Analysis of low energy heavy ion multiparticle transfer reactions in terms of entrance and exit doorways

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A projection operator method is developed for the analysis of low energy heavy ion multiparticle transfer reactions. Entrance doorways introduce structure which is transferred sequentially via reaction (or exit) doorways to the reaction channel of interest. The theory treats both gross and intermediate structure, and the coupling from entrance to exit doorways introduces additional spreading and rather narrow structure in the reaction cross section. Consistent with experiment, energy shifts of reaction channel resonances relative to each other and to the elastic are present. The theory is suggested as a means of studying, for example, $^{12}C+^{12}C\rightarrow^{20}Ne^{*}+\alpha$ in the range of 10—30 MeV (c.m.) where the 0_2^+ state of $^{12}C^{*}$ would play an important role.

[NUCLEAR REACTIONS Light ions, projection operator intermediate structure theory, entrance and reaction doorways, multiparticle transfer.]

I. INTRODUCTION

Considerable experimental evidence has accumulated for the occurrence of intermediate structure (IS) in low energy heavy ion reactions. This IS has been manifested by the apparent occurrence in several cases of nuclear molecular formations. Such simple patterns of nuclear motion at relatively high excitation energies in certain nuclear systems have not only stimulated additional experimental efforts, but have offered theorists a chance to examine the nuclear dynamics that leads to identifiable structure in an otherwise complex array of data.

The collisions found to be most amenable to the excitation of clear molecular states are $^{12}C+^{12}C$ and $^{12}C+^{16}O$. While many reaction products are possible and have been investigated experimentally, most theoretical analyses have centered on the elastic and inelastic channels.[1–4] The theoretical emphasis on these exit channels is understandable because of their relative simplicity in comparison to other reactions. The few attempts at calculating cross sections other than elastic and inelastic have usually been of the direct reaction type. An exception is the work of Adhikari[5] who studied exit doorway states in nuclear reaction. While bearing some similarity to our work, there are striking differences between our approach and interpretation and that of Ref. 5. These will be referred to later in this paper. The major impetus for our work was the strong need for a theoretical microscopic description of more complex reactions that not only reproduces the broad resonances but also yields the underlying observed IS.

The projection operator method based on the Feshbach doorway approach[6] has been successful in explaining $^{12}C+^{12}C$ and $^{12}C+^{16}O$ elastic scattering data[1] as well as $^{12}C+^{12}C$ inelastic scattering to the 2^+, 3^-, and 0_2^+ excited states of ^{12}C.[2,3] In this paper we extend this work to the study of cross sections for more complicated reactions by employing a model that uses hallway states (one step more complex than the entrance doorway) as reaction or exit doorways to the reaction channel of interest. Cosman *et al.*[7] have indicated that the resonances associated with a particular entrance doorway may be correlated from channel to channel and to the elastic channel. In our model the coupling from entrance to exit doorways introduces additional spreading and structure in the reaction cross section. In addition, energy shifts of reaction channel resonances relative to each other and to the elastic channel are present. This would be consistent with experimental observations[7] for $^{12}C+^{12}C$. Because of the additional requirement of passing through exit doorways, rather narrow resonances can be obtained. The additional structure coming from the spreading from exit doorways to even more complicated states will be averaged over.

In Sec. II the general theory and procedure are developed in a model independent manner. The work of Wang and Shakin[8,9] on secondary doorways serves as a valuable impetus for our efforts. A shape elastic entrance doorway subspace is introduced along with an exit doorway subspace in order to account for the gross and intermediate structure. We suggest that the specific experimental measurements of $^{12}C+^{12}C\rightarrow^{20}Ne^{*}+\alpha$ in the energy range 10—30 MeV (c.m.) (e.g., Greenwood *et al.*[10] would be an interesting test case). Special emphasis would be placed on the 7.68 MeV 0_2^+ level in ^{12}C and exit doorways would be taken as loosely bound states of the $(^{8}Be+\alpha)$ system resulting from the alpha decay of the aforementioned 0_2^+ state. Detailed numerical calculations are in progress and will be published at a later date.

