Influence of the nuclear autocorrelation function on the positron production in heavy-ion collisions

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The influence of a nuclear reaction on atomic positron production in heavy-ion collisions is investigated. Using statistical concepts, we describe the nuclear S matrix for a heavy-ion induced reaction as a statistically fluctuating function of energy. The positron production rate is then dependent on the autocorrelation function of this S matrix, and on the ratio of the "direct" versus the "fluctuating" part of the nuclear cross section. Numerical calculations show that in this way, current experimental results on positron production in heavy-ion collisions can be reproduced in a semiquantitative fashion.

[NUCLEAR REACTIONS Positron emission, nuclear correlation width, heavy-ion collisions.]

I. INTRODUCTION

With the advent of heavy-ion accelerators, atomic ionization and rearrangement processes caused by the collision of two heavy atoms have become a focal point of experimental and theoretical investigations. Particular attention has been devoted to the production of positrons in such collisions. In most theoretical calculations, the motion of the two atomic nuclei was described in terms of a Coulomb trajectory. The possibility of a *nuclear* collision, and its influence upon the atomic excitation process, was considered in Ref. 1. It was argued that a nuclear collision causes a time delay τ and an associated phase difference between the amplitude for atomic excitation *prior* to the nuclear collision on the one hand, and the amplitude for atomic excitation *subsequent* to the nuclear collision, on the other. Numerical calculations[1] showed that this phase difference may markedly affect the spectrum of atomic positrons. Recent experimental studies of such heavy systems[2] as U + U have indicated that the atomic positron spectrum is indeed different from the one calculated under neglect of nuclear collision processes, and qualitatively similar to the one calculated in Ref. 1, although the value of the nuclear delay time

$$\tau=(3\cdots 10)\times 10^{-21}\ \text{sec}$$

used there seems unexpectedly large from the point of view of nuclear reaction theory. This situation calls for a more detailed investigation of the influence of nuclear reactions on atomic positron production, and the present paper addresses this question.

Our starting point is an expression for the total (atomic *and* nuclear) transition amplitude derived in the accompanying paper by one of us.[3] This expression is valid if the spatial regions in which atomic and nuclear excitation processes, respectively, take place are clearly separated, and if in the spatial domain of atomic excitation a semiclassical approximation for the motion of the atomic nuclei is justified. The first of these conditions is met if $R \gg R_N$ where R_N is the typical internuclear distance at which nuclear reaction processes set in, while R is the typical radius of the relevant atomic orbits. Because of the relativistic contraction of the innermost atomic orbits, the inequality $R \gg R_N$ is not really well fulfilled in the cases to which we apply the theory, and a generalization of our starting-point formula is called for. This is an open problem. A further approximation used in the derivation of the starting-point formula is the neglect of transfer of angular momentum from relative nuclear motion into nuclear excitation. This approximation is probably justified since, experimentally, all scattering events are counted in which the loss of kinetic energy of the two heavy ions does not exceed a few tens of MeV, and since at 10 or 20 MeV intrinsic excitation energy the relevant nuclear spin values are very small in comparison with typical angular momentum values of relative motion.

Our expression for the total transition amplitude contains as factors the elements of the nuclear scattering matrix. To evaluate further the resulting expression for the cross section, we use concepts of nuclear fluctuation theory and write the nuclear S matrix as a sum of two terms, an average matrix (averaged over energy) and a fluctuating part with a well-defined autocorrelation function. In this way we succeed in expressing the energy-averaged cross section in terms of the nuclear autocorrelation function and the associated mean width Γ_c. So far, there does not appear to exist a theoretical analysis that would yield an expression for Γ_c in the case of heavy-ion induced reactions. Therefore, Γ_c appears as a free parameter in our theory, comparable to the quantity $\hbar/\tau$ of Ref. 1.

In Sec. II, the arguments sketched above are presented in detail. Section III contains some simplifications, Sec. IV the atomic theory, Sec. V the results of our numerical calculations, and Sec. VI a brief summary.

II. THEORETICAL DEVELOPMENT

If the interaction between the two atomic nuclei is given by the Coulomb potential of two point charges, the probability per unit energy interval of observing a positron of energy E_- is given by

$$\frac{dP_{e^+}}{dE_-}=\sum_{E_i>E_F}|c_{l;E_i,E_-}(-\infty,+\infty)|^2 . \tag{2.1}$$

