Alpha particle photoeffect

I. Elminyawi and J. S. Levinger
Rensselaer Polytechnic Institute, Troy, New York 12181
(Received 30 September 1982)

Using sum rules we calculate four energy weighted moments for the ^{4}He electric dipole photoeffect for a Volkov spin-independent potential with Serber exchange. Our results are $\sigma_{-1}=2.83$ mb, $\sigma_0=119$ MeV mb, $\sigma_1=5430$ MeV^2 mb, and $\sigma_2=2.76\times10^5$ MeV^3 mb. We use only the first term in the hyperspherical expansions of the potential and wave function. We invert either two, three, or four of these moments to determine the photoeffect cross section. Results with different numbers of moments agree with each other to about 15%. We find reasonable agreement with measurements of the total photoeffect cross section.

[NUCLEAR REACTIONS Photodisintegration of ^{4}He, moments of photoeffect, hyperspherical harmonics.]

I. INTRODUCTION

Many workers have calculated the photoeffect cross section for ^{4}He, using ground state and continuum wave functions. Earlier work is reviewed by Gari and Hebach,[1] who also present their shell model calculation, for photon energies above 40 MeV. In Fig. 52 of their paper they show that their calculation agrees with experiment, within the experimental uncertainties, up to an energy of 140 MeV.

In a preliminary calculation, Levinger[2] used hyperspherical harmonics (h.h.) with grand orbital $L=0$ for the initial state and $L=1$ for the continuum final state to calculate the electric dipole photoeffect. The cross section has a peak value of 4.0 mb at 27 MeV. At high energy the cross section has a long tail but is only about $\frac{1}{3}$ of that calculated by Gari and Hebach.

In our model calculation[3] for the nuclear photoeffect we found the cross section near threshold $\sim E^{3/2}$ and asymptotically $\sim\exp[-(E/D)^{1/2}]$, where E is the energy above threshold. Motivated with that we introduced a new orthonormalized set $S_n(x)$ with weighting function $x^{1/2}\exp(-x^{1/2})$ to use them for inversion of the moments. We give $S_n(x)$ and the coefficients of inversion in the Appendix. We found the inverted cross section within 5% of the exact one and the agreement was better the more moments we used. We assumed that the photoeffect for few nucleon systems follows the same asymptotic behavior, and applied this technique to invert triton moments; we found[3] reasonable agreement with the experimental data.

In Sec. II we use the formalism introduced by Fabre and used by Levinger[2] for ^{4}He to calculate the ground state wave function along with the binding energy (we found -28.705 MeV). The potential used is Volkov spin independent with a fraction of Majorana exchange called x.

Then we use sum rules to calculate the energy-weighted moments σ_{-1} to σ_2 as functions of x. We emphasize the Serber exchange ($x=\frac{1}{2}$). Our results for σ_{-1} and σ_0 agree within 15% with other calculations.[4–6] We agree with experiments[7,8] within 25%. As far as we know σ_1 and σ_2 were not reported before. We use calculational methods similar to those used by Clare[9] and Lally,[10,11] for the triton.

In Sec. III we invert two moments, three moments, and four moments using "S inversion" to get the cross sections. Our inversions agree with each other and are in fair agreement with experimental data.[8,12] The two moment inversion agrees better with experiment than the others. Three and four moments cross sections have shifted peaks and go slightly negative at high energy. We discuss that and future work in Sec. IV.

II. CALCULATION OF MOMENTS

The Hamiltonian of the α particle in its initial (ground) state is

$$H=T+\bar{\bar{V}}\ , \tag{1}$$

where

$$T=-(\hbar^2/2M)(\nabla_{\xi_1}{}^2+\nabla_{\xi_2}{}^2+\nabla_{\xi_3}{}^2)\ . \tag{2}$$

Note that the factor $\frac{1}{2}$ is missing in the work of Levinger,[2] but this does not effect other calculations in that paper.[2] M is the nucleon mass. The Laplacian operators are for Jacobi coordinates $\vec{\xi}_1$, $\vec{\xi}_2$, and $\vec{\xi}_3$ used by Levinger.[2] The hyper-radius ξ is defined as

$$\xi^2=2(\xi_1{}^2+\xi_2{}^2+\xi_3{}^2) \ , \tag{3}$$

$$\bar{\bar{V}}=\sum_{i<j} V(r_{ij})(1-x+xP_{ij}) \ , \tag{4}$$

where $(1-x)$ is the fraction of Wigner exchange force, and x is the fraction of Majorana exchange.

