Meson exchanges in the harmonic-oscillator quark model

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Meson exchanges are introduced into the harmonic oscillator quark model along with a lower quark wave function. The mechanism is based on quark correlations that derive from quark exchange scattering via gluon exchange at short distances. This method allows avoiding spurious van der Waals forces at long range. Meson-baryon coupling constants and vertex form factors are calculated. They are found to be in agreement with those obtained from other quark models, phenomenological potentials, and the broken SU(3) flavor symmetry. The resulting S-wave NN phase shifts are compared with those from quark-molecular models.

[NUCLEAR REACTIONS Meson exchanges, $E_{\text{lab}} \leq 300$ MeV. Coupling constants, vertex form factors calculated from harmonic-oscillator quark interchange model. Comparison of NN S-wave phase shifts with those of potential models.]

I. INTRODUCTION

Quantum chromodynamics[1] (QCD) is now widely accepted as a non-Abelian gauge theory of the strong interactions. Through asymptotic freedom the effective quark-gluon coupling of color dynamics becomes weaker at short distances. This novel property is expected to make the nucleon-nucleon (NN) interaction accessible to a perturbative treatment at short range. At larger distances, when the nucleons no longer overlap much, the NN interaction still remains one of the most challenging problems in nuclear physics. Here the quark-gluon degrees of freedom are effectively frozen in by color confinement, and meson exchanges give a good parametrization of the low and medium energy phenomenology. But conventional meson-field theoretic models take the hadrons to be pointlike, whereas they are now known to have a finite size of the order of ~ 1 fm and to consist of valence quarks predominantly.

The MIT bag model[2] realizes color confinement by means of boundary conditions. This allows for the independent, shell-model-like, movement of relativistic valence quarks inside hadrons that is consistent with high-energy deep inelastic electron scattering. It has proved remarkably successful for the single-hadron spectroscopy in the low mass region. The color magnetic interaction, which at short range derives from gluon exchange and is treated perturbatively, provides the spin dependent two-body force.

An alternative, equally successful model involving nonrelativistic massive (constituent) quarks in a harmonic oscillator confinement potential with a two-body color hyperfine interaction has been developed by Isgur and Karl.[3] The validity of this constituent quark model (CQM) is limited at long range by unphysical van der Waals forces[4] and at short distances by the nonrelativistic wave functions of massive quarks. In order to circumvent these difficulties, we make explicit use of a lower quark wave function which already occurs implicitly in the (assumed Dirac) magnetic moments[3] and in the two-body pair current through electromagnetic gauge invariance.[5]

Futhermore, at long range we eliminate the spurious van der Waals forces, replacing them by meson exchanges (obtained from transforming the color singlet u-channel gluon exchange between quarks instead of the t-channel gluon exchange in second order perturbation theory). This treatment parallels our use of a quark interchange or pairing mechanism (QPM) to generate effective meson exchanges between nonoverlapping MIT bags.[6,7] Both additions should extend the validity of the CQM.

In the subsequent comparison of meson theoretic calculations we use an updated version of Ref. 8. This includes a πNN vertex form factor with a dipole regulator Λ_π different from that of all other exchanged mesons in order to reproduce the measured asymptotic D/S ratio of the deuteron ground state. This parameter is now known[9] to depend almost exclusively on the one-pion exchange potential (OPEP). Such a modification of Λ_π alone was suggested recently[10] and appears to be consistent with QCD ideas according to which the spontaneous breakdown of chiral symmetry, the transition of the "current" (or Lagrangian mass) into "constituent" quarks, and the formation of the pion as a Nambu-Goldstone boson may occur at shorter distances than the color confinement processes.[11]

II. CONSTITUENT QUARK MODEL

The (CQM) incorporates massive colored quarks in the confining potential

$$V=\tfrac{1}{2}K\sum_{i<j}(\vec{r}_i-\vec{r}_j)^2=\tfrac{3}{2}K(\rho^2+\lambda^2)\ , \tag{1}$$

where the harmonic oscillator constant K contains the color matrix element $\langle \lambda_i\cdot\lambda_j\rangle$ with the SU(3) color generators λ^a, $a=1,2,\ldots,8$. The relative quark coordinates in Eq. (1) are defined as

$$\vec{\rho}=\frac{1}{\sqrt{2}}(\vec{r}_1-\vec{r}_2),\quad \vec{\lambda}=\frac{1}{\sqrt{6}}(\vec{r}_1+\vec{r}_2-2\vec{r}_3)\ , \tag{2}$$

and the center of mass (c.m.) coordinate is

$$\vec{R}=\tfrac{1}{3}\sum_i\vec{r}_i\ . \tag{3}$$

Hence

$$\sum_i\vec{r}_i^{\,2}=\lambda^2+\rho^2+3R^2 \tag{4}$$

and

$$\vec{r}_1=\vec{R}+\frac{1}{\sqrt{2}}\vec{\rho}+\frac{1}{\sqrt{6}}\vec{\lambda}\ ,$$
$$\vec{r}_2=\vec{R}-\frac{1}{\sqrt{2}}\vec{\rho}+\frac{1}{\sqrt{6}}\vec{\lambda}\ , \tag{5}$$
$$\vec{r}_3=\vec{R}-\sqrt{2/3}\vec{\lambda}\ .$$