This formula was derived in Ref. 4 (hereafter referred to as I); the transition amplitude $c_{l;E_i,E_-}(t_i,t)$ is given as a solution of the coupled differential Eqs. (6.2) of I with the initial condition

$$c_{l;E_i,E}(t_i,t)=\delta_{E_i,E} \quad \text{at } t=t_i . \tag{2.2}$$

The index l, not used in I, denotes the orbital angular momentum of relative motion of the two nuclei and thus specifies the Coulomb trajectory as well as the distance of closest approach. The symbol $\sum$ implies both a summation over discrete states and an integration over continuum states throughout this paper, and the sum in Eq. (2.1) must be taken over all single-electron states which are initially unoccupied. A generalization of these formulae to the case of nuclear interactions was given in Ref. 3 (hereafter referred to as II). Under assumptions summarized in the Introduction, the total transition amplitude T from an initial atomic state E_i to a final state E_-, and from an initial nuclear state 0 to a final nuclear state N, induced by the collision of two nuclei with relative orbital angular momentum l and c.m. kinetic energy E, is given by [cf. Eqs. (3.13) in II]

$$T_{0N}(l;E;E_i,E_-)=\sum_{E_n}c_{l,0;E_i,E_n}(-\infty,0)S^l_{0N}(E-E_n)c_{l,N;E_n,E_-}(0,\infty) . \tag{2.3}$$

Here, $S^l_{0N}(E)$ is the nuclear S-matrix element, and the coefficients $c_{l,N;E,E'}$ are defined as before except that the additional index N takes account of the reduction of the kinetic energy of relative motion of the two nuclei due to excitation of the nuclear state N. The argument of the nuclear S matrix takes account of the loss of energy due to atomic excitation and should more correctly be written as $E-(E_n-E_i)$. We suppress the term E_i in the sequel in order to simplify the notation, and because it does not appear anyway in the final expressions. Similarly, k_{NE_-} and η_{NE_-} in Eqs. (2.4) below and thereafter should read $k_{N(E_--E_i)}$ and $\eta_{N(E_--E_i)}$, respectively. This is again suppressed.

The total scattering amplitude can then be written as [cf. Eq. (3.14) in II]

$$f_{NE_-}(\theta)=\frac{i}{2}(k_{00}k_{NE_-})^{-1/2}\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)\{\delta_{0N}\delta_{E_iE_-}-T_{0N}(l;E;E_i,E_-)\exp[i\sigma_l(\eta_{00})+i\sigma_l(\eta_{NE_-})]\} , \tag{2.4}$$

where $k_{00}(k_{NE_-})$ and $\eta_{00}(\eta_{NE_-})$ are the modulus of the wave vector of relative motion and the Sommerfeld parameter prior to (after) the scattering, respectively. The symbol θ denotes the nuclear scattering angle, and σ_l is the Coulomb phase shift for angular momentum l. The coincidence cross section for inelastic nuclear scattering into the angle θ and for production of positrons with energy E_- is given by

$$\frac{d^2\sigma_{0\to N}}{d\Omega\, dE_-}=\sum_{E_i>E_F}|f_{NE_-}(\theta)|^2 . \tag{2.5}$$

Collisions between heavy ions have been described in terms of statistical theories.[5] This description has the attractive feature that it leads automatically to transport equations for energy, angular momentum, and mass under inclusion of the diffusive terms. Transport equations of this type have proved to be very successful in the phenomenological analysis of the data. Although it is not certain that an approach along these lines is suitable for the transfer of the first 10 or 20 MeV from relative motion into intrinsic excitation (the problem with which we are here concerned), we adopt it as a working hypothesis and are thus naturally led to view each element of the nuclear S matrix as a stochastic process, i.e., as a randomly fluctuating function of energy. In keeping with such ideas we decompose the nuclear S matrix into an average part $\langle S^l_{0N}(E)\rangle$ and a fluctuating part $S^{l,fl}_{0N}(E)$, the latter being characterized by its autocorrelation function,

$$\langle S^{l,fl}_{0N}(E_1)S^{l',fl*}_{0N}(E_2)\rangle=\delta_{ll'}\langle S^{l,fl}_{0N}(E_1)S^{l,fl*}_{0N}(E_1)\rangle\frac{\Gamma_c}{\Gamma_c+i(E_2-E_1)} . \tag{2.6}$$

The form (2.6) is taken directly from standard compound-nuclear theory.[6] We emphasize that a statistical analysis of the nuclear S matrix in the frame of stochastic theories of heavy-ion reactions has not yet been given, and that Eq. (2.6) must be considered as being *ad hoc.* We obviously also have no estimate of the correlation width Γ_c. One might speculate that $\hbar/\Gamma_c$ is related to the diabatic-adiabatic relaxation time introduced recently.[7] In the present context, we consider Γ_c as a free parameter, and the entire stochastic approach embodied in Eq. (2.6) as a hypothesis of which we explore the consequences.