For the Volkov potential, the lowest hypermultipole[2] is

$$V_0(\xi)=11410f(x_1)-6570f(x_2) \ , \tag{5}$$

where

$$f(x)=(1/4x^4-15/8x^6)\exp(-x^2) + \tfrac{1}{2}\pi^{1/2}(1/2x^3-3/2x^5+15/8x^7)\,\mathrm{erf}(x) \ , \tag{6}$$

and $x_1=\xi/0.82$ and $x_2=\xi/1.6$.

The hyperspherical wave function Ψ_L for a given grand orbital L is conveniently written in terms of the hyper-radial function $u_L(\xi)$ as follows:

$$\Psi_L(\vec{\xi})=\psi_L(\xi)H_{[L]}(\Omega) = u_L(\xi)\xi^{-4}H_{[L]}(\Omega) \ . \tag{7}$$

$\vec{\xi}$ is a vector in nine-dimensional space, with length ξ given by (3), and direction given by the eight angles Ω.

We have a completely spatially symmetric ground state i ($L=0$), i.e.,

$$P_{ij}\,|\,i\,\rangle=|\,i\,\rangle;\ \langle i\,|\,V(r_{ij})\,|\,i\,\rangle=(\tfrac{1}{6})\langle i\,|\,V_0\,|\,i\,\rangle \ . \tag{8}$$

Solving the hyperspherical Schrödinger equation we get $E=-28.705$ MeV and find $u_0(\xi)$ which peaks around $\xi=4$ fm and decreases exponentially for large ξ. We confirm Ballot's[13] calculation.

Then we proceed to calculate the moments using only the term with the grand orbital zero in the h.h. expansion of $|\,i\,\rangle$. We use the notation, for $f(\xi,\Omega)$,

$$\langle i\,|f\,|\,i\,\rangle=\int_0^\infty \Psi_0{}^2 f\xi^8 d\xi\, d^8\Omega \ .$$

After integrating over the eight angles, we obtain $g(\xi)$ with

$$\langle \psi_0\,|\,g\,|\,\psi_0\rangle=\int_0^\infty \psi_0{}^2 g\xi^8\, d\xi \ ,$$

$$\langle u_0\,|\,g\,|\,u_0\rangle=\int_0^\infty u_0{}^2 g\, d\xi \ , \tag{9}$$

$$\sigma_{-1}=(4\pi^2/\hbar c)\langle i\,|\,D^2\,|\,i\,\rangle \ ,$$

where D is the dipole operator $=e\xi_{1z}$. We integrate over angles and find

$$\langle i\,|\,D^2\,|\,i\,\rangle=\langle u_0\,|\,e^2\xi^2/18\,|\,u_0\rangle \ .$$

Thus

$$\sigma_{-1}=(2\pi^2\alpha/9)\langle u_0\,|\,\xi^2\,|\,u_0\rangle=2.83 \text{ mb} \ ,$$

which is in good agreement with many other calculations. For example, Goldhammer[4] found $\sigma_{-1}=2.73$ mb for a potential with a soft core and a tensor component. Experimentally σ_{-1} can be obtained from elastic scattering of electrons. Frosch[7] found that the r.m.s. radius of the proton distribution is 1.66 fm, giving $\sigma_{-1}=2.7$ mb.