II. THEORY

As in our earlier work on elastic[1] and inelastic[2,3] scattering we divide the Hilbert space into two orthogonal subspaces $\{P\}$ and $\{Q\}$ representing, respectively, the shape elastic continuum space of the reduced mass and the rest of the Hilbert space. The projection operators P and Q satisfy

$$P+Q=1\ . \tag{1}$$

The diagram of Fig. 1 illustrates this decomposition. The

FIG. 1. Total Hilbert space subdivision into single continuum state (P) and more complicated space (Q).

subspaces can be further subdivided by introducing the orthogonal operators p, R, d, and q so that

$$p+R=P \tag{2a}$$

and

$$d+q=Q\ . \tag{2b}$$

The subdivision into p and R follows the procedure of Lev and Beres,[11–13] where R produces the subspace of shape elastic continuum resonances of the reduced mass projectile and p produces the orthogonal nonresonance subspace. The operator d produces the subspace that contains the doorways, i.e., states which involve a nuclear interaction in addition to an optical potential and are one step more complex than $\{R\}$. The coupling between P and d must be calculable within the context of whatever model is chosen to represent the nuclei. The operator q produces the states more complicated than d. In addition, the doorway space $\{d\}$ is further subdivided into an entrance doorway space $\{d_1\}$ and a reaction or exit doorway space $\{d_2\}$, i.e.,

$$d=d_1+d_2\ , \tag{2c}$$

where d_1 and d_2 are the respective operators. The breakups equations (2a)—(2c) are illustrated in Fig. 2.

Letting $\mathscr{H}$ be the total Hamiltonian and $\psi^{(+)}$ the scatttering wave function, the Schrödinger equation is

$$(E-\mathscr{H})\psi^{(+)}=0\ . \tag{3}$$

Making the usual doorway assumption that $\mathscr{H}_{pq}$ is zero, and hence does not contribute to our calculation, we introduce the effective Hamiltonian H via

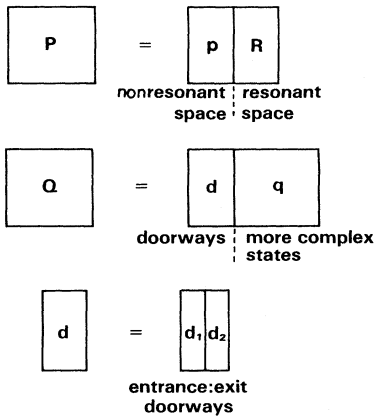

FIG. 2. Subdivision of the single continuum space $\{P\}$, more complicated space $\{Q\}$, and doorway space $\{d\}$.

$$H=\mathscr{H}_{(P+d)(P+d)}+\mathscr{H}_{dq}\frac{1}{E-H_{qq}}\mathscr{H}_{qd}\ . \tag{4}$$

We assume that within our doorway formalism,

$$H_{pd_1}=H_{pd_2}=H_{pq}=0\ , \tag{5a}$$

$$H_{Rd_2}=H_{Rq}=0\ , \tag{5b}$$

and

$$H_{d_1q}=0\ . \tag{5c}$$

Even though H_{pd_1} is set equal to zero in the formal development of the theory, its possible higher order effects are not neglected in the calculation of the continuum widths of the state $|d_1\rangle$ due to virtual transitions to the continuum. Using Eqs. (1), (2), and (5) we obtain the coupled equations

$$(E-H_{pp})p\psi^{(+)}=H_{pR}R\psi^{(+)}\ , \tag{6a}$$

$$(E-H_{RR})R\psi^{(+)}=H_{Rp}p\psi^{(+)}+H_{Rd_1}d_1\psi^{(+)}\ , \tag{6b}$$

$$(E-H_{d_1d_1})d_1\psi^{(+)}=H_{d_1R}R\psi^{(+)}+H_{d_1d_2}d_2\psi^{(+)}\ , \tag{6c}$$

and

$$(E-H_{d_2d_2})d_2\psi^{(+)}=H_{d_2d_1}d_1\psi^{(+)}\ . \tag{6d}$$

The wave functions $q\psi^{(+)}$ are not explicitly treated and have been dropped from Eqs. (6). The usefulness of the coupled equations resulting from the breakup of P [Eq. (2a)] and the subsequent restriction that only H_{pR} can be nonzero [Eq. (5a)] depends on the nature of the R states in the energy region being studied.