The nonrelativistic S-wave harmonic oscillator wave function is given by

$$g(\rho,\lambda)=\left[\frac{\alpha}{\pi}\right]^{3/2}\exp\left[-\frac{\alpha}{2}(\rho^2+\lambda^2)\right]\ , \tag{6}$$

where

$$\alpha^{1/2}=(3Km_q)^{1/4}\simeq 0.32\ \text{GeV}\ . \tag{7}$$

The one-body Hamiltonian is (with the units $\hbar=1=c$)

$$H_0=\frac{1}{2m_q}\vec{p}_\rho^{\,2}+\frac{1}{2m_q}\vec{p}_\lambda^{\,2}+\tfrac{3}{2}K(\rho^2+\lambda^2)\ . \tag{8}$$

The constituent quark mass

$$m_u=m_d=m_q\simeq 0.33\ \text{GeV}/c^2 \tag{9}$$

is expected to arise in the spontaneous breakdown of chiral symmetry. Using a Bardeen-Cooper-Schrieffer (BCS) model for the QCD vacuum it has been suggested[12] that a pairing instability develops for sufficiently strong quark-gluon coupling, $\alpha_s=g^2/4\pi\gtrsim\frac{9}{8}$, which leads to the formation of $q\bar{q}$ condensates. The small current (or Lagrangian) quark mass grows into the constituent quark mass as the quark moves through and interacts with this physical vacuum. Thereby a 0^- bound $q\bar{q}$ state forms with the properties of a Goldstone boson of which the pion is the realization in a perturbed state. Thus, except for the presence of valence quarks with their polarization of the vacuum (implying the presence of virtual mesons) there is no difference between the hadronic interior and the surrounding QCD vacuum in the CQM. This situation is in contrast to a widely assumed normal vacuum in the hadronic interior of an MIT bag, where pions, let alone vector mesons, may not exist, while quarks and gluons interact perturbatively.[13]

Although successful in the single-hadron spectroscopy, the CQM as a nonrelativistic quark model without the pion exchange is severely restricted at long and short distances. Moreover, in second-order perturbation theory, a two-body potential, e.g., the confining potential of Eq. (1) or the color hyperfine interaction,

$$\frac{2\alpha_s}{3m_q^2}\frac{8\pi}{3}\sum_{i>j}\tfrac{1}{4}\vec{\sigma}_i\cdot\vec{\sigma}_j\delta(\vec{r}_i-\vec{r}_j)+\text{tensor part}\ , \tag{10}$$

which contains the same color factor as Eq. (1), implies spurious van der Waals forces at long range.

Since the photon couples directly only to the quarks rather than the hadrons, gauge invariance requires the presence of the two-body pair current in hadrons.[5] With the Dirac notation for the quark wave function

$$\psi(\vec{r})=\begin{bmatrix} g(r) \\ -if(r)\vec{\sigma}\cdot\hat{r}\end{bmatrix}\chi,\quad f(r)=\frac{1}{2m_q}\frac{dg}{dr},\quad \hat{r}=\frac{\vec{r}}{r}\ , \tag{11}$$

Eq. (11) contains the lower radial quark wave function f that is implicit in (assumed Dirac) magnetic moments and the pair current. With $m_q\simeq\alpha^{1/2}$, the upper and lower radial wave functions are comparable,

$$-f(r_p)=\tfrac{1}{2}g(r_p)\ ,$$

at the proton mean square radius $r_p=\alpha^{-1/2}$. Thus including f may well extend the qualitative validity of the CQM to shorter distances.