The integrated cross section is

$$\begin{aligned}\sigma_0&=(2\pi^2/\hbar c)\langle i\,|\,[D,[H,D]]\,|\,i\,\rangle\\ &=(2\pi^2/\hbar c)\langle i\,|\,[D,[T,D]]\,|\,i\,\rangle\\ &\quad+(2\pi^2/\hbar c)\langle i\,|\,[D,[\bar{\bar{V}},D]]\,|\,i\,\rangle \ .\end{aligned} \tag{10}$$

We define

$$a=[T,D]=-(e\hbar^2/M)\partial/\partial\xi_{1z} \tag{11}$$

and

$$b=[\bar{\bar{V}},D]=-(eV_0x/6)(Z_{12}P_{12}+Z_{13}P_{13} -Z_{24}P_{24}-Z_{34}P_{34}) \ . \tag{12}$$

Here Z_{ij} is the z component of the relative coordinates between particles i and particle j, i.e., $Z_{ij}=z_i-z_j$.

The first term on the right hand side of Eq. (10) is given by the Thomas-Reiche-Kuhn (TRK) sum rule:

$$2\pi^2\alpha\hbar^2/M=59.7 \text{ MeV mb} \ .$$

In the second term of Eq. (10), and in all subsequent calculations, we got rid of the operator P_{ij} either by pushing it to the right or to the left through different quantities, according to convenience, until it disappeared as in Eq. (8). Also, by definition of the coordinates,

$$Z_{12}+Z_{13}-Z_{24}-Z_{34}=4\xi_{1z} \ . \tag{13}$$

Thus Eq. (10) gives

$$\sigma_0=59.7-(8\pi^2x/3\hbar c)e^2\langle i\,|\,V_0\xi_{1z}{}^2\,|\,i\,\rangle \ . \tag{14}$$

After angular integration in (14) we find

$$\sigma_0=59.7-(4x\pi^2\alpha/27)\langle u_0\,|\,V_0\xi^2\,|\,u_0\rangle \ . \tag{15}$$

Then

$$\sigma_0=59.7+119.4x \text{ MeV mb} \ .$$

For $x=\frac{1}{2}$, $\sigma_0=119.4$ MeV mb. This agrees well with Levinger's[2] value $\sigma_0=120$ MeV mb for a Volkov potential. Davey and Valk[5] calculated $\sigma_0=107$ MeV mb for a super soft-core tensor potential with the Serber mixture. Gari[6] got 127 MeV mb for σ_0 using the Reid potential. All three agree well with our present result. Gorbunov[8] measured

$$\int_0^{170}\sigma(E_\gamma)dE_\gamma$$

as 103 MeV mb.

$$\begin{aligned}\sigma_1 &= -(4\pi^2/\hbar c)\langle i\mid [H,D]^2\mid i\rangle\\ &=(-4\pi^2/\hbar c)\langle i\mid a^2\mid i\rangle\\ &\quad -(4\pi^2/\hbar c)\langle i\mid ab+ba\mid i\rangle\\ &\quad -(4\pi^2/\hbar c)\langle i\mid b^2\mid i\rangle\ .\end{aligned}\tag{16}$$

The first term on the right of (16) is

$$\begin{aligned}-(4\pi^2\alpha\hbar^4/M^2)\langle i\mid \partial^2/\partial\xi_{1z}{}^2\mid i\rangle\\ =(\tfrac{8}{9})(\pi^2\alpha\hbar^2/M)\langle i\mid H-V_0\mid i\rangle\\ =1289.\ \text{MeV}^2\ \text{mb}\ .\end{aligned}\tag{17}$$

The second term in (16) is evaluated as

$$\begin{aligned}-(4\pi^2/\hbar c)\langle i\mid ab+ba\mid i\rangle\\ =-(2\pi^2\alpha\hbar^2x/3M)\langle i\mid \partial/\partial\xi_{1z}(V_0\xi_{1z})\mid i\rangle\ .\end{aligned}\tag{18}$$