If the incident energy is within the width of an elastic continuum resonance, then the reduced mass has a greatly enhanced probability of being near the origin. Physically this means that the nuclei will be close for a longer time than would be the case if a resonant condition were not satisfied in the elastic channel. The likelihood of nuclear interactions will, consequently, be greatly increased. If, in addition, there are distinct states involving nuclear excitations which can readily couple to the incident channel, then the probability of their formation would be greatly enhanced provided their angular momenta and parity are also the same as the resonating partial wave.

This means that the coupling H_{d_1R} will play a dominant role in doorway formation. The set of Eqs. (6) proves to be very helpful since it makes the process more transparent. The set (6) is particularly useful in the energy region above the Coulomb barrier. The reason for this is that the shape resonances in this region tend to be broad (gross) due to their large continuum escape widths. Consequently, just about any energy in this region is close to or within the width of one of these resonances. Also, the broadness of these states increases the likelihood that there will be doorways which can couple to them.

Equation (6d) allows $d_2\psi^{(+)}$ to be determined in terms of $d_1\psi^{(+)}$ as

$$d_2\psi^{(+)}=\frac{H_{d_2d_1}d_1\psi^{(+)}}{E-H_{d_2d_2}}\ . \tag{7}$$

Substitution of Eq. (7) into Eq. (6c) gives

$$d_1\psi^{(+)}=\frac{1}{E-H_{d_1d_1}-H_{d_1d_2}[1/(E-H_{d_2d_2})]H_{d_2d_1}}H_{d_1R}R\psi^{(+)}\,, \tag{8}$$

and placing Eq. (8) in Eq. (6b) yields

$$R\psi^{(+)}=\frac{1}{E-H_{RR}-H_{Rd_1}(1/\{E-H_{d_1d_1}-H_{d_1d_2}[1/(E-H_{d_2d_2})]H_{d_2d_1}\})H_{d_1R}}H_{Rp}p\psi^{(+)}\,. \tag{9}$$

The solutions of Eqs. (9) and (6a) for $R\psi^{(+)}$ and $p\psi^{(+)}$ can be obtained by introducing the non-Hermitian one body optical Hamiltonian h which will be defined as

$$h=H_{PP}+H_{Pd_1}\frac{1}{E-H_{d_1d_1}-H_{d_1d_2}[1-/(E-H_{d_2d_2})]H_{d_2d_1}}H_{d_1P}\,. \tag{10}$$

Using $H_{pd_1}=0$ it can be seen from inspection that

$$h_{pp}=H_{pp}\,, \tag{11a}$$

$$h_{pR}=H_{pR}\,, \tag{11b}$$

$$h_{Rp}=H_{Rp}\,, \tag{11c}$$

and

$$h_{RR}=H_{RR}+H_{Rd_1}\frac{1}{E-H_{d_1d_1}-H_{d_1d_2}[1/(E-H_{d_2d_2})]H_{d_2d_1}}H_{d_1R}\,. \tag{11d}$$

The use of h allows Eqs. (6a) and (9) to be rewritten as

$$(E-h_{pp})p\psi^{(+)}=h_{pR}R\psi^{(+)} \tag{12a}$$

and

$$(E-h_{RR})R\psi^{(+)}=h_{Rp}p\psi^{(+)}\,. \tag{12b}$$

The formal solutions of this set of coupled equations have been obtained by Lev and Beres and are[13]

$$p\psi^{(+)}=\tilde{\psi}^{(+)}+\tilde{G}_p^{(+)}h_{pR}\frac{1}{E-h_{RR}-h_{Rp}\tilde{G}_p^{(+)}h_{pR}}h_{RP}\tilde{\psi}^{(+)} \tag{13a}$$

and

$$R\psi^{(+)}=\frac{1}{E-h_{RR}-h_{Rp}\tilde{G}_p^{(+)}h_{pR}}h_{Rp}\tilde{\psi}^{(+)}\,, \tag{13b}$$

where $\tilde{G}_p^{(+)}$ is the Green's function operator for the Hamiltonian h_{pp}, i.e.,

$$\tilde{G}_p^{(+)}=(E-h_{pp})^{-1}\,. \tag{14}$$

The wave function $\tilde{\psi}^{(+)}$ is a solution of the homogeneous equation

$$(E-h_{pp})\tilde{\psi}^{(+)}=0\,. \tag{15}$$

The solutions for $d_1\psi^{(+)}$, Eq. (8), and $d_2\psi^{(+)}$, Eq. (7), become simplified by defining another non-Hermitian one body optical Hamiltonian h' as

$$h'=H_{dd}+H_{(R+d_1)d_2}\frac{1}{E-H_{d_2d_2}}H_{d_2(R+d_1)}\,, \tag{16}$$

from which it can be seen that

$$h'_{d_1d_2}=H_{d_1d_2}\,, \tag{17a}$$

$$h'_{d_2d_2}=H_{d_2d_2}\,, \tag{17b}$$

and

$$h'_{d_1d_1}=H_{d_1d_1}+H_{d_1d_2}\frac{1}{E-H_{d_2d_2}}H_{d_2d_1}\,. \tag{17c}$$