The decomposition of S into $\langle S\rangle$ and S^{fl} results in a similar decomposition of $f_{NE_-}(\theta)$,

$$f_{NE_-}(\theta)=\langle f_{NE_-}(\theta)\rangle+f^{fl}_{NE_-}(\theta) , \tag{2.7}$$

with

$$f^{fl}_{NE_-}(\theta)=-\frac{i}{2}(k_{00}k_{NE_-})^{-1/2}\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)\exp[i\sigma_l(\eta_{00})+i\sigma_l(\eta_{NE_-})] \times\sum_{E_n}c_{l,0;E_i,E_n}(-\infty,0)S^{l,fl}_{0N}(E-E_n)c_{l,N;E_n,E_-}(0,+\infty) \quad (2.8)$$

and

$$\langle f_{NE_-}(\theta)\rangle=\frac{i}{2}(k_{00}k_{NE_-})^{-1/2}\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)\{\delta_{0N}\delta_{E_iE_-}-\exp[i\sigma_l(\eta_{00})+i\sigma_l(\eta_{NE_-})] \times\langle S^{l}_{0N}(E)\rangle c_{l,0;E_i,E_-}(-\infty,+\infty)\}\ . \quad (2.9)$$

Equation (2.9) is obtained under the assumption that $\langle S\rangle$ does not vary appreciably over a range of roughly 1 MeV corresponding to atomic excitation energies, so that

$$\langle S(E-E_n)\rangle\cong\langle S(E)\rangle\ ,$$

and that

$$c_{l,N;E_n,E_-}(0,\infty)\cong C_{l,0;E_n,E_-}(0,\infty)\ ,$$

i.e., the atomic amplitudes depend only weakly on the total kinetic energy, for energy differences of a few tens of MeV.

Experimental conditions are such that any heavy-ion experiment always involves an average over an interval of incident energies. Taking this average in Eq. (2.5) and using Eq. (2.7), we obtain

$$\left\langle\frac{d^2\sigma_{0\to N}}{d\Omega\, dE_-}\right\rangle=\sum_{E_i>E_F}[\,|\langle f_{NE_-}(\theta)\rangle|^2+\langle\,|f^{fl}_{NE_-}(\theta)|^2\rangle]\ . \quad (2.10)$$

III. SIMPLIFYING ASSUMPTIONS

A complete evaluation of Eq. (2.10) obviously requires a detailed dynamical theory for the nuclear reaction. In this exploratory paper we wish to avoid such complications. To evaluate the average scattering amplitude of Eq. (2.9), we use the fact[5] that the collision of two heavy nuclei can often well be approximated in classical terms. We assume accordingly that, for given N, the l summation can be evaluated with the help of the usual semiclassical approximation,[8] and that for fixed N and given scattering angle θ only a single point of stationary phase $l_0(\theta,N)$ contributes to the saddle-point evaluation of the resulting integral over dl. We assume further that the l dependence of $c_{l,0;E,E'}(t,t')$ is smooth compared to that of the other factors, and accordingly replace $c_{l,0;E,E'}(t,t')$ everywhere by $c_{l_0;E,E'}(t,t')$ where at the same time we have simplified the notation. This assumption is physically plausible but deserves further investigation. Using these assumptions, introducing the average *nuclear* scattering amplitude $\langle f^n_N(\theta)\rangle$ (defined as in Eq. (2.9) but without any atomic excitation, i.e., for $c_{E_i,E_n}(t,t')=\delta_{E_i,E_n}$), and assuming that the scattering angle is different from zero, we find

$$\sum_{E_i>E_F}|\langle f_{NE_-}(\theta)\rangle|^2\cong|\langle f^n_N(\theta)\rangle|^2 \times\sum_{E_i>E_F}|c_{l_0;E_i,E_-}(-\infty,+\infty)|^2\ . \quad (3.1)$$

The evaluation of Eq. (2.8) is more complicated as $S^{l,fl}_{0N}$ fluctuates with energy, and probably also with angular momentum. In the classical approximation, such fluctuations give rise to a diffusionlike process.[5] If the relative contribution of the last term in Eq. (2.10) is small—and this is probably the case in the quasielastic regime—we expect to be able to approximate this diffusion process in terms of the mean trajectory. This implies that scattering into a given angle θ is again dominated by some fixed angular momentum $l'_0(\theta,N)$. We assume again that the l dependence of $c_{l,0;E_iE'}(t,t')$ near $l=l'_0$ is sufficiently smooth, and we use Eq. (2.6). Defining $f^{n,fl}_N(\theta)$ as the *nuclear* fluctuating scattering amplitude, i.e., as in Eq. (2.8) but without any atomic excitation, we find

$$\sum_{E_i>E_F}\langle\,|f^{fl}_{NE_-}(\theta)|^2\rangle\cong\langle\,|f^{n,fl}_N(\theta)|^2\rangle H(l'_0,E_-)\ , \quad (3.2)$$

where

$$H(l_0,E_-)=\sum_{E_i>E_F}\sum_{E_m,e_n}\frac{\Gamma_c}{\Gamma_c+i(E_m-E_n)}c_{l_0;E_i,E_m}(-\infty,0)c_{l_0;E_m,E_-}(0,+\infty)c^*_{l_0;E_i,E_n}(-\infty,0)c^*_{l_0;E_n,E_-}(0,+\infty)\ . \quad (3.3)$$