Here the operator $\partial/\partial\xi_{1z}$ works only on $V_0\xi_{1z}$. Then Eq. (18) becomes

$$\begin{aligned}&-(2\pi^2\alpha\hbar^2x/3M)\langle i\mid V_0+2\xi_{1z}{}^2V_0'/\xi\mid i\rangle\ ,\\ &-(2\pi^2\alpha\hbar^2x/3M)\{\langle u_0\mid V_0\mid u_0\rangle\\ &\qquad+(\tfrac{1}{9})\langle u_0\mid \xi V_0'\mid u_0\rangle\}\ .\end{aligned}\tag{19}$$

Here,

$$V_0'=dV_0/d\xi\ .\tag{20}$$

The second term in (16)$=5295.\ x$ MeV mb. The third term in (16) is

$$\begin{aligned}-(4\pi^2\alpha/9)x^2\langle i\mid -V_0{}^2(z_1-z_2-z_3+z_4)^2\mid\\ =(16\pi^2\alpha x^2/81)\langle i\mid V_0{}^2\xi^2\mid i\rangle\\ =5959x^2\ \text{MeV}^2\ \text{mb}\ .\end{aligned}\tag{21}$$

Thus,

$$\sigma_1=(1289+5295x+5959x^2)\ \text{MeV}^2\ \text{mb}\ ,\tag{22}$$

and this gives $\sigma_1=5427.$ MeV^{2}mb for the case $x=\frac{1}{2}$.

The next moment is

$$\sigma_2=(2\pi^2/\hbar c)\langle i\mid [[H,[H,D]],[H,D]]\mid i\rangle\ .\tag{23}$$

We use Eq. (12) for $[H,D]$. Since a commutes with T,

$$\sigma_2=(2\pi^2/\hbar c)\langle i\mid [([T,b]+[\bar{\bar{V}},a]+[\bar{\bar{V}},b]),(a+b)]\mid i\rangle\ ,\tag{24}$$

which could be reduced to

$$\sigma_2=(4\pi^2/\hbar c)\langle i\mid baV_0+V_0a^2-a\bar{\bar{V}}a-b\bar{\bar{V}}a+E_0b^2-bTb+V_0ab-a\bar{\bar{V}}b-b\bar{\bar{V}}b\mid i\rangle\ .\tag{25}$$

After considerable work we find

$$\begin{aligned}\sigma_2=(2\pi^2\alpha\hbar^2/M)\{&-(4\hbar^2/9M)\langle\psi_0\mid V_0'\mid\psi_0'\rangle+(8x/27)(E_0\langle u_0\mid V_0\mid u_0\rangle+\langle u_0\mid V_0{}^2\mid u_0\rangle\\ &+(\hbar^2/M)\langle\psi_0\mid V_0'\mid\psi_0'\rangle-\langle u_0\mid \xi V_0V_0'\mid u_0\rangle)\\ &+(8x^2/81)(-(\tfrac{1}{2})\langle u_0\mid V_0V_0''\xi^2\mid u_0\rangle-3\langle u_0\mid V_0V_0'\xi\mid u_0\rangle-\langle\psi_0\mid \xi^2V_0V_0'\mid\psi_0'\rangle\\ &-\langle\psi_0\mid V_0{}^2\xi\mid\psi_0'\rangle+9\langle\psi_0\mid V_0{}^2\mid\psi_0\rangle)\\ &-(8x^3/243)(M/\hbar^2)\langle u_0\mid V_0{}^3\xi^2\mid u_0\rangle\}\ .\end{aligned}\tag{26}$$