Moreover, meson exchanges arise from correlated quarks at turning points to replace the perturbative van der Waals forces at longer range. For this to happen meson exchanges are required to have some inherently nonperturbative features. This is the case for the correlated quark reflections in nonoverlapping hadronic MIT bags.[6–8]

To illustrate how this comes about consider six valence quarks in Fig. 1(a) that are far from each other,

$$|\vec{r}_i-\vec{r}_j|\gg\Lambda_{\text{QCD}}^{-1}\ ,$$

except for one pair with the quantum labels c and d. These quarks c and d at the location $x\simeq y$ could exchange a gluon and thus interact via the S-wave δ piece of the color hyperfine interaction (10), were they not constrained by color confinement. As the two quark triplets move apart in Fig. 1(b), t-channel gluon exchange can no longer occur between quarks c and d because of color confinement. This eliminates the van der Waals mechanism. Instead, the color singlet u-channel gluon exchange in Fig. 1(c) that mediates quark exchange scattering at short range ($|\vec{x}-\vec{y}|\approx 0$) may be sustained at medium and longer distances if mesonic ($q\bar{q}$) modes dominate. The u-channel gluon exchange involves the color transitions $c_i\to d_f$ and $d_i\to c_f$. If it is expressed in terms of the direct t-channel transitions $c_i\to c_f$ and $d_i\to d_f$ as follows,

$$2\bar{\psi}_{d_f}(x)\gamma_\mu\tfrac{1}{2}\lambda_a\psi_{c_i}(x)\bar{\psi}_{c_f}(x)\gamma^\mu\tfrac{1}{2}\lambda_a\psi_{d_i}(x)=(\tfrac{2}{3})^2[\bar{\psi}_{c_f}(x)O_\alpha\psi_{c_i}(x)][\bar{\psi}_{d_f}(x)O_\alpha\psi_{d_i}(x)]+\text{color octet term}\ , \tag{12}$$

with ψ of Eq. (11) and Dirac's γ_μ of the quark-gluon coupling, and where summations over repeated indices are implied as usual, then the colors in the quark labels c_i and c_f (and similarly d_i and d_f) are the same but the flavors, helicities, or momenta are not necessarily. The relevant color singlet transition operators O_α defined by Eq. (12),

$$O_\alpha=(1,i\gamma_5,\frac{i}{\sqrt{2}}\gamma_\alpha,\frac{i}{\sqrt{2}}\gamma_5\gamma_\alpha)\times(1,\underline{\tau},U_\pm,V_\pm,1_s)\ , \tag{13}$$

are in one-to-one correspondence with the four meson nonets of spin-parity 0^+, 0^-, 1^-, and 1^+. As the quarks c at x and d at y move farther apart, each held within its quark triplet, they continue to interact directly, i.e.,

$$c_i\xrightarrow{O_\alpha}c_f$$

and

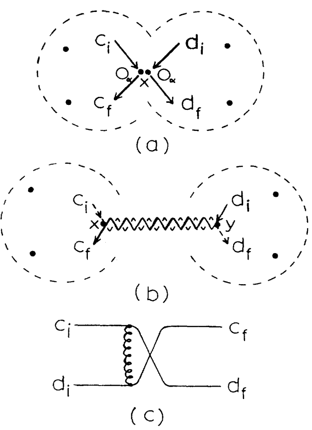

FIG. 1. Colliding nucleons with one quark of each triplet at the location $x \approx y$ representing (a) correlated quark reflections $c_i \rightarrow c_f$ and $d_i \rightarrow d_f$, (b) mediated by mesonic $q\bar{q}$ exchange when x is far from y, and (c) initiated by u-channel gluon exchange when x is close to y or $x = y$.

$$d_i \xrightarrow{O_\alpha} d_f$$

via effective quark interchange. The color octet term in Eq. (12) drops out. The gauge dependent term $\alpha q^\mu q^\nu / q^2$ of the current-current coupling does not enter in Eq. (12) because $q_0 = 0$ and

$$\int d^3x\, e^{i\vec{q}\cdot\vec{x}}\, \vec{\nabla}\cdot\bar{\psi}(x)\vec{\gamma}\psi(\vec{x}) \propto \vec{q}\cdot(\vec{\sigma}\times\hat{q}) = 0\ . \tag{14}$$

Through the transformation to the direct transitions the gluon degrees of freedom are frozen in. The color matrix element $(\frac{2}{3})^2$ becomes an overall factor. If the t-channel poles are saturated by (Yukawa) mesons with the physical mass spectrum m_α, this leads naturally to the effective one-boson exchange (OBE) quark interaction

$$V_Q = (\tfrac{2}{3}g)^2 \frac{N^2}{2} \sum_\alpha O_\alpha \langle T(\phi_\alpha \phi_\alpha^\dagger)\rangle_0 O_\alpha\ . \tag{15}$$

