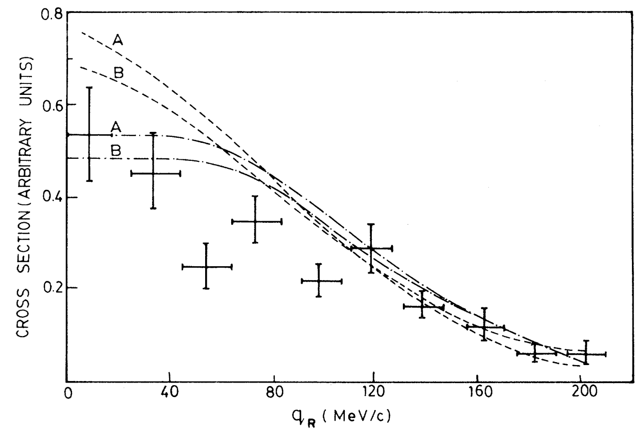

FIG. 3. Coincidence cross section for the positive side of the recoil momentum compared with experimental data of Ref. 34. The dashed curve is obtained using the approxiation of Epp and Griffy and the dotted-dashed using the $\vec{\nabla}$ operator. Curves A and B correspond to ES-GK and GR-GR potentials, respectively.

placed by the asymptotic momentum. The effect of the gradient operator on the cross section for both the positive and negative sides of $\vec{q}_R$ is shown in Figs. 3 and 4, respectively, and compared with the experimental data.[34] While the $\vec{\nabla}$ operator decreases the cross section throughout the momentum region on the negative side, the cross section shows an increase for higher values of $\vec{q}_R$ on the positive side for both the potentials. When compared with the negative side of $\vec{q}_R$, the results obtained with the operator for the positive side show a better agreement with the experimental data.[34] The same data are used for both the sides of $\vec{q}_R$ and the curve corresponding to the GK potential with the $\vec{\nabla}$ operator normalized to the experimental peak.

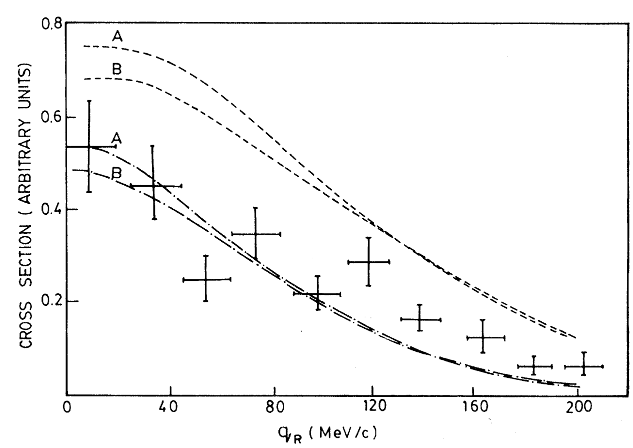

FIG. 4. Same as in Fig. 3 for the negative side of the recoil momentum.

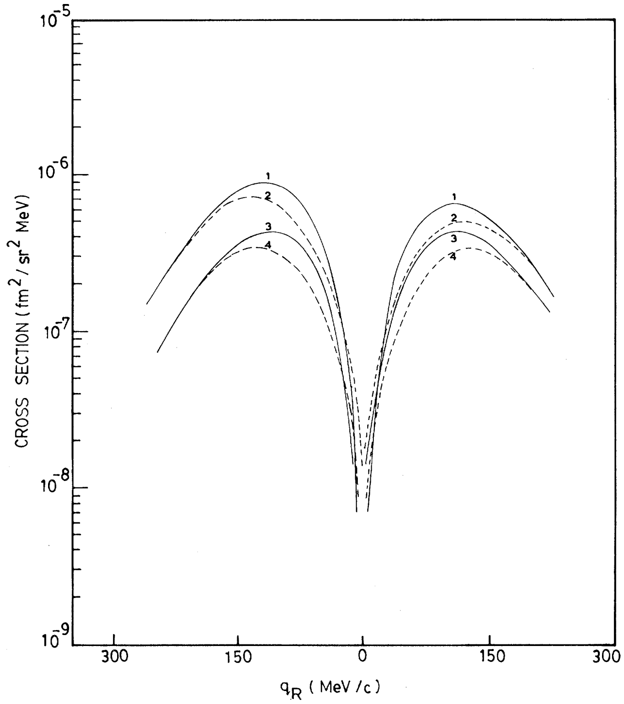

FIG. 5. Coincidence cross section for the $p_{3/2}$ state. The continuous and the dashed curves represent the PWIA and DWIA (ES-JA potentials) calculations, respectively. Curves 1 and 2 include all the terms in the matrix element while 3 and 4 include only the Coulomb term.