Experimentally, positron production is measured in coincidence with heavy ions scattered into a particular solid angle; the loss of kinetic energy (and, therefore, the intrinsic excitation energy) is limited to a few tens of MeV. To calculate the coincidence rate, we must sum Eq. (2.10) over all final states N within the experimentally admitted interval (this sum is denoted by $\sum'_N$), and divide by the singles count rate. In summing Eq. (2.10), we use Eqs. (3.1) and (3.2). We assume, moreover, that for the comparatively small excitation energies considered here, l_0 and l'_0 depend little on N and can be replaced by the angular momentum $l_c(\theta)$ of a Coulomb trajectory leading to the same scattering angle. The singles count rate is obtained by a similar summation, all atomic amplitudes being put unity. The probability per unit energy interval to observe a positron of energy E_- coincident with nuclear scattering into the angle θ is then given by

$$\frac{dP_{e^+}}{dE_-}=\alpha(\theta)\sum_{E_i>E_F}|c_{l_c;E_i,E_-}(-\infty,+\infty)|^2+[1-\alpha(\theta)]H(l_c,E_-)\ , \tag{3.4}$$

where $0\le\alpha(\theta)\le 1$ and

$$\alpha(\theta)=\sum_N{}'\,|\langle f_N^n(\theta)\rangle|^2\left[\sum_N{}'\,|\langle f_N^n(\theta)\rangle|^2+\sum_N{}'\langle|f_N^{nfl}(\theta)|^2\rangle\right]^{-1}. \tag{3.5}$$

Obviously, $\alpha(\theta)=1$ if $f^{nfl}=0$, in which case our expression (3.4) reduces to Eq. (2.1). The *novel* feature is the occurrence of the expression (3.3) in the coincidence rate (3.4). It is similar in form to terms discussed in Ref. 9. By virtue of the autocorrelation function

$$\Gamma_c/[\Gamma_c+i(E_m-E_n)]$$

the coherent summation over the energies E_m and E_n is restricted. For $\Gamma_c\to\infty$, the function $H(l_c,E_-)$ obviously becomes equal to the first sum on the right-hand side of Eq. (3.4), as it must, and then Eqs. (3.4) and (2.1) coincide. The function (3.3) has a similarity also with the time-delay phase factors introduced in Ref. 1. This is seen by writing

$$\frac{\Gamma_c}{\Gamma_c+i(E_m-E_n)}=\frac{\Gamma_c}{|\Gamma_c+i(E_m-E_n)|}e^{i\varphi}\ ,$$

$$\varphi=\arctan[(E_n-E_m)/\Gamma_c]\ . \tag{3.6}$$

For $\Gamma_c\gg|E_n-E_m|$, the phase factor is

$$\exp\{i(E_n-E_m)/\Gamma_c\}$$

which, with $\tau=\hbar/\Gamma_c$, is just the form used in Refs. 1.

The steps leading to Eq. (3.4) are clearly rough approximations to a more complete theory. We use them in order to obtain a first orientation over the kind of results which are to be expected in the present framework. In the terminology of compound-nucleus theory,[6] the quantity $\alpha(\theta)$ is the fraction of reaction processes corresponding to "direct" nuclear scattering. One might speculate that this is the fraction of processes in which the sudden (diabatic) nuclear potential is relevant, while $|1-\alpha(\theta)|$ is the other fraction, i.e., processes in which the nuclear potential changes during the collision, so that the process takes a circuitous path through the potential landscape before it

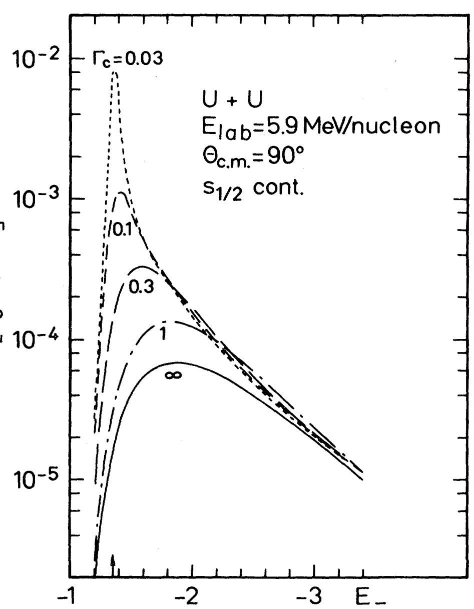

FIG. 1. The function $H[l_c(\theta),E_-]$ defined in Eq. (3.3) [in units of $(m_ec^2)^{-1}$, with m_e the mass of the electron] is plotted versus E_- (in units of m_ec^2) for the reaction U + U at 5.9 MeV/nucleon, a scattering angle $\theta_{c.m.}=90°$ and various values of Γ_c as indicated (in units m_ec^2) at each curve. Only the $s_{1/2}$ contribution (as defined in the text) is shown. The arrow indicates the position of the resonance $E_0=-1.36$ at the distance of closest approach, $R_{min}=21$ fm.