Thus the spin structure of the OBE interactions results from the vector nature of the exchanged gluon which then implies chiral invariance, while the mixed singlet-octet SU(3) flavor content of the OBE's comes from the flavor independence of the quark-gluon vertex in Eq. (12). The mesonic quark transitions,

$$\bar{\psi}_f(x) O_\alpha \psi_i(x) \phi_\alpha(x)\ ,$$

are taken to be local and the meson fields ϕ_α pointlike in the long wavelength limit (and mean field approximation). At this stage the OBE mechanism involves one *single*-quark matrix element in each of the colliding hadrons, which is consistent with the independent motion of the quarks despite the OBE's deeper origin in correlations with coherence length m_α^{-1} in each channel α. The close contact situation depicted in Fig. 1(a) corresponds to high momentum transfer $q = p' - p$ between the active quarks of the pair which, in the QPM, is that between the nucleons. If this happens in the hadronic surface regions, where it is more likely, the hadronic c.m. distance will remain fairly large so that the colliding hadrons are by no means highly overlapping. (Nonetheless, high momentum transfer q is often translated into short c.m. distances of extended composite nucleons using the uncertainty principle.) In fact, if such quark reflections are restricted to the hadronic surface of colliding MIT bags, then the pion couplings of chiral bags result from this quark pairing mechanism.[14,15] Thus the effective meson-quark vertices,

$$\tfrac{2}{3}g \frac{N}{\sqrt{2}} \int d^3r\, e^{i\vec{q}\cdot\vec{r}}\, \bar{\psi}(\vec{r}) O_\alpha \psi(\vec{r})\ , \tag{16}$$

that follow from Eq. (12) and correspond to the bag model formulas, contain an unrestricted spatial integration. The effective quark-gluon coupling constant $\alpha_s = g^2/4\pi \simeq 1.6$ at low q^2 is adjusted to the spin splitting in the hadron masses.[3] The normalization factor N is independent of the meson channel α. For MIT bags N^2 $(=2)$ occurs because the overlap of the perturbative bag interior and the QCD vacuum differs from 1. Here $N^2 \simeq 5.7$ follows from the value $g^2_{\pi \mathrm{NN}}/4\pi = 13.4$ (i.e., $N^2/N^2_{\mathrm{bag}} = 2.84$, $m_q/m_{q,\mathrm{bag}} = 0.33/0.108$).

When the lower quark component is included, translation invariance is violated. For a more satisfactory treatment of the c.m. motion, recoil and c.m. corrections are required. Since they are small at low q^2 compared with the quark-model dependence of coupling constants and vertex form factors, and are straightforward but tedious to calculate,[16] they will not be pursued further here. For nonrelativistic baryons in the initial and final states with plane c.m. motion, the c.m. part of the O_α-transition amplitude,

$$\int d^3R\, e^{i\vec{q}\cdot\vec{R}}\, e^{i(\vec{p}_1 - \vec{p}\,'_1)\cdot\vec{R}} = (2\pi)^3 \delta(\vec{q} - \vec{p}\,'_1 + \vec{p}_1)\ , \tag{17}$$

would simply give momentum conservation provided the lower quark components were neglected.

The corresponding matrix elements in the relative quark coordinates,

$$\Gamma_\alpha \equiv \langle O_\alpha \rangle = \int d^3\rho\, d^3\lambda\, \bar{\psi}(\rho,\lambda) O_\alpha \psi(\rho,\lambda) \times \exp\left[i\vec{q}\cdot\left[\frac{1}{\sqrt{2}}\vec{\rho} + \frac{1}{\sqrt{6}}\vec{\lambda}\right]\right]\ , \tag{18}$$

are calculated in the following.

The scalar effective meson-quark vertex with $O_\alpha = 1$ or $\underline{\tau}, \ldots$ in Eq. (18) takes the form [upon using $\vec{r} = \vec{r}_1$ of Eq. (5) and setting $\vec{R} = 0$ in Eq. (11)]

$$\begin{aligned}
\Gamma_s \equiv \langle 1 \rangle &= \left[1 + \frac{\alpha}{4m_q^2}\right]^{-1} \left[\frac{\alpha}{\pi}\right]^3 \int d^3\lambda \exp(i\vec{q}\cdot\vec{\lambda}/\sqrt{6} - \alpha\lambda^2) \\
&\quad \times \int d^3\rho \exp(i\vec{q}\cdot\vec{\rho}/\sqrt{2} - \alpha\rho^2) \left[1 - \frac{\alpha}{4m_q^2}\left[\tfrac{1}{2}\rho^2 + \tfrac{1}{6}\lambda^2 + \frac{1}{\sqrt{3}}\vec{\rho}\cdot\vec{\lambda}\right]\right] \\
&= \left[1 + \frac{\alpha}{4m_q^2}\right]^{-1} e^{-\vec{q}^{\,2}/6\alpha} \left[1 - \frac{\alpha^2}{4m_q^2}\left[\frac{1}{\alpha} - \frac{\vec{q}^{\,2}}{9\alpha^2}\right]\right]\ .
\end{aligned} \tag{19}$$