Thus Eqs. (8) and (7) become

$$d_1\psi^{(+)}=\frac{1}{E-h'_{d_1d_1}}H_{d_1R}R\psi^{(+)} \tag{18}$$

and

$$d_2\psi^{(+)}=\frac{1}{E-h'_{d_2d_2}}h'_{d_2d_1}d_1\psi^{(+)}\,, \tag{19}$$

where $R\psi^{(+)}$ is given by Eq. (13b). A complete normalized set of states $\{\phi_R^i\}$ in $\{R\}$ are introduced with eigenvalues $\{\epsilon_R^i\}$ which satisfy

$$(\epsilon_R^i-h_{RR})\phi_R^i=0\,, \tag{20}$$

where the complex eigenvalues ϵ_R^i are given in terms of their real and imaginary parts as

$$\epsilon_R^i=E_R^i-i\Gamma_R^{ii}/2\,. \tag{21}$$

Near a resonance the elastic portion of the scattering state will overlap strongly with a state in $\{R\}$. This indicates that the radial portions of the states $\{\phi_R^i\}$ should be concentrated in the interaction region since this characterizes a resonance of the scattering state. Thus the part of an R state describing the reduced mass should look very much like a bound state wave function.

The operator R can now be written as

$$R=\sum_i |\phi_R^i\rangle\langle\phi_R^i|\,. \tag{22}$$

Furthermore, the assumption is made that the energy E is close to the energy E_R^i of an isolated single particle resonance. In that case only one term in Eq. (22) contributes, and Eq. (13a) can be rewritten as

$$p\psi^{(+)}=\tilde{\psi}^{(+)}+\frac{\tilde{G}_p^{(+)}h_{pR}\,|\,\phi_R\rangle\langle\phi_R\,|\,h_{Rp}\,|\,\tilde{\psi}^{(+)}\rangle}{E-\langle\phi_R\,|\,h_{RR}\,|\,\phi_R\rangle-\langle\phi_R\,|\,h_{Rp}\tilde{G}_p^{(+)}h_{pR}\,|\,\phi_R\rangle}\ , \tag{23}$$

where the subscript i has been dropped for clarity. The quantities Δ_R, Γ_R, $\Gamma_R^{\downarrow}$, and $\Gamma_R^{\uparrow}$ are introduced as the single particle resonance energy shift, total width, spreading width, and escape to the continuum, respectively, and are defined as

$$E_R=\mathrm{Re}\langle\phi_R\,|\,h_{RR}\,|\,\phi_R\rangle\ , \tag{24a}$$

$$\Delta_R=\mathrm{Re}\langle\phi_R\,|\,h_{Rp}\tilde{G}_p^{(+)}h_{pR}\,|\,\phi_R\rangle\ , \tag{24b}$$

$$\Gamma_R=\Gamma_R^{\downarrow}+\Gamma_R^{\uparrow}\ , \tag{24c}$$

$$\Gamma_R^{\downarrow}=-2\,\mathrm{Im}\langle\phi_R\,|\,h_{RR}\,|\,\phi_R\rangle\ , \tag{24d}$$

and

$$\Gamma_R^{\uparrow}=-2\,\mathrm{Im}\langle\phi_R\,|\,h_{Rp}\tilde{G}_p^{(+)}h_{pR}\,|\,\phi_R\rangle\ . \tag{24e}$$

It is clear that the definitions of E_R and $\Gamma_R^{\downarrow}$ are the same as in Eqs. (20) and (21). Because of h_{RR} [Eq. (11d)], $\Gamma_R^{\downarrow}$ has an energy dependence. The expression for $p\psi^{(+)}$, Eq. (23), can now be written as

$$p\psi^{(+)}=\tilde{\psi}^{(+)}+\frac{\tilde{G}_p^{(+)}h_{pR}\,|\,\phi_R\rangle\langle\phi_R\,|\,h_{Rp}\,|\,\tilde{\psi}^{(+)}\rangle}{E-E_R-\Delta_R+i\Gamma_{R/2}}\ . \tag{25}$$