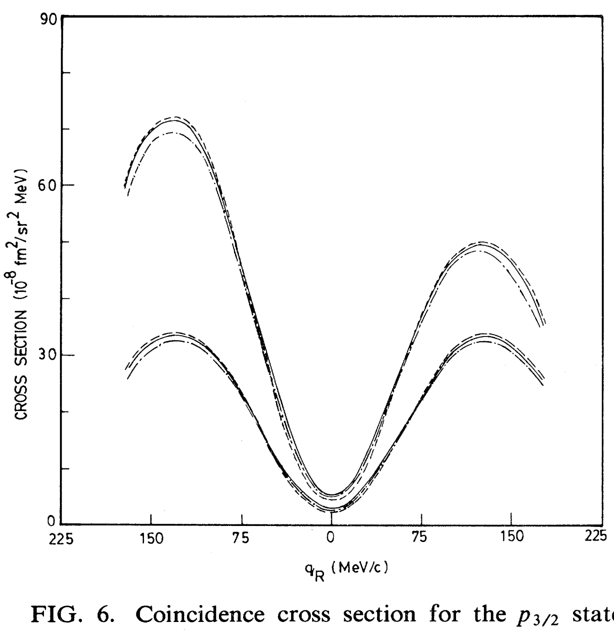

FIG. 6. Coincidence cross section for the $p_{3/2}$ state. The continuous, the dashed, and the dotted-dashed curves represent the DWIA calculations with ES-GK, ES-JA, and GR-GR potentials. The symmetric curves are obtained only with the Coulomb term while the asymmetric curves include all the terms in the matrix element.

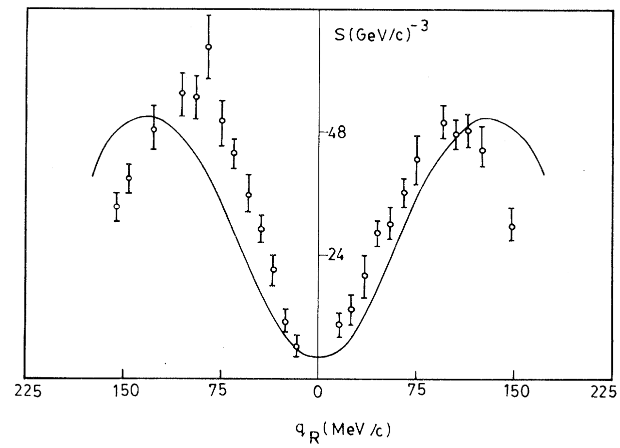

FIG. 7. Momentum distributions for the $p_{3/2}$ state obtained from the QQ^* term of the matrix element in the perpendicular kinematics. Experimental data are taken from Ref. 19.

In Figs. 5 and 6 the coincidence cross section for the $p_{3/2}$ state is plotted as a function of the recoil momentum in the perpendicular kinematics. The cross section shows a symmetric distribution on both sides when the predominant Coulomb term of the matrix element alone is used in the calculation. The inclusion of the other terms produces an asymmetry in the cross section which is exhibited in Fig. 5. The peaks of the DWIA curves are shifted towards the higher momentum values and this is a specific effect of the distortion. The influence of the choice of bound and scattering state potentials is shown in Fig. 6, wherein the cross sections for three different types of potentials ES-GK, GR-GR, and ES-JA are plotted. The effect of the imaginary component of the spin-orbit term ($W_s = -2.57$ MeV) in the energy dependent potentials of Jackson and Abdul-Jalil[12] is displayed in the figure. Besides the deviation at the peaks, the ES-JA potential gives a lower minimum when compared with other potentials.

We have also calculated the DWIA spectral function distribution using the QQ^* term in the matrix element. The results in Figs. 7 and 8 correspond to the calculations in perpendicular and parallel kinematics, respectively, and they are compared with the recent experimental data from the Saclay group.[19] It is evident from the figures that there is close agreement between our results and the experimental values over a wide range of the recoil momentum $\vec{q}_R$.

The effect of the nuclear deformation on the cross section is shown for the $p_{3/2}$ state in Figs. 9 and 10. The calculation has been done for the following input values: $E_i = 605$ MeV, $E_f = 475$ MeV,

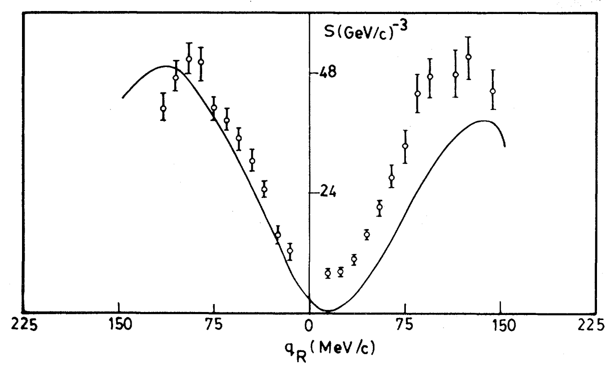

FIG. 8. Same as Fig. 7 but for the parallel kinematics.