Applying the Wigner-Eckart theorem for symmetric S-wave functions, the spin-isospin matrix elements translate the quark one-body spin-isospin operators into those of nucleons according to the standard rules

$$\sum_{Q=1}^{3}\langle 1\rangle=3\,,\quad \sum_{Q}\langle\vec{\sigma}_Q\rangle=\vec{\sigma}_N\,,\quad \sum_{Q}\langle\vec{\sigma}_Q\underline{\tau}_Q\rangle=\tfrac{5}{3}\vec{\sigma}_N\underline{\tau}_N\,. \tag{20}$$

As a consequence, the coupling constants

$$g_{\epsilon NN}=3\tfrac{2}{3}g\frac{N}{\sqrt{2}}\left[1-\frac{\alpha}{4m_q^2}\right]\left[1+\frac{\alpha}{4m_q^2}\right]^{-1}, \tag{21}$$

$$g_{\delta NN}=\tfrac{2}{3}g\frac{N}{\sqrt{2}}\left[1-\frac{\alpha}{4m_q^2}\right]\left[1+\frac{\alpha}{4m_q^2}\right]^{-1}, \tag{22}$$

are obtained, setting $q^2=0$ in Eq. (19). Their numerical values are listed in Table I together with those of other quark confinement models for comparison. Since the S^*NN coupling vanishes, the scalar-meson-octet $F/(F+D)$ ratio $\alpha_s=1$ is found as in Ref. 7, i.e., pure F-type coupling which determines the remaining scalar-nonet couplings. The πNN vertex is obtained similarly from the single-quark transition operator $i\gamma_5\underline{\tau}$ and, more generally, the pseudoscalar meson nonet vertex. The integrals involved in Γ_5 can again be solved analytically. Hence

$$\Gamma_5\equiv\langle\gamma_5\rangle=\frac{5}{3}\frac{i\alpha}{m_q}\left[\frac{\alpha}{\pi}\right]^3\int d^3\lambda\exp(i\vec{q}\cdot\vec{\lambda}/\sqrt{6}-\alpha\lambda^2)\left[1+\frac{\alpha}{4m_q^2}\right]^{-1}\int d^3\rho\exp(i\vec{q}\cdot\vec{\rho}/\sqrt{2}-\alpha\rho^2)\vec{\sigma}_N\cdot\left[\frac{\vec{\rho}}{\sqrt{2}}+\frac{\vec{\lambda}}{\sqrt{6}}\right]$$
$$=\frac{5}{3}\frac{2m_N}{3m_q}e^{-\vec{q}^{\,2}/6\alpha}\bar{u}(p')i\gamma_5\underline{\tau}_N u(\vec{p})\left[1+\frac{\alpha}{4m_q^2}\right]^{-1} \tag{23}$$

to lowest order in p and p' using plane-wave nucleon spinors, and

$$g_{\pi NN}=\tfrac{5}{3}(\tfrac{2}{3}g)\frac{N}{\sqrt{2}}\frac{2m_N}{3m_q}\left[1+\frac{\alpha}{4m_q^2}\right]^{-1}, \tag{24}$$

$$g_{\eta NN}=\tfrac{2}{3}g\frac{N}{\sqrt{2}}\frac{2m_N}{3m_q}\left[1+\frac{\alpha}{4m_q^2}\right]^{-1}. \tag{24'}$$

For their numerical values we refer to Table I.

The vertex form factor $\exp(q^2/6\alpha)$ is common to all meson-baryon vertices. If this Gaussian is approximated by the dipole shape $[\Lambda^2/(\Lambda^2+\vec{q}^{\,2})]^2$ up to $q=1$ GeV/c, the cutoff $\Lambda\simeq0.89$ GeV/c is found which agrees with the value obtained with the MIT bag model by Bakker *et al.*[8]

As in Ref. 7, one obtains the $F/(F+D)$ ratio $\alpha_5=\frac{2}{5}$ from $g_{\eta'NN}=0$ and $g_{\pi NN}$, $g_{\eta NN}$ given in Eqs. (24) and (24').