Similarly using Eqs. (24) $R\psi^{(+)}$, Eq. (13b), becomes

$$R\psi^{(+)}=\frac{|\,\phi_R\rangle\langle\phi_R\,|\,h_{Rp}\,|\,\tilde{\psi}^{(+)}\rangle}{E-E_R-\Delta_R+i\Gamma_R/2}\ , \tag{26}$$

where again the subscript i has been dropped for clarity.

Up to this point little has been said about the doorways themselves. As was done for R, Eq. (20), complete orthonormal sets of states $\{\phi_{d_1}^i\}$ and $\{\phi_{d_2}^i\}$ are introduced with corresponding eigenvalues $\{\epsilon_{d_1}^i\}$ and $\{\epsilon_{d_2}^i\}$, respectively, which satisfy

$$(\epsilon_{d_1}^i-h'_{d_1d_1})\phi_{d_1}^i=0 \tag{27a}$$

and

$$(\epsilon_{d_2}^i-h'_{d_2d_2})\phi_{d_2}^i=0\ , \tag{27b}$$

where the relationship between h and H is expressed in Eqs. (17). The complex eigenvalues are given in terms of their real and imaginary parts as

$$\epsilon_{d_1}^i=E_{d_1}^i-i\Gamma_{d_1}^i/2 \tag{28a}$$

and

$$\epsilon_{d_2}^i=E_{d_2}^i-i\Gamma_{d_2}^i/2\ . \tag{28b}$$

The exact nature of the entrance and exit doorways will depend on the particular model used to represent the interaction of the nuclei. However, they should satisfy the following criteria:

(1) The two doorway spaces should involve nuclear excitations and be orthogonal to each other.

(2) The coupling of d_1 to the entrance channel and d_2 to the exit channel should be direct and calculable. The continuum width of each $\phi_{d_1}^i$ involves virtual transitions to the continuum. By a suitable choice of $\{d_1\}$ and H_{d_1R}, these widths can be calculated using an equation similar to Eq. (24e).

(3) The spatial extension of ϕ_{d_1} should be similar to that of a bound state of the compound system. In other words its usefulness lies in the fact that it helps describe the scattering state in the interaction region.

The operators d_1 and d_2 can be written analogously to Eq. (22) as

$$d_1=\sum_s\,|\,\phi_{d_1}^s\rangle\langle\phi_{d_1}^s\,| \tag{29a}$$

and

$$d_2=\sum_t\,|\,\phi_{d_2}^t\rangle\langle\phi_{d_2}^t\,|\ . \tag{29b}$$

The wave functions $d_1\psi^{(+)}$ and $d_2\psi^{(+)}$, Eqs. (18) and (19), then become, respectively,

$$d_1\psi^{(+)}=\sum_s\frac{|\,\phi_{d_1}^s\rangle\langle\phi_{d_1}^s\,|\,H_{d_1R}\,|\,R\psi^{(+)}\rangle}{E-h'_{d_1d_2}} \tag{30a}$$

and

$$d_2\psi^{(+)}=\sum_t\frac{|\,\phi_{d_2}^t\rangle\langle\phi_{d_2}^t\,|\,h'_{d_2d_1}\,|\,d_1\psi^{(+)}\rangle}{E-h'_{d_2d_2}}\ , \tag{30b}$$

where $R\psi^{(+)}$ is given by Eq. (26).

Letting H_i be the interaction Hamiltonian, the T matrix can now be written using the expressions for $p\psi^{(+)}$, $R\psi^{(+)}$, $d_1\psi^{(+)}$, and $d_2\psi^{(+)}$, Eqs. (25), (26), (30a), and (30b), respectively, to give

$$T=\langle\chi_f\,|\,H_I\,|\,\psi^{(+)}\rangle=T_1+T_2+T_3+T_4+T_5\ , \tag{31}$$