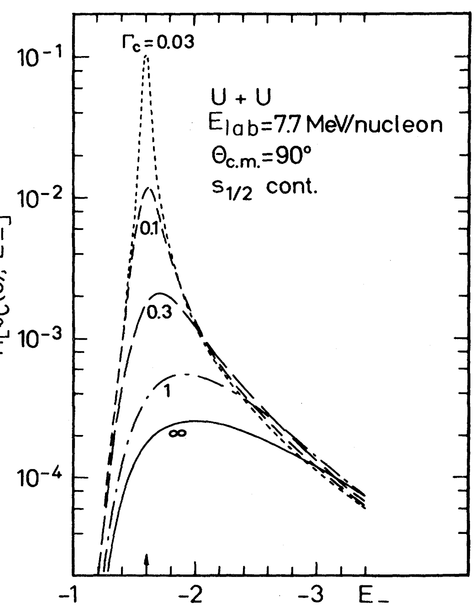

FIG. 2. The same as Fig. 1, for the same reaction, but at 7.7 MeV/nucleon, with $E_0=-1.60$ and $R_{min}=16.1$ fm.

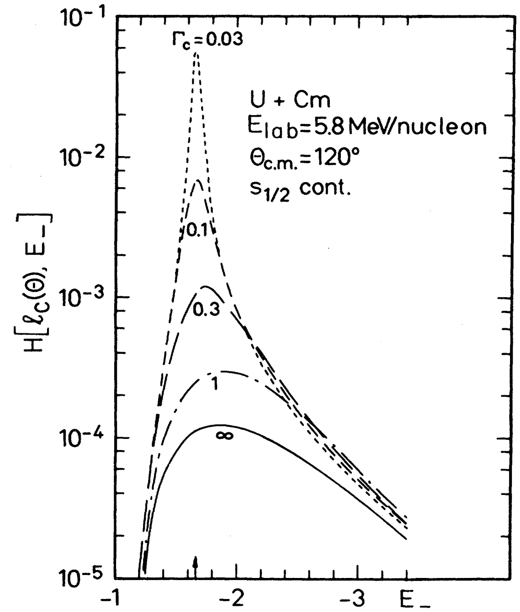

FIG. 3. The same as Fig. 1 for the reaction U + Cm at 5.8 MeV/nucleon and $\theta_{c.m.}=120°$, with $E_0=-1.66$ and $R_{min}=19.5$ fm.

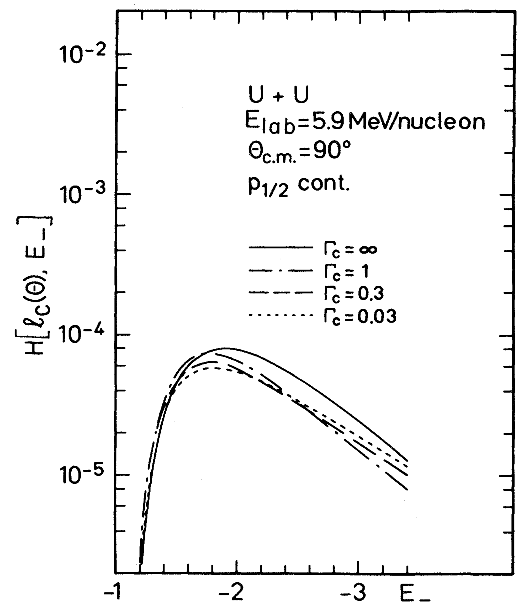

FIG. 4. The same as Fig. 1 but for the $p_{1/2}$ contribution.

returns to the elastic or near-elastic channels N. We make these comments here in order to demonstrate that the physical picture used above, although speculative, is not totally without possible significance. From a practical point of view, it is clear that our theory contains two parameters, $\alpha(\theta)$ and Γ_c. It is obvious that Γ_c may depend both on θ through l_c and on the experimentally admissible energy interval entering into the summation $\sum'_N$. It should be expected that Γ_c increases very rapidly with excitation energy. This suggests that an observation of effects associated with Γ_c is possible only for experiments which filter out scattering events with small energy losses, and that the results of such observations may depend sensitively upon the size of the experimental energy interval.

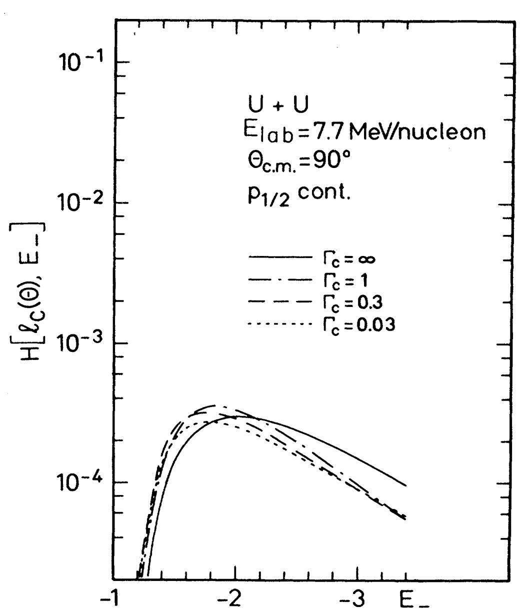

FIG. 5. The same as Fig. 2 but for the $p_{1/2}$ contribution.