We integrate by parts terms with the ket $\mid\psi_0'\rangle=d\psi_0/d\xi$ to get

$$\begin{aligned}\sigma_2=(2\pi^2\alpha\hbar^2/M)\{&(\tfrac{2}{9})(\hbar^2/M)\langle\nabla^2V_0\rangle+(8x/27)(E_0\langle V_0\rangle+\langle V_0{}^2\rangle-\tfrac{1}{2}(\hbar^2/M)\langle\nabla^2V_0\rangle-\langle\xi V_0V_0'\rangle)\\ &+(8x^2/81)(\langle(V_0')^2\xi^2\rangle/2+3\langle V_0V_0'\xi\rangle+(\tfrac{27}{2})\langle V_0{}^2\rangle)-(8x^3/243)(M/\hbar^2)\langle V_0{}^3\xi^2\rangle\}\ ,\end{aligned}\tag{27}$$

where

$$\nabla^2 V_0 \equiv (\partial^2/\partial\xi^2 + (8/\xi)\partial/\partial\xi) V_0$$

and the expectation value

$$\langle F \rangle \equiv \int_0^\infty u_0{}^2 F(\xi) d\xi \ .$$

Using Simpson's rule to integrate from zero to 15 fm, the expectation values of the different quantities needed for the moments are the following:

$$\begin{aligned} \langle V_0 \rangle &= -77.3 \text{ MeV} \ , \\ \langle V_0{}^2 \rangle &= 6707 \text{ MeV}^2 \ , \\ \langle V_0'/\xi \rangle &= 5.49 \text{ MeV fm}^{-2} \ , \\ \langle V_0'' \rangle &= 10.4 \text{ MeV fm}^{-2} \ , \\ \langle \xi V_0 V_0' \rangle &= -7203 \text{ MeV}^2 \ , \\ \langle \xi^2 (V_0')^2 \rangle &= 12\,200 \text{ MeV}^2 \ , \\ \langle \xi^2 V_0{}^3 \rangle &= -7.03 \times 10^6 \text{ MeV}^3 \text{ fm}^2 \ . \end{aligned} \tag{28}$$

Then,

$$\sigma_2 = (2\pi^2 \alpha \hbar^2/M)(501 + 3131x + 7411x^2 + 5581x^3) \ . \tag{29}$$

For $x=\frac{1}{2}$, $\sigma_2 = 2.76 \times 10^5$ MeV^3 mb. It is obvious that our moments obey the inequality:

$$\sigma_2/\sigma_1 > \sigma_1/\sigma_0 > \sigma_0/\sigma_{-1} \quad \text{for } 0 < x < 1 \ . \tag{30}$$

(That inequality stems from the requirement of the cross section being always ≥ 0.)

III. INVERSION OF THE MOMENTS OF THE ALPHA PHOTOEFFECT

In Ref. 3 we express the quantity $\sigma(E_\gamma)/E_\gamma$ in terms of a truncated sum of orthonormalized polynomials $S_n(x)$ as follows:

$$\sigma(E_\gamma)/E_\gamma = \exp(-x^{1/2}) x^{1/2} \sum_{n=0}^{N} \lambda_n S_n(x) \ . \tag{31}$$

Here

$$x = (E_\gamma - B)/D = E/D \ , \tag{32}$$

with $B = 19.82$ MeV for *two-body* breakup. The functions $S_n(x)$ and equations to determine the coefficients λ_n are given in the Appendix. The parameter D is adjusted such that

$$\sum_{n=0}^{N} \lambda_n S_n(0) = 0 \ , \tag{33}$$

thus giving us $E^{3/2}$ dependence of σ near threshold. Our results are given in Table I.

The three cross sections found using the values

TABLE I. Energy parameter and coefficients. The parameter D and coefficients λ_n are used in Eqs. (31) and (32). Also see Eqs. (A2) and (33).

	D(MeV)	λ_0(mb)	λ_1(mb)	λ_2(mb)	λ_3(mb)
two moments	0.746	1.895	−2.321		
three moments	1.037	1.363	−0.890	−0.886	
four moments	1.269	1.114	−0.428	−0.581	−0.529

from the table are shown in Figs. 1 and 2. We also show experimental values compiled by Gibson[12] for $E_\gamma < 50$ MeV. For $E_\gamma > 50$ MeV we show Gorbunov's[8] measurements.