where

$$T_1=\langle\chi_f\,|\,H_I\,|\,\tilde{\psi}^{(+)}\rangle\ , \tag{32a}$$

$$T_2=\frac{\langle\chi_f\,|\,H_I\tilde{G}_p^{(+)}h_{pR}\,|\,\phi_R\rangle\langle\phi_R\,|\,h_{Rp}\,|\,\tilde{\psi}^{(+)}\rangle}{E-E_R-\Delta_R+i\Gamma_R/2}\ , \tag{32b}$$

$$T_3=\frac{\langle\chi_f\,|\,H_I\,|\,\phi_R\rangle\langle\phi_R\,|\,h_{Rp}\,|\,\tilde{\psi}^{(+)}\rangle}{E-E_R-\Delta_R+i\Gamma_R/2}\ , \tag{32c}$$

$$T_4=\frac{\langle\chi_f\,|\,H_I(E-h'_{d_1d_1})^{-1}H_{d_1R}\,|\,\phi_R\rangle\langle\phi_R\,|\,h_{Rp}\,|\,\tilde{\psi}^{(+)}\rangle}{E-E_R-\Delta_R+i\Gamma_R/2}\ , \tag{32d}$$

and

$$T_5=T'_5\frac{\langle\phi_R\,|\,h_{RP}\,|\,\tilde{\psi}^{(+)}\rangle}{E-E_R-\Delta_R+i\Gamma_R/2}\ , \tag{32e}$$

where

$$T_5' = \sum_{s,t} \frac{\langle \chi_f \mid H_I \mid \phi_{d_2}^t \rangle}{E - E_{d_2} + i(\Gamma_{d_2}/2)} \frac{\langle \phi_{d_2}^t \mid h_{d_2 d_1}' \mid \phi_{d_1}^s \rangle}{E - E_{d_1} + i(\Gamma_{d_1}/2)} \langle \phi_{d_1}^s \mid H_{d_{1R}} \mid \phi_R \rangle \ . \tag{32f}$$

The five terms in the T matrix have simple physical interpretations. The term T_1, Eq. (32a), represents direct transfer from the nonresonant part of the single particle continuum wave function to the final state. This term is expected to be a relatively smooth function of energy and is illustrated in Fig. 3. In complicated reactions it would provide the pure DWBA contribution. In this figure and Figs. 4–6 solid connecting lines refer to direct couplings in expressions (32) while dotted lines refer to the implicit couplings appearing in the denominators. The second term T_2, Eq. (32b), describes the situation in which the reduced mass enters a single particle resonance (SPR) which then couples with the many possible continuum levels before scattering to the final state. This is a higher order process that is expected to give a very small contribution to the T matrix. Consequently, its effect will not be considered.

The term T_3, Eq. (32c), is what would be the single particle resonance scattering term in an elastic scattering process. In the first part of the reaction the reduced mass enters a single particle resonance (SPR) state. It is unlikely to provide the main avenue for advancement to final states involving multiparticle breakup. The process is illustrated in Fig. 4. The term T_4, Eq. (32d), represents the case in which the single particle resonance state forms, and subsequently couples to entrance doorways which in turn couple to the final reaction state. This term is illustrated in Fig. 5. The effects of entrance doorways enter explicitly in the numerator of this term, whereas in T_3 they only enter indirectly via the spreading width. Since T_4 is a second order term, it is expected to be small unless the energy is close to an entrance doorway energy. As was seen in our earlier studies of elastic and inelastic scattering, the widths of these doorway states are smaller than the SPR widths and, consequently, T_4 exhibits sharper structure than T_3. Note also that in the figure the effects of q enter implicitly via the spreading of the doorways. However, in the case of complex reactions involving multiparticle breakup it is not clear how an entrance doorway state d_1 could lead to the final state.

We expect in our theory that the final term T_5, Eqs. (32e) and (32f), contains the appropriate coupling to the final state from the entrance doorway d_1 via the processes $d_1 \rightarrow d_2$ and $d_2 \rightarrow f$. The reaction is illustrated in Fig. 6. The IS related to the entrance and exit doorways $|\phi_{d_1}\rangle$ and $|\phi_{d_2}\rangle$ is present in the resonance terms in the denominator of Eq. (32f). The widths can be calculated as in Refs. 1–3, and a particle-vibration model, for example, may be conveniently used to determine the states $|\phi_{d_1}\rangle$. The effective Hamiltonian H, Eq. (4), could then be taken to be of the form[14]