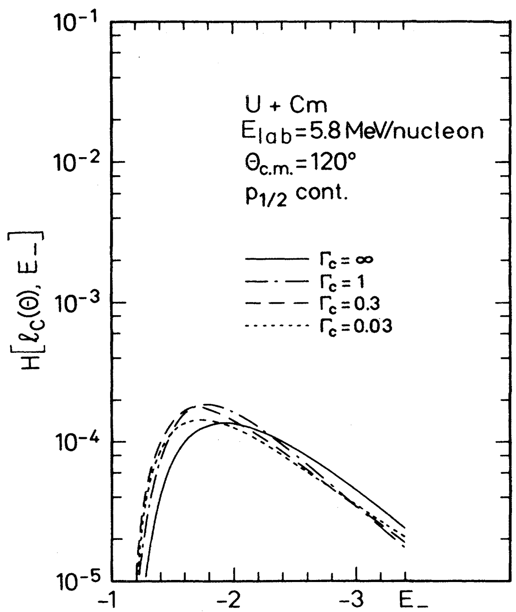

FIG. 6. The same as Fig. 3 but for the $p_{1/2}$ contribution.

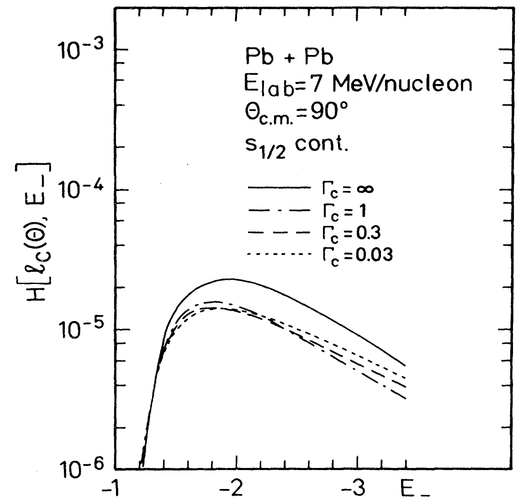

FIG. 7. The same as Fig. 1 for the reaction Pb + Pb at 7.0 MeV/nucleon and $\theta_{\rm c.m.}=90°$ with $R_{\min}=16.1$ fm.

IV. THE SUPERCRITICAL CASE

Given $\alpha(\theta)$ and Γ_c, the evaluation of Eq. (3.4) for undercritical atoms is straightforward. For supercritical atoms (with total charge so large that the $1s$ bound state "dives" into the positron continuum) we use the formalism developed in I. We have to pay attention to the fact that the modified basis introduced in I does not consist of eigenstates of the Hamiltonian at a fixed internuclear distance, whereas the energies E_m, E_n in Eq. (3.3) are by definition eigenenergies. We must therefore transform back from the modified basis to the unmodified one. We use the notation and definitions of I and consider by way of example the sum (we omit the index l_c)

$$\sum_{E_m} c_{E_i,E_m}(-\infty,0)c_{E_m,E_-}(0,\infty)\frac{\Gamma_c}{\Gamma_c+i(E_m-E_n)} \tag{4.1}$$

which we write in the form

$$\sum_{E_m>-1} c_{E_i,E_m}(-\infty,0)c_{E_m,E_-}(0,\infty)\frac{\Gamma_c}{\Gamma_c+i(E_m-E_n)}$$
$$+\sum_{E'_-}\Bigg[c_{E_i,E_0}(-\infty,0)c_{E_0,E_-}(0,\infty)|a_{E'_-}|^2+\sum_{E''_-,E'''_-}c_{E_i,E''_-}(-\infty,0)c_{E'''_-,E_-}(0,\infty)b_{E''_-E'_-}b^*_{E'''_-E'_-}$$
$$+\sum_{E''_-}c_{E_i,E''_-}(-\infty,0)c_{E_0,E_-}(0,\infty)b_{E''_-E'_-}a^*_{E'_-}+\sum_{E''_-}c_{E_i,E_0}(-\infty,0)c_{E''_-,E_-}(0,\infty)a_{E'_-}b^*_{E''_-E'_-}\Bigg]\frac{\Gamma_c}{\Gamma_c+i(E'_--E_n)}\,. \tag{4.2}$$

Here, the coefficients c_{E_i,E_0}, etc. are solutions of the system (6.2) in I while the coefficients a and b are given by

$$a_{E'_-}=\langle\varphi_{E'_-}|\varphi_r\rangle=\begin{cases}0, & \text{if } E'_-<E_c\,,\\ [\Gamma(E'_-,E_0)/2\pi]^{1/2}\left[E_0-E'_--\frac{i}{2}\Gamma(E'_-,E_0)\right]^{-1}, & \text{if } E_c<E'_-<-1\,,\end{cases} \tag{4.3}$$

$$b_{E_-,E'_-}=\langle\varphi_{E'_-}|\chi_{E_-}\rangle=\delta(E_--E'_-)+(E_--E'_--i\eta)^{-1}a_{E'_-}[\Gamma(E_-,E_0)/2\pi]^{1/2}\,.$$