IV. DISCUSSION

We do not show Gibson's estimates of the experimental error in Fig. 1, for the following reason. There are large disagreements between different workers, far outside the quoted experimental errors, for the (γ,n) cross section, so we cannot apply

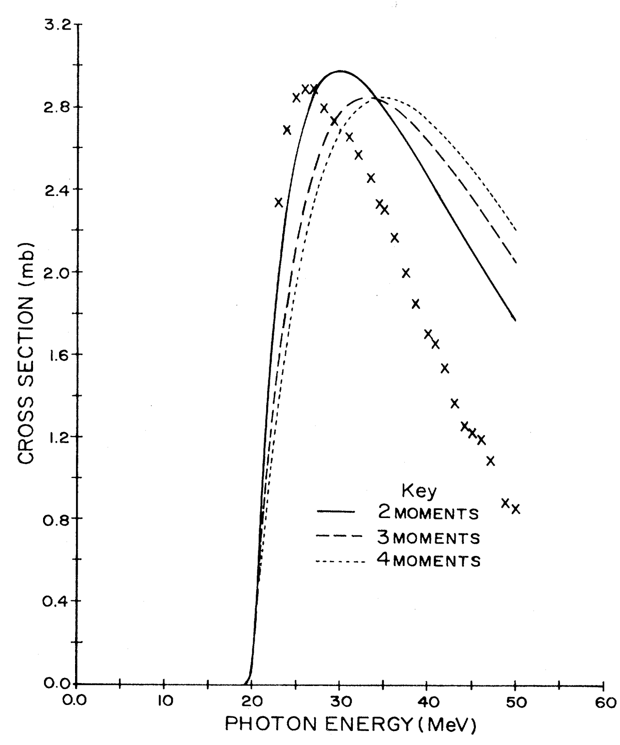

FIG. 1. Photoeffect cross section for alpha particle in mb versus photon energy in MeV. The curve marked "2" shows our result for S inversion using two moments σ_{-1} and σ_0; "3" shows S inversion with three moments, and "4" with four moments. The $\times$'s show Gibson's compilation of experimental results.