For the vector mesons ω and ρ, the quark matrix elements of γ_0 and $\vec{\gamma}$ are evaluated first. They are given by the expressions

$$\langle\gamma_0(\underline{\tau})^T\rangle=\Gamma_{0T}(\underline{\tau}_N)^T\,,\quad \langle\vec{\gamma}(\underline{\tau})^T\rangle=\Gamma_T(\underline{\tau}_N)^T i\vec{\sigma}_N\times\hat{q} \tag{25}$$

for $T=0$ or 1, and

$$\Gamma_{0T}=\left[1+\frac{\alpha}{4m_q^2}\right]^{-1}\times3^{1-T}\left[1+\frac{\alpha}{4m_q^2}\left[1-\frac{\vec{q}^{\,2}}{9\alpha}\right]\right]e^{-\vec{q}^{\,2}/6\alpha}\,, \tag{26}$$

$$\Gamma_T=\left[1+\frac{\alpha}{4m_q^2}\right]^{-1}\frac{q}{3m_q}(\tfrac{5}{3})^T e^{-\vec{q}^{\,2}/6\alpha}\,. \tag{26'}$$

Hence the Sachs's form factors are

$$G_{E,T}=\Gamma_{0T},\quad G_{M,T}=\frac{2m_N}{q}\Gamma_T \tag{26''}$$

because, to lowest order in $\vec{p}$ and $\vec{p}\,'$, with $q^2=-\vec{q}^{\,2}$, and Eqs. (19)–(23) of Ref. 6, the vector-meson-nucleon coupling for $T=0,1$ takes the form

TABLE I. NN-meson coupling strengths $g^2(\vec{q}^{\,2}=0)/4\pi$ and coupling ratios f/g for the constituent quark model (CQM), the constituent quark model with meson coupling to quark 3 (CQM$_3$), the MIT bag model (Ref. 8), and a linear confinement potential (Ref. 8).

Meson (J^π,T)	Mass (GeV/c^2)	CQM	$g^2/4\pi(f/g)$ CQM$_3$	MIT bag	Linear confinement potential
$\epsilon(0^+,0)$	1.2	7.07	1.07	3.89	4.53
$\delta(0^+,1)$	0.96	0.79	0.12	0.44	0.50
$\eta(0^-,0)$	0.5485	4.82	4.82	4.87	5.32
$\pi(0^-,1)$	0.1385	13.4	13.4	13.4	14.8
$\omega(1^-,0)$	0.7823	9.22(−0.49)	6.4(−0.40)	6.0(−0.37)	3.78(−0.49)
$\rho(1^-,1)$	0.763	1.02(1.56)	0.7(2.04)	0.67(2.16)	0.42(3.18)
$D(1^+,0)$	1.285	0.57(−0.97)	0.2(−1.46)	0.57(−1.45)	0.22
$A_1(1^+,1)$	1.1	1.58(−0.97)	0.55(−1.46)	0.97(−1.45)	0.6

$$\Gamma^{\mu}_{T} \equiv \langle \gamma^{\mu}(\underline{\tau})^{T} \rangle = \left[1-\frac{q^2}{4m_N^2}\right]^{-1} \bar{u}(\vec{p}\,') \left[\left[\Gamma_{0T}+\frac{q}{2m_N}\Gamma_T\right]\gamma^{\mu} - \left[\Gamma_{0T}-\frac{2m_N}{q}\Gamma_T\right] i\sigma^{\mu\nu}q_{\nu}/2m_N \right] (\underline{\tau}_N)^{T} u(\vec{p}) \; . \tag{27}$$

Consequently, the ω and ρ vector (g) and tensor (f) coupling constants become

$$g_{\omega\mathrm{NN}} = \frac{3}{\sqrt{2}}\frac{2}{3}g\frac{N}{\sqrt{2}}, \quad g_{\rho\mathrm{NN}} = \frac{1}{\sqrt{2}}\frac{2}{3}g\frac{N}{\sqrt{2}} \; , \tag{28}$$

$$f_{\omega\mathrm{NN}}/g_{\omega\mathrm{NN}} = -1 + \frac{2m_N}{9m_q}\left[1+\frac{\alpha}{4m_q^2}\right]^{-1} ,$$

$$f_{\rho\mathrm{NN}}/g_{\rho\mathrm{NN}} = -1 + \frac{10m_N}{9m_q}\left[1+\frac{\alpha}{4m_q^2}\right]^{-1} . \tag{28$'$}$$

The numerical values are given in Table I.

It is worth emphasizing that these vector meson couplings are consistent with the vector meson dominance model and, as single-quark operators, they are also consistent with the SU(6) symmetry of the constituent quark model. Thus they give the standard value $-\frac{3}{2}$ for the proton-neutron magnetic moment ratio. And just as these magnetic moments are renormalized by pionic effects[13] (and recoil corrections[6,16]), the OBE is renormalized by the two-boson exchange (TBE) plus higher order iterations. As shown for the bag model in Ref. 8, the two-pion exchange significantly increases the tensor coupling of the ρ exchange and the low-mass strength of the scalar-isoscalar (effective σ_0) exchange. Both effects are well known from the dispersion theoretic treatment of the NN interaction.