They are the overlap matrix elements between the modified (φ_r,χ_{E_-}) and unmodified (φ_{E_-}) basis functions. As in Refs. 4 and 10, we neglect the unimportant coupling among the continuum positron states χ_{E_-} and end up with integrals an example of which is

$$\lim_{\Delta E\to 0}\int_{E_--\Delta E}^{E_-+\Delta E}dE'_-\int d\epsilon\int d\epsilon'|a_\epsilon|^2 b_{E_-,\epsilon}b^*_{E'_-,\epsilon'}\frac{\Gamma_c}{\Gamma_c+i(\epsilon-\epsilon')}\,.$$

Such integrals can be approximately evaluated analytically under the condition that $\Gamma(E_-,E_0)$ is a slowly varying function of E_- [cf. (4.4) and (5.8) in I]. We thus obtain

$$\lim_{\Delta E\to 0}\int_{E_--\Delta E}^{E_-+\Delta E}dE'_-\int_{E_--\Delta E}^{E_-+\Delta E}dE''_-\int d\epsilon\int d\epsilon' b_{E_-,\epsilon}b^*_{E'_-,\epsilon}b_{E_-,\epsilon'}b^*_{E''_-,\epsilon'}\frac{\Gamma_c}{\Gamma_c+i(\epsilon-\epsilon')}\cong 1\,, \tag{4.4a}$$

$$\lim_{\Delta E\to 0}\int_{E_--\Delta E}^{E_-+\Delta E}dE'_-\int d\epsilon\int d\epsilon'|a_\epsilon|^2 b_{E_-,\epsilon}b^*_{E'_-,\epsilon'}\frac{\Gamma_c}{\Gamma_c+i(\epsilon-\epsilon')}\cong\frac{\Gamma_c}{\Gamma_c+i[E_0-(i/2)\Gamma(E_0,E_0)-E_-]}\,, \tag{4.4b}$$

$$\int d\epsilon \int d\epsilon' |a_\epsilon|^2 |a_{\epsilon'}|^2 \frac{\Gamma_c}{\Gamma_c + i(\epsilon-\epsilon')} \cong \frac{\Gamma_c}{\Gamma_c+\Gamma(E_0,E_0)} , \tag{4.4c}$$

$$\lim_{\Delta E\to 0} \int_{E_- -\Delta E}^{E_- +\Delta E} dE'_- \int d\epsilon \int d\epsilon' b_{E_-\epsilon} a^*_\epsilon b_{E_-\epsilon'} b^*_{E'_-\epsilon'} \frac{\Gamma_c}{\Gamma_c+i(\epsilon-\epsilon')} \cong [\Gamma(E_-,E_0)/2\pi]^{1/2} \left[E_0 - E_- - \frac{i}{2}\Gamma(E_0,E_0) - i\Gamma_c \right]^{-1} , \tag{4.4d}$$

$$\int d\epsilon \int d\epsilon' b_{E_-\epsilon} a^*_\epsilon b_{E_-\epsilon'} a^*_{\epsilon'} \frac{\Gamma_c}{\Gamma_c+i(\epsilon-\epsilon')}$$

$$\cong [\Gamma(E_-,E_0)/2\pi][(E_- - E_0)^2 + \tfrac{1}{4}\Gamma^2(E_0,E_0)]^{-1} \left\{ \frac{\Gamma_c}{\Gamma_c+\Gamma(E_0,E_0)} + 1 - 2\,\mathrm{Re}\left[\frac{\Gamma_c}{\Gamma_c + \frac{1}{2}\Gamma(E_0,E_0) + i(E_0 - E_-)} \right] \right\} , \tag{4.4e}$$

$$\int d\epsilon \int d\epsilon' b_{E\epsilon} a^*_\epsilon |a_{\epsilon'}|^2 \frac{\Gamma_c}{\Gamma_c+i(\epsilon-\epsilon')} \cong [\Gamma(E_-,E_0)/2\pi]^{1/2} \frac{\Gamma_c}{\Gamma_c+\Gamma(E_0,E_0)} \left[E_- - E_0 - \frac{i}{2}\Gamma(E_0,E_0) - i\Gamma_c \right]^{-1} . \tag{4.4f}$$

The numerical calculation of the amplitudes c_{E,E_-} which appear in Eqs. (4.2) has shown[10] that the typical energy range over which these amplitudes change significantly is of order $m_e c^2$. This together with the analytical expressions given in Eqs. (3.3), (4.2), (4.4b), and (4.4d)–(4.4f) shows that a resonancelike peak with width $\lesssim \Gamma(E_0,E_0)+2\Gamma_c$ is expected to show up at the energy E_0 of the 1s resonance in the positron spectrum. It is caused by the fact that the only amplitude with a sharp energy dependence in the entire positron production process is the amplitude a_{E_-} for this resonance. The energy E_0 is the energy of this resonance at the point of nuclear contact or, in the present approximation in terms of Coulomb trajectories, at the distance of closest approach. Clearly, E_0 is a function of the nuclear scattering angle θ.