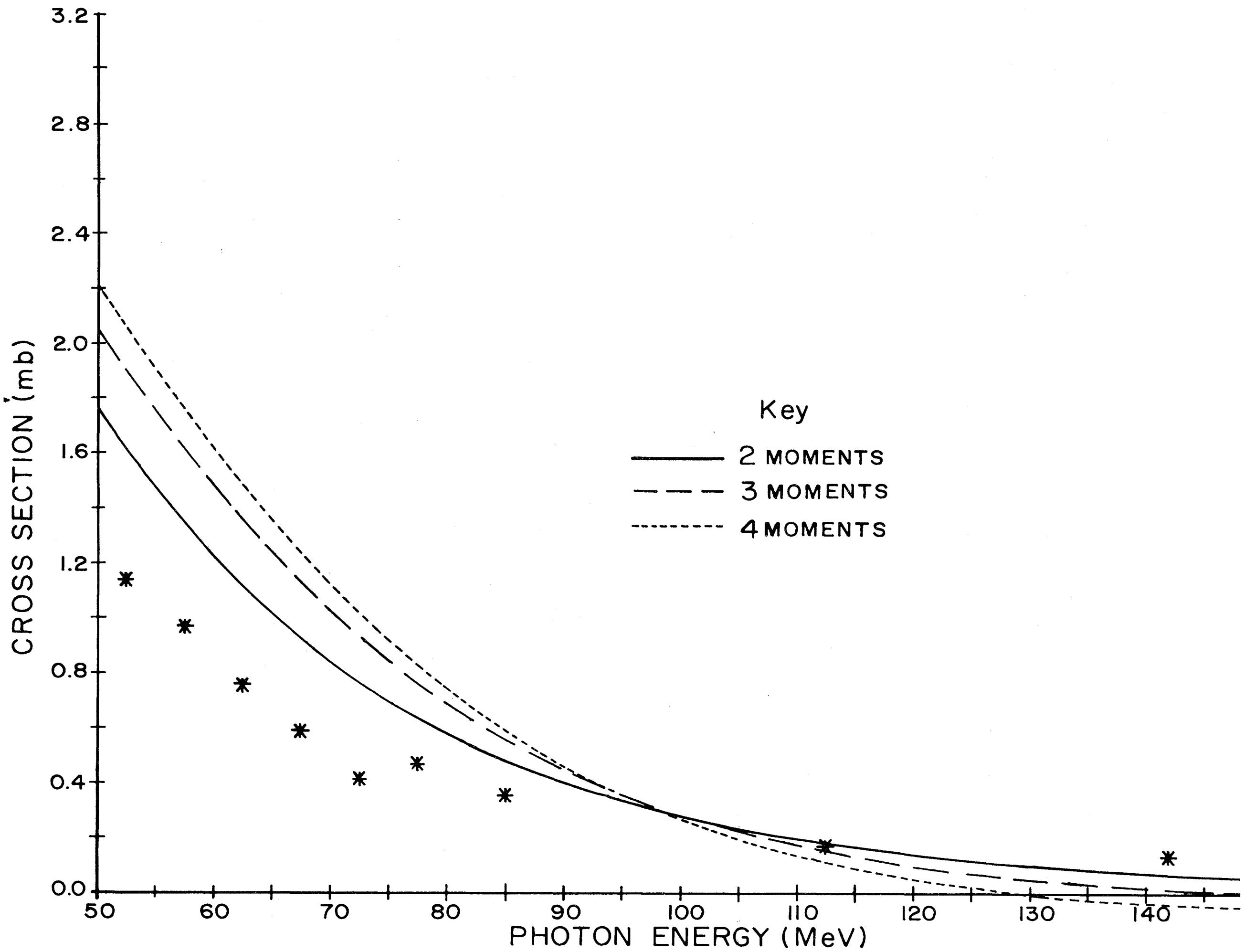

FIG. 2. Photoeffect cross section versus photon energy. The curves marked "2," "3," and "4" show our results for S inversion using two, three, and four moments, respectively. The asterisks show Gorbunov's measurements.

standard statistical methods to find the statistical error of the measurement. The disagreement between our inversions in Fig. 1 and Gibson's compilation is the same order of magnitude as his quoted "errors."

We also do not show Gorbunov's quoted statistical errors in our Fig. 2; they are 10 percent at 50 MeV, increasing to 50 percent at 140 MeV. Our disagreements with Gorbunov in Fig. 2 are somewhat larger than his quoted errors. Our two-moment inversion is in fair agreement with Gari's Fig. 52 for the calculated cross section from 40 to 140 MeV.

How well does S inversion converge as we increase the number of moments used? How accurate are our moment calculations in this paper for our assumed Volkov potential? How sensitive are the moments to the assumed potential? Is it significant in Fig. 1 that the more moments we use the further are our calculations from Gibson's compilation of experimental data?

An answer to one question depends on answers to others: e.g., the rate of convergence depends on the numerical values of the moments. Our *preliminary* answers are as follows. First, convergence appears satisfactory though not excellent. Table I shows that $|\lambda_n|$, the absolute value of the coefficients, tends to decrease with n. But the convergence is clearly less rapid than for our model,[3] where for four-moment S inversion, the coefficients $|\lambda_2|$ and $|\lambda_3|$ are each less than 10% of λ_0. The behavior of the coefficients is of course reflected in the convergence of the cross sections with N: Figs. 1 and 2 show a spread of 15% as we increase N from 2 to 4, for photon energies between 25 and 100 MeV, while we found a spread of less than 5% in our model.

In this paper we severely truncated the h.h. expansions of potential energy and of the ground state wave function. Calculations by Clare[9] and Lally[10,11] on moments for the triton showed that corresponding truncation led to errors of order 10%.

We must calculate moments with other potentials to determine the sensitivity of moments to the assumed potential. As noted above, our value of the integrated cross section agreed with that of other workers[5,6] within 10%; but the higher moments may be more sensitive to assumptions made concerning the potential.