$$H = (-\hbar^2/2m)\nabla^2 + H_1 + H_2 + V + iW + H_{12} \ , \tag{33}$$

where, respectively, H_1 and H_2 represent the internal Hamiltonian of each separate ion, $V+iW$ represents an effective complex potential, and H_{12} couples the relative motion to the vibrational degrees of freedom of each ion. Because H_{pd_1} is assumed to be zero in Eq. (5), W represents the absorption due to the coupling of d to q in the second term of Eq. (4).[1] However, the loss in flux due to virtual transitions from $\{d_1\}$ to the continuum can be included in W by including the higher order effects of not setting H_{pd_1} equal to zero. The approach of Lev and Beres[15] provides a simple procedure for obtaining such an energy dependent local imaginary potential.

In practice, the sets of states $\{\phi_R^i\}$, Eq. (20), and $\{\phi_{d_1}^i\}$, Eq. (27a), are constructed from the scattering state solution at appropriate energies. To see this more clearly consider the procedure for obtaining ϕ_{RJ}, an SPR state of the Jth partial wave. Upon inspecting the set of solutions [(25) and (26)] it can be seen that ϕ_{RJ} should be nearly proportional to $P\psi^{(+)}$ near a resonance. Letting $\psi_J^{(+)}$ represent the Jth partial wave of $P\psi^{(+)}$, the following method can thus be used to produce ϕ_{RJ}.

(1) Find the resonance energies of $\psi_J^{(+)}$. The resonance energies are conventially obtained by scanning the S matrix or phase shift. A more efficient alternate method based on the projection operator formalism is given by Larry and Beres.[16]

(2) Solve the Schrödinger equation for $\psi_J^{(+)}$ at this energy.

(3) Let ϕ_{RJ} be proportional to $\psi_J^{(+)}$ out to some cutoff radius corresponding to the range of the nuclear interaction.

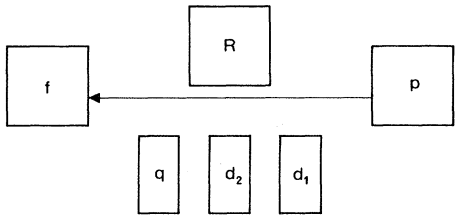

FIG. 3. Direct process, where ← denotes nuclear interaction.

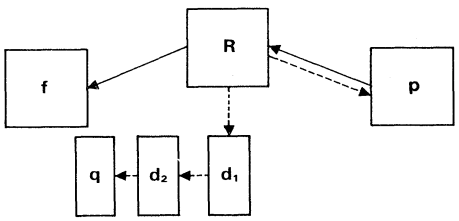

FIG. 4. Gross structure from virtual resonances, where ← denotes nuclear interaction and ←— the width and shift.

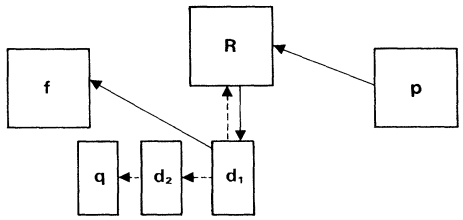

FIG. 5. Intermediate structure from double resonance mechanism, where ← denotes nuclear interaction and ←— the width and shift.

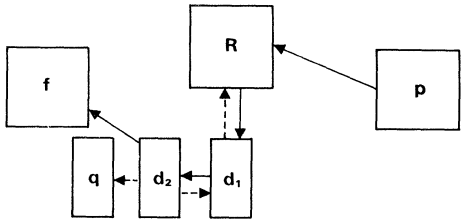

FIG. 6. Intermediate structure from single particle resonances leading sequentially to entrance doorways, exit doorways, and the final state. The notation ← denotes nuclear reaction and ←— indicates the width and shift.

(4) Normalize ϕ_{RJ}.

(5) Evaluate the terms in the denominator of Eqs. (32) using the set of equations (24).

(6) The matrix elements involving $h'_{d_2 d_1}$ and H_I, Eq. (32f), are to be calculated within the model chosen to describe the reaction of interest.

The differential and total cross sections can be obtained from the square of the T matrix with the appropriate kinematic factors.