$\theta_e = 50.2°$, and the oscillator parameter $b = 1.67$ fm. The bound state proton wave function is obtained from the Nilsson model for ^{12}C with oblate deformation parameter $\eta = -6$. The cross section has been calculated from the matrix element using the Nilsson wave function with and without the coupling between N and $N+2$ states. The magnitude of the cross section and the position of the peak are affected by the nuclear deformation, as evident from Fig. 9. The effect of the N and $N+2$ coupling on the cross section is displayed in Fig. 10. When com-

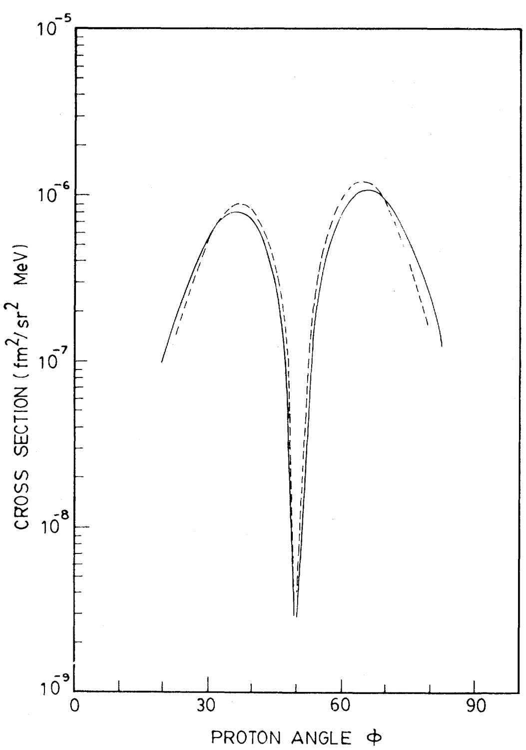

FIG. 9. Coincidence cross section versus the proton angle for the $p_{3/2}$ state. The continuous and the dashed curves correspond to the zero deformation and oblate deformation ($\eta = -6$), respectively.

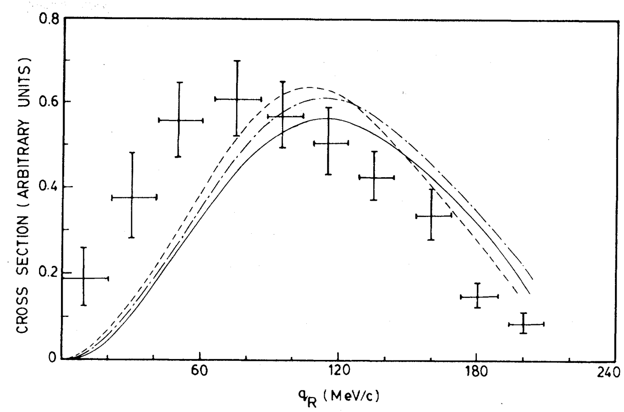

FIG. 10. Coincidence cross section for the $p_{3/2}$ state compared with experimental data of Ref. 34. The continuous curve corresponds to the zero deformation, the dashed curve to the oblate deformation ($\eta = -6$) with N and $N+2$ coupling, and the dotted-dashed curve to the oblate deformation without the coupling.

pared with zero deformation, the cross sections obtained with the coupling is greater in the low momentum region up to the peak and smaller in the high momentum region, whereas the cross section without the coupling is greater throughout the momentum region considered. It is found that there is better agreement between the results with $N+2$ coupling and the experimental data.[34] But the lateral shift of the theoretical curves from the experimental data still remains unexplained, although Radhakant[5] obtains a closer agreement with the experimental curve by changing the bound state wave function from the harmonic oscillator to that of Woods-Saxon with the ES parameters. However, we did not get this large lateral shift when we repeated the calculations with the same wave functions and our results are similar to those of Epp and Griffy.[4]

VI. CONCLUSION

The present study of the quasifree scattering process brings out the importance of the off-shell effects and the nuclear deformation on the coincidence cross section. The results establish that the gradient operator $\vec{\nabla}$ for the proton momentum reduces the cross section over a wide range, whereas in the case of photoproduction of pions, it has been found[35] that the $\vec{\nabla}$ operator for the pion momentum increases the photopion cross section. It has also been found that the effect of the gradient operator in the quasifree process is to shift the peak of the coincidence cross section towards the positive side of $\vec{q}_R$. Further, the cross section is considerably affected by the deformed bound state wave function and there is an improvement of the results towards the experimental data. The deformed intrinsic state for the ^{12}C ground state is consistent with many theoretical calculations.[21–26] As far as the phenomenological potentials are concerned, the effect of the different optical potentials used is found to be well pronounced at the peaks of the cross section. The imaginary component of the spin-orbit term in the optical potential of JA gives a lower minimum for the cross section at the center ($\vec{q}_R = 0$), besides the deviation at the peaks.

Hitherto the experimental study of the quasifree scattering of electrons has been mainly aimed at the confirmation of the shell model picture of the nucleus by estimating the separation energies and determining the momentum distribution of the protons from the various shells. Henceforth it is hoped that the study of the off-shell effects and other finer details in the quasifree scattering process will assume greater importance and the experimentalists will pay more attention to these aspects in the years to come.

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