The axial-vector single-quark transition matrix elements $\langle \gamma_5\gamma_\mu \rangle$ are calculated similarly,

$$\langle \gamma_5\gamma_0 \rangle = 0 \; ,$$

because

$$\bar{\psi}\gamma_5\gamma_0\psi = -(g, if\vec{\sigma}\cdot\hat{r})\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}g\\ -if\vec{\sigma}\cdot\hat{r}\end{bmatrix} = 0 \; ,$$

while

$$\bar{\psi}\gamma_5\vec{\gamma}\psi = -(g^2-\tfrac{1}{3}f^2)\vec{\sigma} - 2f^2(\hat{r}\vec{\sigma}\cdot\hat{r}-\tfrac{1}{3}\vec{\sigma}) \; .$$

Using plane-wave nucleon spinors, to lowest order in $\vec{p},\vec{p}\,'$,

$$\bar{u}(\vec{p}\,')\gamma_5\vec{\gamma}u(\vec{p}) = -\chi^{\dagger}\vec{\sigma}_N\chi \; ,$$

$$\bar{u}(\vec{p}\,')\gamma_5 u(\vec{p}) = -\chi^{\dagger}\frac{\vec{\sigma}_N\cdot\vec{q}}{2m_N}\chi \; ,$$

the axial-vector matrix element is obtained in the usual form

$$\langle \gamma_5\gamma_\mu(\tau)^T \rangle = \bar{u}(\vec{p}\,')(g_{A,T}\gamma_5\gamma_\mu\underline{\tau}_N^T + g_{P,T}\gamma_5 q_\mu \underline{\tau}_N^T/m_N)u(\vec{p}) \; , \tag{29}$$

where $q^2 = -\vec{q}^{\,2}$, and $T=0,1$. Hence

$$g_{A,T} = (\tfrac{5}{3})^T \left[1-\frac{\alpha}{12m_q^2}\right]\left[1+\frac{\alpha}{4m_q^2}\right]^{-1} ,$$

$$g_{A_1\mathrm{NN}} = \tfrac{1}{3}gNg_{A,1}, \quad g_{D\mathrm{NN}} = \tfrac{3}{5}g_{A_1\mathrm{NN}} \; ,$$

$$g_{P,0} = -\frac{m_N^2}{9m_q^2}\left[1+\frac{\alpha}{4m_q^2}\right]^{-1} , \quad g_{P,1} = \tfrac{5}{3}g_{P,0} \; . \tag{30}$$

Thus $g_{A,1} = g_A = 1.244$. The resulting A_1NN and DNN coupling constants are given in Table I. Overall there is a remarkable resemblance between the values of the meson-nucleon coupling constants given in Table I for the Isgur-Karl harmonic oscillator model and those of the bag model or a linear confinement potential. This is so despite the confinement potential being expressed in the relative quark coordinates in the harmonic oscillator model, compared to the distance between the quark and hadron center of the others.

III. DISCUSSION

At short distances where nucleons overlap, the core of the NN interaction is expected to be accessible to a perturbative treatment of gluon and quark exchanges. Such calculations will be abbreviated as quark molecular in the following. It is an open question to what extent, if at all, the heavier vector-meson exchanges, viz., $\rho,\omega,\ldots$, which in phenomenological meson field theories are crucial for the

TABLE II. $^1S_0$ and $^3S_1$ NN phase shifts, scattering lengths a, and effective ranges r from Arndt (Ref. 25), QPM (Ref. 8) (strong ρ tensor case with $\Lambda_\pi = 1.083$ GeV/c to reproduce the measured deuteron D/S ratio $\eta = 0.0274$, $g^2_{\sigma_0}/4\pi = 5.384$, $g^2_{\sigma_1}/4\pi = 0.097$), the constituent quark model (CQM with $m_\sigma = 0.495$ GeV/c^2, $g^2_{\sigma_0}/4\pi = 5.897$, $g^2_{\sigma_1}/4\pi = 0.179$), and a Bonn potential (EHM) (Ref. 26).