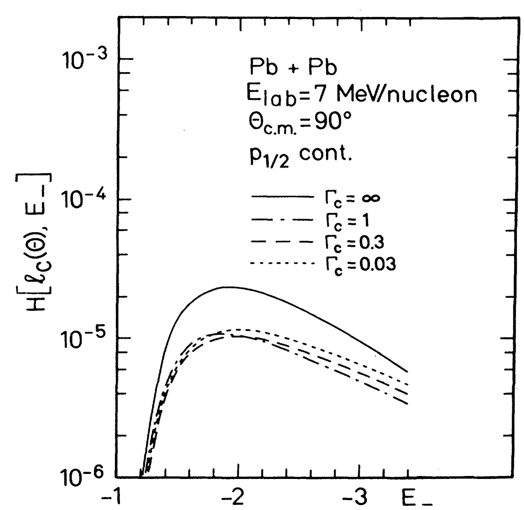

FIG. 8. The same as Fig. 1 but for the $p_{1/2}$ contribution.

V. NUMERICAL RESULTS

We have calculated numerically the function $H(l_c,E_-)$ defined in Eq. (3.3) for various systems and scattering angles, and several choices of Γ_c. We recall that for $\Gamma_c=\infty$, this function gives the positron spectrum in the absence of any nuclear reaction. The method of calculation of the amplitudes $c_{l;E,E'}(t,t')$ is described in Ref. 10. Since the monopole approximation for the electrostatic potential produced by two heavy ions is used, electronic states of different angular momenta become decoupled. The positron spectrum is therefore given as an incoherent sum of the contributions from different channels specified by the electronic angular momentum. Only the most important channels, $s_{1/2}$ and $p_{1/2}$, are considered in the present calculation, and the Fermi energy E_F is chosen such that all the levels up to the $3s_{1/2}$ or $4p_{1/2}$ state, respectively, are filled initially. Figures 1 to 3 display the results obtained for the systems indicated in the captions. In each case, we have evaluated only the $s_{1/2}$ contribution. The calculation yields the probability of positron production per unit energy interval and for a given scattering angle. Two trends in the figures are remarkable. First, with decreasing values of Γ_c, a sharp peak appears at the energy E_0 which gives the resonance energy at the distance of closest approach. This is to be expected in view of the analytical results given in Sec. IV. Second, with decreasing values of Γ_c, the function H strongly increases—we recall that the plot in Figs. 1–3 is a semilogarithmic one. This means that even if only a small fraction of all nuclear scattering events are affected by the nuclear autocorrelation function, i.e., if $|1-\alpha(\theta)| \ll 1$, a noticeable change of the positron spectrum is to be expected. Quantitatively, and for a fixed choice Γ_0 of Γ_c and of $\alpha(\theta)$, the spectrum is, according to Eq. (3.4), given by

$$\alpha(\theta)H(l_l,E_-)\,|_{\Gamma_c=\infty} + [1-\alpha(\theta)]H(l_c,E_-)\,|_{\Gamma_c=\Gamma_0} .$$

This formula and the graphs in Figs. 1–3 show that for $\Gamma_0 \approx 50$ to 100 keV, patterns of the type observed experimentally[2] can be produced. We emphasize that in the context of the semiclassical formula (2.3), the positron spectrum is expected to have only a *single* peak at or near $E_- = E_0$. This is the most distinct difference between our results and those of Ref. 1. We also remark that a fairly large value of $\Gamma_c \approx 500$ keV leads to an overall enhancement of the spectrum by roughly a factor of 2, which suggests that nuclear effects might also be identified by precise measurements of the absolute rate of positron production.

For the sake of comparison, we show in Figs. 4–6 the positron spectra calculated under the same kinematical conditions as in Figs. 1–3, respectively, but for the $p_{1/2}$ contribution. The strong peaks are absent, in contrast to the $s_{1/2}$ contribution, but the results are still weakly dependent on Γ_c. In Figs. 7 and 8 we show the $s_{1/2}$ and the $p_{1/2}$ contributions, respectively, for the undercritical case of the reaction Pb + Pb. These figures also show no peak, which confirms the statement that the peak is caused by the $s_{1/2}$ resonance in the positron continuum.

VI. SUMMARY

We have shown that the inclusion of a nuclear reaction, described in terms of a fluctuating nuclear S matrix with an autocorrelation function of the Ericson type, is capable of modifying considerably the form of the atomic positron spectrum observed in heavy-ion reactions. Our calculations are based on severe assumptions and drastic approximations. These deserve further exploration. It appears, however, that the observation of positrons produced in coincidence with quasielastically scattered nuclei may provide the long-missing tool for understanding better the mechanism of these nuclear reactions. The tool is provided by the sharp $s_{1/2}$ resonance in supercritical quasiatoms.

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