In the case of $^{12}C + {}^{12}C \rightarrow {}^{20}Ne^* + \alpha$, where

$$|\chi_f\rangle = |{}^{20}Ne^* + \alpha\rangle \ , \tag{34}$$

the 7.68 MeV 0_2^+ level in ^{12}O could be the most important contributor to $|\phi_{d_1}\rangle$. This state is just above the threshold for the alpha decay of ^{12}C into the $\alpha + {}^{20}Ne^*$ channel. In fact, the 0_2^+ excitation is equivalent to the alpha particle model.[17] We could thus assume that the reaction proceeds predominantly through the entrance doorways consisting of reduced mass single particle resonances coupled to the 0_2^+ states, i.e.,

$$|\phi_{d_1}\rangle = |{}^{12}C + {}^{12}C(0_2^+)\rangle \ . \tag{35}$$

The states $|\phi_{d_2}\rangle$ may then be taken as

$$|\phi_{d_2}\rangle = |{}^{12}C + ({}^{8}Be + \alpha)\rangle \ , \tag{36}$$

where one ^{12}C nucleus remains intact and the other ^{12}C nucleus has broken apart into loosely bound states of the $(^8Be + \alpha)$ system. The transition from the entrance doorways $|\phi_{d_1}\rangle$ to the exit doorways $|\phi_{d_2}\rangle$ can take place as an alpha decay of the excited $^{12}C(0_2^+)$ state. This may be assumed to occur in such a fashion that the emitted alpha particle does not leave until the ^8Be nucleus is transferred to the remaining ^{12}C nucleus to form ^{20}Ne. The emitted alpha particle would have an energy which is essentially its share of the total $^{12}C(0_2^+)$ excitation energy. The state $|{}^{12}C + ({}^{8}Be + \alpha)\rangle$, Eq. (36), is an alpha cluster configuration whose energy is shared among more degrees of freedom than the entrance doorway, and it is the preequilibrium component one step more complicated than ϕ_{d_1}. The widths Γ_{d_2} in the denominator of Eq. (32f) can be estimated to be at least an order of magnitude less than the widths Γ_{d_1}. While these preequilibrium states can proceed to a compound nucleus, with varying possible decay modes at each stage, one may assume that there is also the probability that the states ϕ_{d_2} can couple to the reaction continuum. Thus, the molecular and α cluster structures in the case of $^{12}C + {}^{12}C$ will produce branching ratios to the different reaction channels. This is a nonstatistical process and reaction resonances should have a large percentage of intermediate structure. The study of $^{12}C + {}^{12}C \rightarrow {}^{20}Ne^* + \alpha$ represents just one possible reaction channel. The matrix element $\langle \chi_f | H_I | \phi_{d_2} \rangle$ in Eq. (32f) may be extracted for a specified angular momentum from a DWBA code which allows the calculation of multiparticle transfer reaction cross sections taking into account finite range effects.

Cosman *et al.*[6] have provided a comparison of $^{12}C + {}^{12}C$ intermediate structure data in the vicinity of 20 MeV (c.m.) for several reaction channels, including a sum over 22 $^{20}Ne^*$ levels in $^{12}C + {}^{12}C \rightarrow {}^{20}Ne^* + \alpha$ observed in the experiment of Greenwood *et al.*[9] Our previous calculation[3] shows structure for $^{12}C + {}^{12}C \rightarrow {}^{12}C(0_2^+) + {}^{12}C$ (g.s.) in the same energy range. The T matrix representing the sequential development $p \rightarrow R \rightarrow d_1$ in Eqs. (32e) and (32f) and Fig. 6 is exactly the matrix element used in producing the inelastic cross section for 0_2^+. This provides an impetus for the calculation of the $d_1 \rightarrow d_2 \rightarrow f$ steps as indicated by Eq. (32f) and Fig. 6. It is reasonable that the 0_2^+ related intermediate structure will be carried through to the final channel and provide a correlation between the $^{20}Ne^* + \alpha$ cross section and the cross sections for other $^{12}C + {}^{12}C$ reactions. Extensive calculations are now in progress and the results will be published at a later time.

Finally we point out that while we agree with Adhikari[5] that the exit and entrance doorways do not have to be the same, *our exit doorways are related to the entrance channel,* unlike those of Ref. 5. In fact, as is stressed in our paper, the entrance doorways in our model introduce structure which is transferred sequentially via the reaction doorways to the reaction channel of interest. Reference 5, on the other hand, expects intermediate structure to arise from the coupling of exit channels to exit doorways that have no inherent relationship to the entrance doorways.

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