Partial wave	Ref.	50	$\rightarrow E_{\mathrm{lab}}$ (MeV) 100	200	300	a (fm)	r (fm)
$^1S_0$	Arndt	35°	23°	3°	−10°	−23.7	2.73
$^1S_0$	QPM	31°	16°	−2°	−12°	−23.7	3.21
$^1S_0$	CQM	35°	21°	6°	−3°	−22.8	3.08
$^1S_0$	Bonn	38°	26°	7°	−6°	−15.46	2.80
$^3S_1$	Arndt	53°	38°	15°	0°	5.42	1.75
$^3S_1$	QPM	55°	35°	14°	2°	5.56	2.03
$^3S_1$	CQM	57°	39°	21°	11°	5.90	1.97
$^3S_1$	Bonn	57°	38°	14°	−3°	5.50	1.86

TABLE III. $^1S_0$ and $^3S_1$ NN phase shifts from the bag model (QPM) and the constituent quark model (CQM) without the OPE + TPE, a quark-molecular CQM calculation (FFLS) (Ref. 19) and a two-center bag model (FF) (Ref. 21).

Partial wave	Ref.	$\rightarrow E_{\rm lab}$ (MeV) 50	100	200	300
$^1S_0$	QPM-π-σ	−29°	−40°	−52°	−59°
$^1S_0$	CQM-π-σ	−34°	−46°	−58°	−64°
$^1S_0$	FFLS	−10°	−17°	−25°	−30°
$^1S_0$	FF	+19°	+19°	+15°	+13°
$^3S_1$	QPM-π-σ	−14°	−21°	−31°	−40°
$^3S_1$	CQM-π-σ	−28°	−37°	−46°	−51°
$^3S_1$	FFLS	−10°	−13°	−20°	−23°
$^3S_1$	FF	+21°	+21°	+19°	+16°

short range repulsion, and spin-orbit splittings of the NN interaction, are included in such a "quark-molecular" description. The long range tails of the $\rho,\omega,\ldots$ exchanges, though, are essential for the correct values of low energy parameters such as scattering lengths, effective ranges, and the deuteron bound state properties, which are quite sensitive to small changes of these couplings.

Since nonoverlapping MIT bags do not exert any force on one another, two-center quark-molecular calculations with this model are not valid at medium to long ranges, unless the OPE and the TPE interactions are included. This leads to chiral bag models (CBM),[15] where the pointlike (Goldstone) pion continues the axial-vector current across the bag surface. The QPM contains the CBM as a special case where the correlated quark reflections that cause mesonic interactions are restricted to the hadronic surface.[14] In contrast to the CBM where only the pion is introduced for the continuity of the axial current, the QPM requires all four meson nonets for chiral invariance to hold.

In the harmonic oscillator model the color hyperfine interaction induces van der Waals forces which disagree with the data at long range. However, when the lower quark wave functions are included, and meson exchanges are invoked at longer range to replace perturbative contributions from QCD, quark-molecular calculations with the CQM may extend the NN force to shorter ranges.

Such calculations have been carried out by several groups in recent years. Liberman[17] was among the first to deal with the six-quark system using a harmonic oscillator confinement potential and a two-body quark interaction. These calculations were extended more recently.[18,19] In the framework of a two-center MIT bag model, DeTar[20] extracted a three-quark cluster deformation energy as a crude approximation for the NN potential at short range. These calculations were improved by including the [4,2] quark configuration in addition to the totally symmetric [6].[21]

There appears to be agreement that a repulsive core exists when equivalent local potentials are constructed, and the nearly linear energy dependence and the magnitude of the repulsion agree qualitatively with the short range parametrization of the Paris potential.[22] Such a comparison suggests that at short range the NN interaction is indeed of quark-molecular origin and that meson exchanges replace them at longer distances. This is anticipated in the QPM, and other hybrid models.[23] Others[24] believe that meson exchanges will be explained by quark-molecular mechanisms except for the pion exchange because, at short distances, the gluon exchange is attractive only in $(q\bar{q})0^-$ states. If valid, S-wave phase shifts (cf. Table II) should then become comparable to the quark-molecular ones when the OPE and TPE are deleted from them. This is not the case. The NN S-wave phase shifts from the resonating group method within the harmonic oscillator model are small and repulsive (cf. Table III). The comparison in Table III indicates that a good deal or all of the repulsion supplied by the vector and axial-vector mesons is needed in addition to the CQM quark-molecular contribution to reproduce the measured S-wave phase shifts.

Two-center bag model calculations[21] do not give similar results except for the soft repulsion at short range. These S-wave phase shifts are small and attractive (cf. Table III). Only if the OPE and a significant portion of the dominant and attractive TPE (effective σ_0 exchange) are added can these S-wave phase shifts be made to agree with the data at low energy. For laboratory energies above 100 MeV there is not enough repulsion at short range, even if all heavy meson exchanges are included.

The inconsistency of these quark-molecular results may be due to violations of local color-gauge invariance when gluons are treated only via effective quark interactions. This point was emphasized recently by Lipkin.[27] Before this problem is resolved it is difficult to extend meson exchanges to shorter distances.

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