In view of the inconclusive answers to the first three questions, we cannot assess the significance of the disagreement with Gibson's compilation. As noted above, the experimental error is not well known, and there are several approximations in our present results. In future work we should be able to give more definitive answers to these questions.

We thank Henry Valk for useful discussions of this problem.

APPENDIX

The S inversion[3] uses orthonormal polynomials $S_n(x)$,

$$\begin{aligned} S_0 &= \tfrac{1}{2} \ , \\ S_1 &= 6^{-1/2}(1-x/12) \ , \\ S_2 &= 0.359(1.0-0.141x+0.00192x^2) \ , \\ S_3 &= 0.327(1-0.186x+0.00456x^2 \\ &\quad -0.204x^3/10^4) \ . \end{aligned} \tag{A1}$$

The first four coefficients λ_n for S inversion are found from Eq. (31):

$$\begin{aligned} \lambda_0 &= \sigma_{-1}/(2D) \ , \\ \lambda_1 &= 6^{-1/2}(\sigma_{-1}+yf_1/12)/D \ , \\ \lambda_2 &= 0.359(\sigma_{-1}+0.141yf_1 \\ &\quad +0.00192y^2f_2)/D \ , \\ \lambda_3 &= 0.327(\sigma_{-1}+0.186yf_1+0.00456y^2f_2 \\ &\quad +0.204y^3f_3/10^4)/D \ . \end{aligned} \tag{A2}$$

The dimensionless number is

$$y = B/D \ ,$$

where B is the binding energy and D is an arbitrary parameter. The functions f_1, f_2, f_3, are found from the moments σ_{-1}, σ_0, σ_1, and σ_2 as follows:

$$\begin{aligned} f_1 &= \sigma_{-1}-\sigma_0/B \ , \\ f_2 &= \sigma_{-1}-2\sigma_0/B+\sigma_1/B^2 \ , \\ f_3 &= \sigma_{-1}-3\sigma_0/B+3\sigma_1/B^2-\sigma_2/B^3 \ . \end{aligned} \tag{A3}$$

[1]M. Gari and H. Hebach, Phys. Rep. 72, 1 (1981).

[2]J. S. Levinger, Phys. Rev. C 19, 2083 (1979).

[3]I. Elminyawi and J. S. Levinger, Nucl. Phys. A386, 165 (1982).

[4]P. Goldhammer and H. S. Valk, Phys. Rev. 127, 945 (1962).

[5]P. O. Davey and H. S. Valk, Phys. Rev. 156, 1039 (1967).

[6]M. Gari, J. Hebach, B. Sommer, and J. G. Zabolitsky, Phys. Rev. Lett. 41, 1288 (1978).

[7]P. O. Frosch, R. E. Rand, K. J. Van Costrum, and M. R. Yearian, Phys. Rev. 156, 1039 (1967).

[8]A. N. Gorbunov, in *Photo Nuclear and Photo Mesic Processes,* proceedings of the P. N. Lebedev Physics Institute, 1974, edited by D. V. Skobel'tsyn (Translation Consultant Bureau, New York, 1976), pp. 57—84; B. C. Berman, R. Bergere, and P. Carlos, Phys. Rev. C 26, 304 (1982).

[9]R. B. Clare and J. S. Levinger, Phys. Rev. C 23, 657 (1981).

[10]D. F. Lally and J. S. Levinger, Phys. Rev. C 25, 1061 (1982).

[11]D. F. Lally, M. S. thesis, Rensselaer Polytechnic Institute, 1981 (unpublished).

[12]B. F. Gibson, in *Proceedings of the International Conference on The Few-Body Problem, Eugene, Oregon, 1980,* edited by F. S. Levin (North-Holland, New York, 1981).

[13]J. L. Ballot, M. Beiner, and M. Fabre de la Ripelle, in *Proceedings of the International Symposium on the Present Status and Novel Developments in the Nuclear Many-Body Problem, Rome, 1972,* edited by V. Calgoro and C. Ciofi degli Atti (Editrico Compositori, Bologna, 1974), p. 565.